International Journal of Mathematical Archive-9(5), 2018, 114-118
Available online through www.ijma.info ISSN 2229 – 5046

ON THE SOLUTION OF SIMULTANEOUS LINEAR FUZZY EQUATIONS

R. K. DAS[1], S. N. ADHIKARY[2] AND SANJEEV KUMAR*[3]

[1]P. G. Department of Mathematics, S.K.M. University, Dumka, Jharkhand, India.

[2]Head, Department of Mathematics, S. P. College, Dumka, Jharkhand, India.

[3]Research Scholar, S.K.M. University, Dumka, Jharkhand, India.

(Received On: 17-03-18; Revised & Accepted On: 20-04-18)

ABSTRACT

G. J. Klir and Bo Yuan [2] established a process of solving fuzzy equations of the form $\tilde{A} + X = \tilde{B}$, *and* $\tilde{A} \cdot X = \tilde{B}$, *where* $\tilde{A}$ *and* $\tilde{B}$ *are fuzzy numbers in the form of closed intervals. On the basis of the procedure developed by them, the paper is addressed to the introduction of the solution of simultaneous linear fuzzy equations.*

Keywords: *α-cut. Fuzzy numbers. Fuzzy equations. arithmetic operations.*

1. INTRODUCTION

Fuzzy numbers are close to a given real number or around a given interval of real numbers. To qualify as a fuzzy number, a fuzzy set $\tilde{A}$ on R must be a normal fuzzy set, its α-cut $\tilde{A}\alpha$ must be a closed interval for every $\alpha \in [0,1]$ and the support set $O_{+\tilde{A}}$ must be bounded.

Each fuzzy number is uniquely determined by its α-cut. So arithmetic operations +, - , · and ÷ of fuzzy numbers mean the arithmetic operations on closed intervals.

We define
$$[a, b] + [d, e] = [a + d, b + e]$$
$$[a, b] - [d, e] = [a - e, b - d]$$
$$[a, b] \cdot [d, e] = [\min (ad, ae, bd, be), \max (ad, ae, bd, be)]$$
$$[a, b] \div [d, e] = [a, b] \times [\frac{1}{d}, \frac{1}{e}]$$
$$= [\min\left(\frac{a}{d},\frac{a}{e},\frac{b}{d},\frac{b}{e}\right), \max\left(\frac{a}{d},\frac{a}{e},\frac{b}{d},\frac{b}{e}\right)]$$
Provided $o \notin [d, e]$ [2].

2. SOLUTION OF $\tilde{A} + X = \tilde{B}$; $\tilde{A} \cdot X = \tilde{B}$

In these fuzzy equations, $X = \tilde{B} - \tilde{A}$ and $X = \frac{\tilde{B}}{\tilde{A}}$ are not solutions like ordinary algebra, so α-cut have been introduced like $\tilde{A}\alpha$, $\tilde{B}\alpha$, $\tilde{X}\alpha$ of $\tilde{A}$, $\tilde{B}$ and X, where $X = [x_1, x_2]$ is a closed interval. Then solutions have been found by taking the interval equations $\tilde{A}\alpha + X\alpha = \tilde{B}\alpha$ and $\tilde{A}\alpha \cdot X\alpha = \tilde{B}\alpha$, where $x_1 \leq x_2$

When the solutions exist and the nested property $\alpha \leq \beta \Rightarrow X\beta = X\alpha$ hold, Then $X = \bigcup_{\alpha\in[0,1]} \alpha \, . \, X\alpha$ is the solution [2]

In this paper, we propose to develop the solution of simultaneous linear fuzzy equations.

Corresponding Author: Sanjeev Kumar[3]*
[3]Research Scholar, S.K.M. University, Dumka, Jharkhand, India.

3. SOLUTION OF SIMULTANEOUS LINEAR FUZZY EQUATIONS

Let
$$\left.\begin{array}{l}\tilde{A}X + \tilde{B}Y = \tilde{C}\\ \tilde{D}X + \tilde{E}Y = \tilde{F}\end{array}\right\} \quad (1)$$

be simultaneous linear fuzzy equations where $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}$ are fuzzy numbers in the form of closed intervals and their possible solutions X and Y will also be fuzzy numbers in the form of closed intervals $X = [x_1, x_2]$, $Y = [y_1, y_2]$.

