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RESTRAINED ve- and ev- m-DOMINATION ON S-VALUED GRAPHS

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ABSTRACT

In this paper we discuss the notion of restrained ve- and ev- mixed domination on S-valued graphs.

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Keywords: *S valued graphs, restrained ve-weight m-domination set, restrained ev-weight m-domination set.*

1. INTRODUCTION

The theory of domination in graphs was initiated by Berge [1]. In [7], Chandramouleeswaran *et.al* introduced the notion of Semiring valued graphs (simply S-valued graphs). Motivated by this, we discuss the notion of vertex-edge mixed domination [3] and edge-vertex mixed domination [4] on S-valued graphs. In our previous paper, we introduce and discuss the notion of global ve- m-domination on S-valued graphs. In this paper we discuss the notion of restrained ve- and ev- m-domination on S-valued graphs.

2. PRELIMINARIES

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1: [2] A semi ring (S, +, .) is an algebraic system with a non-empty set S together with two binary operations + and . such that
(1) (S, +, 0) is a monoid.
(2) (S, .) is a semigroup.
(3) For all a, b, c $\in$ S , a . (b + c) = a . b + a . c and (a + b) . c = a . c + b . c
(4) 0 . x = x . 0 = 0 , $\forall$ x $\in$ S.

Definition 2.2:[2] Let (S, +, .) be a semiring. A Canonical Pre-order $\preceq$ in S defined as follows: for a, b $\in$ S, a$\preceq$b if and only if, there exists an element c $\in$ S such that a + c = b.

Definition 2.3: [9] Let $G = (V, E \subset V \times V)$ be a given graph with $V, E \neq \phi$. For any semiring (S, +, .), a semi ring-valued graph (or a S-valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be $\psi(x, y) = \begin{cases} \min\{\sigma(x), \sigma(y)\}, if \sigma(x) \preceq \sigma(y) or \sigma(y) \preceq \sigma(x) \\ 0, otherwise \end{cases}$

For every unordered pair (x,y) of $E \subset V \times V$. We call σ, a S- vertex set and ψ, a S-edge set of G^S.

Definition 2.4: [3] A S− valued graph $G^S = (V, E, \sigma, \psi)$ is said to be a S-Star(S-Wheel) if its underlying graph G is a Star(Wheel) along with S-values.

Definition 2.5: [8] Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. The open neighbourhood of v_i in G^S is defined as the set $N_S(v_i) = \{(v_j, \sigma(v_j)), where (v_i, v_j) \in E, \psi(v_i, v_j) \in S\}$.

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Definition 2.6: [4] The closed neighbourhood of v_i in $G^S = (V, E, \sigma, \psi)$ is defined to be the set $N_S[v_i] = N_S(v_i) \cup \{(v_i, \sigma(v_i))\}$.

Definition 2.7: [4] Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph. Let $e \in E$. The open neighbourhood of e, denoted by $N_S(e)$, is defined to be the set, $N_S(e) = \{(e_i, \Psi(e_i)) / e$ and e_i are adjacent$\}$.

The closed neighbourhood of e, denoted by $N_S[e] = N_S(e) \cup (e_i, \Psi(e_i))$.

Definition 2.8: [5] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. Let D $\subseteq$ V. If every edge of G^S is weight m-dominated by any vertex in D, then D is said to be a ve- weight m- dominating set.

Definition 2.9: [5] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. Let T $\subseteq$ E. If every vertex of G^S is weight m−dominated by any edge in T, then T is said to be a ev−weight m−dominating set.

3. RESTRAINED VE- M-DOMINATION ON S-VALUED GRAPHS

In this section, we introduce the notion of restrained vertex – edge mixed domination on S valued graphs, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 3.1: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. Let D $\subseteq$ V. If every edge of G^S is m-dominated by a vertex in D and also by a vertex in V – D, the D is said to be a restrained ve-weight m-dominating set of G^S.

