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WEAK ISOMORPHISM CONCEPT ON BIPOLAR NEIGHBOURHOOD FUZZY GRAPH

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ABSTRACT

In this Paper for the given fuzzy graph $G_B: (\sigma_B, \mu_B)$, Bipolar Neighbourhood fuzzy graph G_{Bn}: (σ_{Bn},μ_{Bn}) of G_B is defined. Some of its properties under the specific nature of G is discussed.

Keywords: *Neighbourhood fuzzy graph, Bipolar neighbourhood fuzzy graph, Strong bipolar fuzzy graph, Weak isomorphism of bipolar neighbourhood fuzzy graph.*

1. INTRODUCTION

In 1965, Zadeh [1] introduced the notion of a fuzzy subset of a set. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, expert systems, decision making and automata theory. In 1994, Zhang [2] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. A bipolar fuzzy set is an extension of Zadeh's fuzzy set theory whose membership degree range in [-1, 1]. In this paper bipolar neighbourhood fuzzy graph of G_B is defined. Some results related bipolar neighbourhood fuzzy graph is discussed.

2. PRELIMINARIES

In this section, we introduce some basic definitions that are required in the sequel.

Definition 2.1: A fuzzy graph is a pair $G: (\sigma, \mu)$ with the underlying set V. The pair $G_n: (\sigma_n, \mu_n)$is the same underlying set v, it is defined as follows: Two nodes are made as neighbors in G_n iff they have a common neighbours in G, where $\sigma_n(x) = \sigma(x)$ for all x in V.If x and y are made as neighbours in G_n,

$$\mu_n(x,y) = \mu(x,y) \quad if\ (x,y) \in \mu^*$$
$$= \mu^2(x,y)\ if\ (x,y) \notin \mu^*,$$
else $\mu_n(u,v) = 0$

From the above definition of μ_n,

$$\mu_n(x,y) \leq \sigma(x) \wedge \sigma(y)\ for\ all\ (x,y) \in \mu_n^*,$$
$$= \sigma_n(x) \wedge \sigma_n(y) for\ all\ (x,y) \in \mu_n^*.$$

Hence μ_n is a fuzzy relation on the fuzzy set σ_n and the pair $G_n: (\sigma_n, \mu_n)$ is a fuzzy graph, termed as neighbourhood fuzzy graph of G.

Definition 2.2: A bipolar fuzzy graph is a pair $G_B: (\sigma_B, \mu_B)$ with the underlying set V. The pair $G_{Bn}: (\sigma_{Bn}, \mu_{Bn})$ is the same underlying set V, it is defined as follows: Two nodes are made as neighbours in G_{Bn} iff they have a common neighbour in G_B, where $\sigma_{Bn}(x) = \sigma_B(x)$ for all x in V. If x and y are made as neighbours in G_{Bn},

$$\mu_{Bn}^P(x,y) = \mu_B^P(x,y) \qquad if\ (x,y) \epsilon\ \mu_B^*$$
$$= \mu_B^{2^P}(x,y) \qquad if\ (x,y) \notin \mu_B^*$$
$$\mu_{Bn}^N(x,y) = \mu_B^N(x,y) \qquad if\ (x,y) \epsilon\ \mu_B^*$$
$$= \mu_B^{2^N}(x,y) \qquad if\ (x,y) \notin \mu_B^*$$

else
$$\mu_{Bn}^P(x,y) = 0$$
$$\mu_{Bn}^N(x,y) = 0$$

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From the above definition of μ_{Bn}

$$\mu_{Bn}^{P}(x,y) \le \sigma_{B}^{P}(x) \wedge \sigma_{B}^{P}(y) \quad for\ all\ (x,y) \in \mu_{Bn}^{*}$$
$$\le \sigma_{Bn}^{P}(x) \wedge \sigma_{Bn}^{P}(y)\ for\ all\ (x,y) \in \mu_{Bn}^{*}$$

$$\mu_{Bn}^{N}(x,y) \le \sigma_{B}^{N}(x) \vee \sigma_{B}^{N}(y) \quad for\ all\ (x,y) \in \mu_{Bn}^{*}$$
$$= \sigma_{Bn}^{N}(x) \vee \sigma_{Bn}^{N}(y)\ for\ all\ (x,y) \in \mu_{Bn}^{*}$$

Hence μ_{Bn} is a bipolar fuzzy relation on the bipolar fuzzy set σ_{Bn} and the pair $G_{Bn}: (\sigma_{Bn}, \mu_{Bn})$ is a bipolar fuzzy graph, termed as bipolar neighbourhood fuzzy graph of G_B.