It is to be noted that the equations (1) do not have any solution like ordinary algebraic simultaneous equations

$ax + by = c$

and $dx + ey = f$

having $\left(\frac{ce-bf}{ae-bd}, \frac{cd-af}{bd-ae}\right)$ as their solution because if it is so, let $\tilde{A} = [a_1, a_2]$, $\tilde{B} = [b_1, b_2]$, $\tilde{C} = [c_1, c_2]$, $\tilde{D} = [d_1, d_2]$, $\tilde{E} = [e_1, e_2]$, $\tilde{F} = [f_1, f_2]$

Then, $\tilde{A}X + \tilde{B}Y = \tilde{C}$ implies

$[a_1, a_2] \cdot [x_1, x_2] + [b_1, b_2] \cdot [y_1, y_2] = [c_1, c_2]$

or $[\min (a_1x_1, a_1x_2, a_2x_1, a_2x_2), \max (a_1x_1, a_1x_2, a_2x_1, a_2x_2)] + [\min (b_1y_1, b_1y_2, b_2y_1, b_2y_2), \max (b_1y_1, b_1y_2, b_2y_1, b_2y_2)] = [c_1,c_2]$

or $[a_1x_1, a_2x_2] + [b_1y_1, b_2y_2] = [c_1, c_2]$

because $a_1 \leq a_2$, $x_1 \leq x_2 \Rightarrow a_1x_1 \leq a_1x_2 \leq a_2x_1 \leq a_2x_2$

and similarly $b_1y_1 + b_1y_2 \leq b_2y_1 \leq b_2y_2$

or $[a_1x_1 + b_1y_1, a_2x_2 + b_2y_2] = [c_1, c_2]$

$\Rightarrow$ $a_1x_1 + b_1y_1 = c_1$ ------------------ (i)

$a_2x_2 + b_2y_2 = c_2$ ------------------ (ii)

Like wise $\tilde{D}X + \tilde{E}Y = \tilde{F}$ implies

$d_1x_1 + e_1y_1 = f_1$ ----------------- (iii)

$d_2x_2 + e_2y_2 = f_2$ ----------------- (iv)

Converting equations (1) to four simultaneous ordinary algebraic linear equations (i), (ii), (iii) and (iv) instead of two.

So we introduce the α-cut

$\tilde{A}\alpha = [a_{1\alpha}, a_{2\alpha}]$, $\tilde{B}\alpha = [b_{1\alpha}, b_{2\alpha}]$, $\tilde{C}\alpha = [c_{1\alpha}, c_{2\alpha}]$, $\tilde{D}\alpha = [d_{1\alpha}, d_{2\alpha}]$, $\tilde{E}\alpha = [e_{1\alpha}, e_{2\alpha}]$, $\tilde{F}\alpha = [f_{1\alpha}, f_{2\alpha}]$, $X\alpha = [x_{1\alpha}, x_{2\alpha}]$, $Y\alpha = [y_{1\alpha}, y_{2\alpha}]$ of the fuzzy numbers.

Solving (i) and (iii) and that of (ii) and (iv) we have the values

$$x_1 = \frac{c_1e_1 - b_1f_1}{a_1e_1 - b_1d_1}, \quad y_1 = \frac{c_1d_1 - a_1f_1}{b_1d_1 - a_1e_1}$$
$$x_2 = \frac{c_2e_2 - b_2f_2}{a_2e_2 - b_2d_2}, \quad y_2 = \frac{c_2d_2 - a_2f_2}{b_2d_2 - a_2e_2}$$

$\therefore$ $x_1 \leq x_2$, $y_1 \leq y_2$

So, $$\frac{c_1e_1 - b_1f_1}{a_1e_1 - b_1d_1} \leq \frac{c_2e_2 - b_2f_2}{a_2e_2 - b_2d_2} \text{ -}$$

and $$\frac{c_1d_1 - a_1f_1}{b_1d_1 - a_1e_1} \leq \frac{c_2d_2 - a_2f_2}{b_2d_2 - a_2e_2}$$

If we take the interval equations

$$X\alpha = \frac{\tilde{C}\alpha \cdot \tilde{E}\alpha - \tilde{B}\alpha \cdot \tilde{F}\alpha}{\tilde{A}\alpha \cdot \tilde{E}\alpha - \tilde{B}\alpha \cdot \tilde{D}\alpha} \quad (2)$$

$$Y\alpha = \frac{\tilde{C}\alpha \cdot \tilde{D}\alpha - \tilde{A}\alpha \cdot \tilde{F}\alpha}{\tilde{B}\alpha \cdot \tilde{D}\alpha - \tilde{A}\alpha \cdot \tilde{E}\alpha} \quad (3)$$