Example 3.2: Let (S = {0, a, b, c}, +, .) be a semiring with the following Cayley Tables:

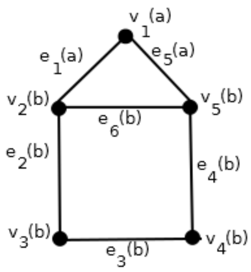
+	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

.	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c

Let $\preceq$ be a canonical pre-order in S, given by

$0 \preceq 0, 0 \preceq a, 0 \preceq b, 0 \preceq c, a \preceq a, a \preceq b, a \preceq c, b \preceq b, c \preceq b, c \preceq c$

Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$



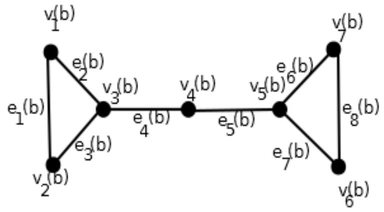
Define $\sigma : V \to S$ by $\sigma(v_1) = a$, $\sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \sigma(v_5) = b$
and $\psi : E \to S$ by $\psi(e_1) = \psi(e_5) = a$, $\psi(e_2) = \psi(e_3) = \psi(e_4) = \psi(e_6) = b$.

Clearly D = {v_2, v_4} is a restrained ve-weight m-dominating set of G^S.

Definition 3.3: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a minimal restrained ve-weight m-dominating set, if

(1) D is a restrained ve-weight m- dominating set.
(2) No proper subset of D is a restrained ve- weight m- dominating set.

Example 3.4: Let (S = {0, a, b, c}, +, .) be a semiring with the canonical preorder given in example 3.2 Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$



Define $\sigma : V \to S$ by $\sigma(v_1) = \sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \sigma(v_5) = \sigma(v_6) = \sigma(v_7) = b$
and $\psi : E \to S$ by $\psi(e_1) = \psi(e_2) = \psi(e_3) = \psi(e_4) = \psi(e_5) = \psi(e_6) = \psi(e_7) = \psi(e_8) = b$.

Clearly $D_1 =\{ v_3 , v_5 \}$, $D_2 =\{ v_1 , v_3 ,v_5\}$, $D_3 =\{ v_3 ,v_5 , v_7\}$, $D_4 =\{ v_1 ,v_4 , v_7 \}$, $D_5 =\{ v_2 ,v_4 , v_6\}$,
$D_6 =\{v_1 ,v_4 , v_6 \}$, $D_7 =\{ v_2 ,v_4 , v_7\}$, $D_8 =\{v_2, v_3 ,v_5 , v_6 \}$, $D_9 =\{v_1, v_3 ,v_5 , v_7\}$, $D_{10} =\{v_1, v_3 ,v_5 , v_6\}$,
$D_{11} =\{ v_2, v_3 ,v_5 , v_7 \}$are all restrained ve-weight m-dominating sets of G^S.
However $D_1 =\{ v_3 , v_5 \}$ is a minimal restrained ve-weight m-dominating sets of G^S.

Definition 3.5: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a maximal restrained ve-weight m-dominating set, if

(1) D is a restrained ve-weight m- dominating set.
(2) There is no restrained ve- weight m- dominating set D' $\subseteq$ V such that D $\subseteq$ D' $\subseteq$ V.

In example 3.4, $D_8 =\{ v_2, v_3 ,v_5 , v_6 \}$, $D_9 =\{v_1, v_3 ,v_5 , v_7 \}$, $D_{10} =\{ v_1, v_3 ,v_5 , v_6 \}$, $D_{11} =\{ v_2, v_3 ,v_5 , v_7 \}$are all maximal restrained ve-weight m-dominating sets of G^S.

Definition 3.6: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a restrained ve-weight m-dominating independent set, if

(1) D is a restrained ve-weight m- dominating set.
(2) If u, v $\in$ D then $N_S(u) \cap (v, \sigma(v)) = \phi$.