Definition 2.3: Let G_1 and G_2 be the bipolar fuzzy graphs. A homomorphism f from G_1 to G_2 is a mapping $f: V_1 \to V_2$ which satisfies the following conditions.

a) $\mu_{A_1}^{P}(x_1) \le \mu_{A_2}^{P}(f(x_1),\ \mu_{A_1}^{N}(x_1) \ge \mu_{A_2}^{N}(f(x_1))for\ all\ x_1 \in V_1$
b) $\mu_{B_1}^{P}(x_1, y_1) \le \mu_{B_2}^{P}(f(x_1)f(y_1)),\ \mu_{B_1}^{N}(x_1, y_1)) \ge \mu_{A_2}^{N}(f(x_1)f(y_1))for\ all\ x_1y_1 \in E_1$

Definition 2.4: Let G_1 and G_2 be bipolar fuzzy graphs. An isomorphism f from G_1 and G_2 is a bijective mapping $f: V_1 \to V_2$ which satisfies the following conditions.

a) $\mu_{A_1}^{P}(x_1) = \mu_{A_2}^{P}(f(x_1)),\ \mu_{A_1}^{N}(x_1) = \mu_{A_2}^{N}(f(x_1))$ for all $x_1 \in V_1$
b) $\mu_{B_1}^{P}(x_1y_1) = \mu_{B_2}^{P}(f(x_1)f(y_1)),\ \mu_{B_1}^{N}(x_1y_1) = \mu_{B_2}^{N}(f(x_1)f(y_1))$ for all $x_1y_1 \in E_1$

We denote $G_1 \cong G_2$ if there is an isomorphism from G_1 to G_2.

Definition 2.5: Let G_1 and G_2 be the bipolar fuzzy graphs. Then a weak isomorphism f from G_1 to G_2 is a bijective mapping $f: V_1 \to V_2$ which satisfies the following conditions.

a) f is homomorphism.
b) $\mu_{A_1}^{P}(x_1) = \mu_{A_2}^{P}(f(x_1),\ \mu_{A_1}^{N} = \mu_{A_2}^{N}(f(x_1))\ for\ all\ x_1 \in V_1$

Definition 2.6: A bipolar fuzzy graph G = (A,B) is called strong if $\mu_{B}^{P}(xy) = \min(\mu_{A}^{P}(x), \mu_{A}^{P}(y))$ and $\mu_{B}^{N}(xy) = \max(\mu_{A}^{N}(x), \mu_{A}^{N}(y))$ for all xy ∈ E

3. PROPERTIES OF BIPOLAR NEIGHBOURHOOD FUZZY GRAPH

Theorem 3.1: A Strong bipolar fuzzy graph $G_B: (\sigma_B, \mu_B)$ need not have its bipolar neighbourhood fuzzy graph G_{Bn} to be strong bipolar fuzzy graph.

Proof: Consider G_{Bn} of the strong bipolar fuzzy graph $G_B: (\sigma_B, \mu_B)$. $Let\ (x,y) \in \mu_{Bn}^{*}$

i.e. The nodes 'x' and 'y' have a common neighbour in G_B.

Case-(i): $If\ (x,y) \in \mu_{B}^{*}\ then\ \mu_{Bn}^{P}(x,y) = \mu_{B}^{P}(x,y) = \sigma_{B}^{P}(x) \wedge \sigma_{B}^{P}(y)$ $and\ \mu_{Bn}^{N}(x,y) = \mu_{B}^{N}(x,y) = \sigma_{B}^{N}(x)\ \sigma_{B}^{N}(y)$ (as G_B is a strong bipolar fuzzy graph).Hence by the definition of G_{Bn}, $\mu_{Bn}^{P}(x,y) = \sigma_{Bn}^{P}(x) \wedge \sigma_{Bn}^{P}(y) forall\ (x,y)\ \mu_{B}^{*}\ and\ \ \mu_{Bn}^{N}(x,y) = \sigma_{Bn}^{N}(x) \vee \sigma_{Bn}^{N}(y)$ for all (x, y) ∈ μ_{B}^{*}.