Then

I $\frac{c_{1\alpha}e_{1\alpha} - b_{1\alpha}f_{1\alpha}}{a_{1\alpha}e_{1\alpha} - b_{1\alpha}d_{1\alpha}} \le \frac{c_{2\alpha}e_{2\alpha} - b_{2\alpha}f_{2\alpha}}{a_{2\alpha}e_{2\alpha} - b_{2\alpha}d_{2\alpha}}$ and $\frac{c_{1\alpha}d_{1\alpha} - a_{1\alpha}f_{1\alpha}}{b_{1\alpha}d_{1\alpha} - a_{1\alpha}e_{1\alpha}} \le \frac{c_{2\alpha}d_{2\alpha} - a_{2\alpha}f_{2\alpha}}{b_{2\alpha}d_{2\alpha} - a_{2\alpha}e_{2\alpha}}$

are the required conditions for the solutions of (1).

II $\frac{c_{1\alpha}e_{1\alpha} - b_{1\alpha}f_{1\alpha}}{a_{1\alpha}e_{1\alpha} - b_{1\alpha}d_{1\alpha}} \le \frac{c_{1\beta}e_{1\beta} - b_{1\beta}f_{1\beta}}{a_{1\beta}e_{1\beta} - b_{1\beta}d_{1\beta}} \le \frac{c_{2\beta}e_{2\beta} - b_{2\beta}f_{2\beta}}{a_{2\beta}e_{2\beta} - b_{2\beta}d_{2\beta}} \le \frac{c_{2\alpha}e_{2\alpha} - b_{2\alpha}f_{2\alpha}}{a_{2\alpha}e_{2\alpha} - b_{2\alpha}d_{2\alpha}}$

and $\frac{c_{1\alpha}d_{1\alpha} - a_{1\alpha}f_{1\alpha}}{b_{1\alpha}d_{1\alpha} - a_{1\alpha}e_{1\alpha}} \le \frac{c_{1\beta}d_{1\beta} - a_{1\beta}f_{1\beta}}{b_{1\beta}d_{1\beta} - a_{1\beta}e_{1\beta}} \le$ ....

where ever $\alpha \le \beta$

If the solution exist I is satisfied and if the next property II also holds, we say that

$X = \bigcup_{\alpha \in [0,1]} \alpha \cdot X\alpha$, $Y = \bigcup_{\alpha \in [0,1]} \alpha \cdot Y\alpha$ is the solution of (1).

Example 3.1: Let $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}$ be six fuzzy numbers given by

$$\tilde{A}(x) = \begin{cases} 0 & for\ x \le 0, x \rhd 2 \\ x & for\ 0 \lhd x \le 1 \\ 2-x & for\ 1 \lhd x \le 2 \end{cases}$$

$$\tilde{B}(x) = \begin{cases} 0 & for\ x \le 1, x \rhd 3 \\ x-1 & for\ 1 \lhd x \le 2 \\ 3-x & for\ 2 \lhd x \le 3 \end{cases}$$

$$\tilde{C}(x) = \begin{cases} 0 & for\ x \le 2, x \rhd 4 \\ x-2 & for\ 2 \lhd x \le 3 \\ 4-x & for\ 3 \lhd x \le 4 \end{cases}$$

$$\tilde{D}(x) = \begin{cases} 0 & for\ x \le 3, x \rhd 5 \\ x-3 & for\ 3 \lhd x \le 4 \\ 5-x & for\ 4 \lhd x \le 5 \end{cases}$$

$$\tilde{E}(x) = \begin{cases} 0 & for\ x \le 4, x \rhd 6 \\ x-4 & for\ 4 \lhd x \le 5 \\ 6-x & for\ 5 \lhd x \le 6 \end{cases}$$

$$\tilde{F}(x) = \begin{cases} 0 & for\ x \le 5, x \rhd 7 \\ x-5 & for\ 5 \lhd x \le 6 \\ 7-x & for\ 6 \lhd x \le 7 \end{cases}$$

We intend to solve $\begin{matrix} \tilde{A}X + \tilde{B}Y = \tilde{C} \\ \tilde{D}X + \tilde{E}Y = \tilde{F} \end{matrix}$

Solution: The **α**-cuts of the given fuzzy numbers are

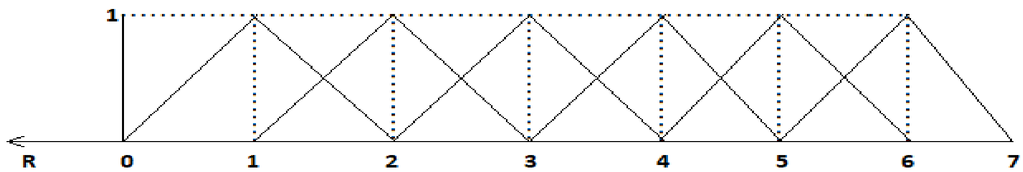

Figure-(a)