In example 3.2, D = $\{v_2 , v_4 \}$ is a restrained ve-weight m-dominating set of G^S.

Also $N_S(v_2) \cap \{(v_4, b)\} = \phi$ and $N_S(v_4) \cap \{(v_2, b)\} = \phi$
Hence D =$\{ v_2 , v_4 \}$ is a restrained ve-weight m-dominating independent set of G^S.

Definition 3.7: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. The restrained vertex-edge mixed domination number of G^S is defined by $\gamma_{RVE}{}^S(G^S) = (|D|_S, |D|)$, where D is a minimal restrained ve-weight m-dominating set.

In example 3.4, restrained vertex-edge mixed domination number of G^S is $\gamma_{RVE}{}^S(G^S) = (|D_1|_S, |D_1|) = (b, 2)$.

Theorem 3.8: For a S-regular Star S_n^S, $\gamma_{RVE}{}^S(S_n{}^S) = (\sigma(v), 1)$ where $\sigma(v) \in S$.

Proof: Let S_n^S be a S-regular Star and let v be the pole of S_n^S.
Then all the edges of S_n^S are m-dominated by the pole v. Also all the edges of S_n^S are m-dominated by a vertex in V − {v}. Hence {v} is a restrained ve-weight m-dominating set. And no proper subset of {v} is a restrained ve-weight m-dominating set. Therefore {v} is a minimal restrained ve-weight m-dominating set. Hence $\gamma_{RVE}{}^S(S_n{}^S) = (\sigma(v), 1)$ where $\sigma(v) \in S$.
Analogously, we can prove the following results,

Corollary 3.9:

(1) For a S-regular Wheel W_n^S, $\gamma_{RVE}{}^S(W_n{}^S) = (\sigma(v), 1)$ where $\sigma(v) \in S$.
(2) For a S-regular Complete Graph K_n^S, $\gamma_{RVE}{}^S(K_n{}^S) = (\sigma(v), 1)$ where $\sigma(v) \in S$.
(3) For a S-regular Complete Bipartite Graph $K_{m,n}^S$, $\gamma_{RVE}{}^S(K_{m,n}{}^S) = \begin{cases} (\sigma(v), n), n \le m \\ (\sigma(v), m), m \le n \end{cases}$ where $\sigma(v) \in S$.

Theorem 3.10: A restrained ve-weight m-dominating set D of a S-valued graph G^S is a minimal restrained ve-weight m-dominating set of G^S iff every vertex v $\in$ D satisfies at least one of the following properties;

(1) There exists a vertex u $\in$ V – D such that $N_S(u) \cap \{D \times S\} = \{(v, \sigma(v))\}$
(2) v is adjacent to no vertex of D.

Proof: Let v $\in$ D. Assume that v $\in$ D satisfies at least one of the above two properties. Then D – {v} is not a restrained ve-weight m-dominating set. Therefore D is a minimal restrained ve-weight m-dominating set.

Conversely, assume that D is a minimal restrained ve-weight m-dominating set. Then for each v $\in$ D, D – {v} is not a minimal restrained ve-weight m-dominating set of G^S. Therefore there exist a vertex u $\in$ V - (D – {v}) that is adjacent to no vertex of (D – {v}).

If u = v, then v is adjacent to no vertex of D.

If u $\neq$ v, then D is a restrained ve-weight m-dominating set and u $\notin$ D $\Rightarrow$ u is adjacent to at least one vertex of D. However u is not adjacent to any vertex of D – {v} $\Rightarrow N_S(u) \cap \{D \times S\} = \{(v, \sigma(v))\}$

Theorem 3.11: A subset D $\subseteq$ V of a S-valued graph G^S is a restrained ve-weight m-dominating independent set iff D is a maximal independent vertex set in G^S.

Proof: Clearly every maximal independent vertex set D in G^S is a restrained ve-weight m-dominating independent set. Conversely, assume that D is restrained ve-weight m-dominating independent set. Then D is independent and every vertex not in D is adjacent to a vertex of D and therefore D is a maximal independent vertex set in G^S.