Case-(ii): If $(x,y) \in \mu_{Bn}^{*}$ but $(x,y) \notin \mu_{B}^{*}$, then $\mu_{Bn}^{P}(x, y) = \mu_{B}^{2^{P}}(x, y)$ = Strength of the strongest path of length 2 between x and y in G_B. $\mu_{Bn}^{N}(x, y) = \mu_{B}^{2^{N}}(x, y)$ = Strength of the strongest path of length 2 between x and y in G_B.

ie) $\mu_{Bn}^{P}(x,y) = \sup\{\mu_{B}^{P}(x, y_i) \wedge \mu_{B}^{P}(y_i, y)/y_i \in Y, y_i$ are common neighbours of x & y in $G_B\}$
$\mu_{Bn}^{N}(x,y) = \inf\{\mu_{B}^{N}(x, y_i) \vee \mu_{B}^{N}(y_i, y)/y_i \in Y, y_i$ are common neighbours of x & y in $G_B\}$

Since G_B is strong

$$\begin{cases} \mu_{Bn}^{P}(x,y) = \text{Sup}\{\sigma_{B}^{P}(x) \wedge \sigma_{B}^{P}(y_i) \wedge \sigma_{B}^{P}(y)\ /y_i \in Y, \text{ are common neighbours of x and y in } G_B\} \\ \mu_{Bn}^{N}(x,y) = \inf\{\sigma_{B}^{N}(x) \vee \sigma_{B}^{N}(y_i) \vee \sigma_{B}^{N}(y)\ /y_i \in Y, \text{ are common neighbours of x and y in } G_B\} \end{cases} \to (1)$$

Case-(ii) (a): If there is at least one common neighbor y_i(say) of x and y, whose weight is greater than $\sigma_{Bn}^{P}(x)\sigma_{Bn}^{P}(y)$ and less than $\sigma_{Bn}^{N}(x)\sigma_{Bn}^{N}(y)$ then the path connecting x and y through y_i is of strength $\sigma_{Bn}^{P}(x)\sigma_{Bn}^{P}(y), \sigma_{Bn}^{N}(x)\sigma_{Bn}^{N}(y)$. All the other paths of length 2 between x and y have their strength $\le (\sigma_{Bn}^{P}(x)\sigma_{Bn}^{P}(y))$ and their strength $\ge (\sigma_{Bn}^{N}(x)\sigma_{Bn}^{N}(y))$.

Using (1) $\mu_{Bn}^{P}(x,y) = \sigma_{Bn}^{P}(x) \wedge \sigma_{Bn}^{P}(y)$
$\mu_{Bn}^{N}(x,y) = \sigma_{Bn}^{N}(x) \vee \sigma_{Bn}^{N}(y)$

Case- (ii) (b): If every common neighbor of x and y have their weights less than $\sigma_{Bn}^{P}(x) \wedge \sigma_{Bn}^{P}(y)$ and their weights less than $\sigma_{Bn}^{N}(x) \vee \sigma_{Bn}^{N}(y)$, then by using (1),
$\mu_{Bn}^{P}(x,y) = \sup\{\sigma_B(y_i) / y_i \in Y, y_i$ are common neighbours of x & y in $G_B\}$
$\mu_{Bn}^{N}(x,y) = \inf\{\sigma_B(y_i) / y_i \in Y, y_i$ are common neighbours of x & y in $G_B\}$.Hence

$$\mu_{Bn}^{P}(x,y) \le \sigma_{Bn}^{P}(x)\sigma_{Bn}^{P}(y) \text{ and}$$
$$\mu_{Bn}^{N}(x,y) \ge \sigma_{Bn}^{N}(x)\sigma_{Bn}^{N}(y).$$

implies G_{Bn} is not a strong bipolar fuzzy graphs.

Theorem 3.2: If $G_B: (\sigma_B, \mu_B)$ is a bipolar fuzzy graph with its underlying graph being complete then G_{Bn} is same as G_B.