$\tilde{A}\alpha = [\alpha, 2 - \alpha]$
$\tilde{B}\alpha = [\alpha + 1, 3 - \alpha]$
$\tilde{C}\alpha = [\alpha + 2, 4 - \alpha]$
$\tilde{D}\alpha = [\alpha + 3, 5 - \alpha]$
$\tilde{E}\alpha = [\alpha + 4, 6 - \alpha]$
$\tilde{F}\alpha = [\alpha + 5, 7 - \alpha]$

Now, $\tilde{C}\alpha \cdot \tilde{E}\alpha - \tilde{B} \cdot \tilde{F}$

$= [\alpha + 2, 4 - \alpha] \cdot [\alpha + 4, 6 - \alpha] - [\alpha + 1, 3 - \alpha] \cdot [\alpha + 5, 7 - \alpha]$

$= [6\alpha - 13, -16\alpha + 19]$

and $\tilde{A}\alpha \cdot \tilde{E}\alpha - \tilde{B}\alpha \cdot \tilde{D}\alpha = [12\alpha - 15, -12\alpha + 9]$

by arithmetic operations '·' and '-'

∴ By (2),

$$X\alpha = \frac{[16\alpha - 13,\ -16\alpha + 19]}{[12\alpha - 15,\ -12\alpha + 9]}$$

$$= \left[16\alpha - 13,\ -16 + 19\right] \cdot \left[\frac{1}{12\alpha - 15},\ \frac{1}{-12\alpha + 9}\right]$$

$$= \left[\min\left\{\frac{16\alpha - 13}{12\alpha - 15}, \frac{16\alpha - 13}{-12\alpha + 9}, \frac{-16\alpha + 19}{12\alpha - 15}, \frac{-16\alpha + 19}{-12\alpha + 9}\right\}, \max\left\{\frac{16\alpha - 13}{12\alpha - 15}, \frac{16\alpha - 13}{-12\alpha + 9}, \frac{-16\alpha + 19}{12\alpha - 15}, \frac{-16\alpha + 19}{-12\alpha + 9}\right\}\right]$$

$$= \left[\frac{16\alpha - 13}{-12\alpha + 9}, \frac{-16\alpha + 19}{-12\alpha + 9}\right] \text{, by arithmetic operation } \div$$

The range of which is $\left[\frac{-13}{9}, \frac{19}{9}\right]$, ($\alpha = 0$)

$$\therefore X(x) = \begin{cases} 0 & \text{for } x \le \frac{-13}{9}, x \rhd \frac{19}{9} \\ \frac{13 + 9x}{16 + 12x} & \text{for } \frac{-13}{9} \lhd x \le -1 \\ \frac{9x - 19}{-16 + 12x} & \text{for } -1 \lhd x \le \frac{19}{9} \end{cases}$$

∴ X = $\bigcup_{\alpha \in [0,1]} \alpha \cdot X\alpha$ is triangular shaped fuzzy number given by

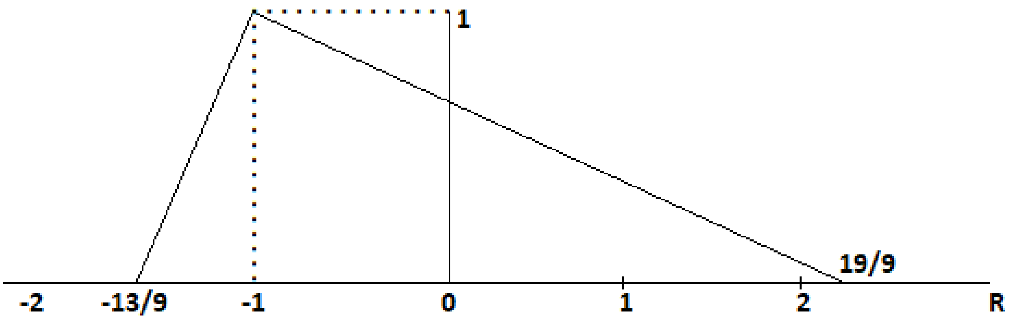

Figure-(b)

Again,

$\tilde{C}\alpha \cdot \tilde{D}\alpha - \tilde{A}\alpha \cdot \tilde{F}\alpha = [14\alpha - 8, -14\alpha + 20]$