Proof: As the underlying bipolar fuzzy graph$G_B^*: (\sigma_B^*, \mu_B^*)$ being complete every pair of nodes in G_B^* have a common neighbour in G_B.So every pair of nodes are made as neighbours in G_{Bn} and by definition

$$\mu_{Bn}^{P}(x,y) = \mu_B^{P}(x,y)$$
$$\mu_{Bn}^{P}(x,y) = \mu_B^{P}(x,y)$$

Hence G_{Bn} is same as G_B.

Theorem 3.3: If $G_B: (\sigma_B, \mu_B)$ is a bipolar fuzzy graph with $\mu_B^{P}(x,y) = c$ (a constant) for all (x,y) in μ_B^* and $\mu_B^{N}(x,y) = c$ (a constant) for all (x,y) in μ_B^* then G_{Bn} is also a bipolar fuzzy graph with all its edge weight as c.

Proof: Let $(x,y) \in \mu_{Bn}^*$ then x and y are made as neighbours in G_{Bn}.

Case-(i): If $(x,y) \in \mu_B^*$ then

$$\mu_{Bn}^{P}(x,y) = \mu_B^{P}(x,y) = c$$
$$\mu_{Bn}^{N}(x,y) = \mu_B^{N}(x,y) = c$$

Case-(ii): If $(x,y) \notin \mu_B^*$ then

$$\mu_{Bn}^{P}(x,y) = \mu_B^{2^P}(x,y) = c$$
$$\mu_{Bn}^{N}(x,y) = \mu_B^{2^N}(x,y) = c$$

Since every arc in the path connecting x and y is of constant weight 'c'. In both cases G_{Bn} is a bipolar fuzzy graph with all its edges of constant weight 'c'.

Theorem 3.4: If $G_B: (\sigma_B, \mu_B)$ is a bipolar fuzzy graph such that every edge in G_B lies in a cycle of length 3, then G_B is weak isomorphic with G_{Bn} by the identity map from $G_B$to G_{Bn}.

Proof: Considering a cycle of length 3, every pair of nodes on that cycle have the third node as their common neighbour, hence every cycle of length 3 is also in $G_{Bn,}$with the edge weight as in G_B. But two nodes which do not lie on the same cycle of length 3,but having a common neighbour in G_B are made as neighbours in G_{Bn} with

$$\mu_{Bn}^{P}(x,y) = \mu_B^{2^P}(x,y) \ as\ (x,y) \notin \mu_B^*$$
$$\mu_{Bn}^{N}(x,y) = \mu_N^{2^P}(x,y) \ as\ (x,y) \notin \mu_B^*$$

Consider the identity map $h: G \to G_{Bn}$ Then,

$$\sigma_B^{P}(x) = \sigma_{Bn}^{P}(h(x)) for\ all\ x\ in\ V.$$
$$\sigma_B^{N}(x) = \sigma_{Bn}^{N}(h(x)) for\ all\ x\ in\ V. \quad (1)$$

For all edges (x, y) on the cycles of length 3

$$\begin{cases} \mu_B^{P}(x,y) = \mu_{Bn}^{P}(x,y) = \mu_{Bn}^{P}(h(x),h(y))\ if\ (x,y) \in \mu_B^* \\ \mu_B^{N}(x,y) = \mu_{Bn}^{N}(x,y) = \mu_{Bn}^{N}(h(x),h(y))\ if\ (x,y) \in \mu_B^* \end{cases} \quad (2)$$

$$\begin{cases} \mu_B^{P}(x,y) = 0 \le \mu_B^{2^P}(x,y) = \mu_{Bn}^{P}(h(x),h(y))\ if\ (x,y) \notin \mu_B^* \\ \mu_B^{N}(x,y) = 0 \ge \mu_B^{2^N}(x,y) = \mu_{Bn}^{N}(h(x),h(y))\ if\ (x,y) \notin \mu_B^* \end{cases} \quad (3)$$

From (1), (2) & (3) h is a weak isomorphism from G to G_{Bn}.

CONCLUSION

In this paper new concept bipolar neighbourhood fuzzy graph is introduced some of its properties are discussed.

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