$\tilde{B}\alpha \cdot \tilde{D}\alpha - \tilde{A}\alpha \cdot \tilde{E}\alpha = [12\alpha - 9, -12\alpha + 15]$

as previous process

∴ By (3),

$$Y\alpha = \frac{[14\alpha - 8,\ -14\alpha + 20]}{[12\alpha - 9,\ -12\alpha + 15]}$$

$$= [14\alpha - 8,\ -14\alpha + 20]\left[\frac{1}{12\alpha - 9}, \frac{1}{-12\alpha + 15}\right]$$

$$= \left[\min\left\{\frac{14\alpha - 8}{12\alpha - 9}, \frac{14\alpha - 8}{-12\alpha + 15}, \frac{-14\alpha + 20}{12\alpha - 9}, \frac{-14\alpha + 20}{-12\alpha + 15}\right\}, \max\left\{\frac{14\alpha - 8}{12\alpha - 9}, \frac{14\alpha - 8}{-12\alpha + 15}, \frac{-14\alpha + 20}{12\alpha - 9}, \frac{-14\alpha + 20}{-12\alpha + 15}\right\}\right]$$

$$= \left[\frac{-14\alpha + 20}{12\alpha - 9}, \frac{-14\alpha + 20}{-12\alpha + 15}\right]$$

When α = 0, $\frac{-20}{9}$ to $\frac{4}{3}$ is the range. But since $\left[\frac{-20}{9},\frac{4}{3}\right]$ is contained in $\left[\frac{-20}{9},\frac{20}{9}\right]$. we may take the later as the range [5]

Thus

$$Y(x) = \begin{cases} 0 & \text{for } x \le \frac{-20}{9}, x \rhd \frac{20}{9} \\ \frac{20+9x}{14+12x} & \text{for } \frac{-20}{9} \lhd x \le 2 \\ \frac{15x-20}{12x-14} & \text{for } 2 \lhd x \le \frac{20}{9} \end{cases}$$

$\therefore$ Y = $\bigcup_{\alpha\in[0,1]} \alpha \cdot Y\alpha$ is a triangular shaped fuzzy number.

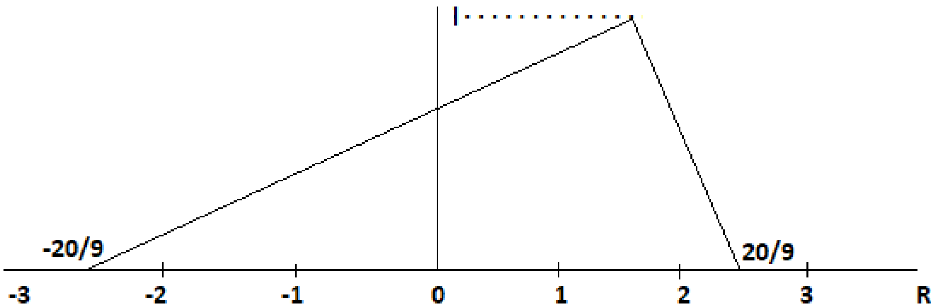

Figure-(c)

4. JUSTIFICATION OF THE SOLUTION

As fuzzy numbers are close to a given real numbers or around a given interval of real numbers, we can say that the fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}$ are around the intervals [0, 2], [1, 3], [2, 4], [3, 4], [4, 6], [5, 7] fig.– (a) indicating the central valued crisp real numbers 1, 2, 3, 4, 5, 6.

So, if the Co-efficient of the equations (1) be respectively these crisp real numbers then simultaneous ordinary equations

X + 2Y = 3

4X + 5Y = 6

Have their solutions X = -1, Y = 2 which are also crisp real numbers around the intervals $\left[\frac{-13}{9},\frac{19}{9}\right]$ and $\left[\frac{-20}{9},\frac{20}{9}\right]$ as shows in figure (b) and (c) respectively.

REFERENCES

1. L. A. Zadeh, Fuzzy Sets, Information and contral 8(3) (1965), 338 – 353.
2. George J. Klir, Bo. Yuan, Fuzzy Sets and fuzzy Logic, (2006), 97 – 117.
3. H. J. Zimmerman, Fuzzy Set Theory and its applications, Allied publishers Limited A – 104.
4. S.K. Pundir, Rimple, Fuzzy Sets and Their applications, First Edition, Pragati Prakashan, Meerut, (2006).
5. Shibroj Singh, Chaman Singh, Fuzzy Set Theory (3rd Edition), Krishna Prakashan, Meerut, (2014).

Source of support: Nil, Conflict of interest: None Declared.