Theorem 3.12: Every maximal independent vertex set D in G^S is a minimal restrained ve-weight m-dominating set.

Proof: Let D be a maximal independent vertex set in G^S. Then by theorem 3.11, D is a restrained ve-weight m-dominating independent set. Since D is independent, certainly every vertex of D is adjacent to no vertex of D. Thus, every vertex of D satisfies the second condition of theorem 3.10. Hence D is a minimal restrained ve-weight m-dominating set.
Combining the above two theorems, we obtain the following theorem,

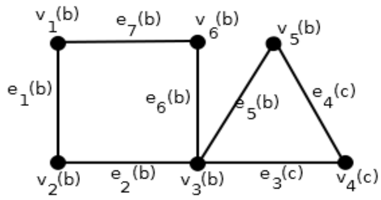
Theorem 3.13: A subset D $\subseteq$ V of G^S is a restrained ve-weight m-dominating independent set iff D is a minimal restrained ve-weight m-dominating set.

4. RESTRAINED EV- M-DOMINATION ON S-VALUED GRAPHS

In this section, we introduce the notion of restrained edge - vertex mixed domination on S valued graphs, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 4.1: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. Let T $\subseteq$ E. If every vertex of G^S is m-dominated by an edge in T and also by an edge in E – T, the T is said to be a restrained ev-weight m-dominating set of G^S.

Example 4.2: Let (S = {0, a, b, c}, +, .) be a semiring with the canonical preorder given in example 3.2 Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$



Define $\sigma : V \to S$ by σ(v_1) = σ(v_2) = σ(v_3) = σ(v_5) = σ(v_6) = b, σ(v_4) = c.
and ψ : E$\to$ S by ψ(e_1) = ψ(e_2) = ψ(e_5) = ψ(e_6) = ψ(e_7) = b, ψ(e_3) = ψ(e_4) = c.

Clearly T_1 = { e_2 }, T_2 = { e_6 }, T_3 = { e_1, e_5 }, T_4 = { e_1, e_2 }, T_5 = { e_1, e_6 }, T_6 = { e_2, e_5 }, T_7 = { e_2, e_6 },
T_8 = { e_2, e_7 }, T_9 = { e_5, e_6 }, T_{10} = { e_5, e_7 }, T_{11} = { e_6, e_7 }, T_{12} = { e_1, e_2, e_5 }, T_{13} = { e_1, e_2, e_6 },
T_{14} = {e_1, e_2, e_5 , e_7 }, T_{15} = { e_1, e_5, e_6, e_7 } are all restrained ev-weight m-dominating sets of G^S.

Definition 4.3: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset T $\subseteq$ E is said to be a minimal restrained ev-weight m-dominating set, if

(1) T is a restrained ev-weight m- dominating set.
(2) No proper subset of T is a restrained ev- weight m- dominating set.

In example 4.2, $T_1 = \{e_2\}$, $T_2 = \{e_6\}$ are the minimal restrained ev-weight m-dominating sets of G^S.

Definition 4.4: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset T $\subseteq$ E is said to be a maximal restrained ev-weight m-dominating set, if

(1) T is a restrained ev-weight m- dominating set.
(2) There is no restrained ev- weight m- dominating set T' $\subseteq$ E such that T $\subseteq$ T' $\subseteq$ E.

In example 4.2, $T_{14} = \{e_1, e_2, e_5, e_7\}$, $T_{15} = \{e_1, e_5, e_6, e_7\}$ are the maximal restrained ev-weight m-dominating sets of G^S.

Definition 4.5: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. A subset T $\subseteq$ E is said to be a restrained ev-weight m-dominating independent set, if

(1) D is a restrained ve-weight m- dominating set.
(2) If e,f $\in$ T then $N_S(e) \cap (f, \psi(f)) = \phi.$

In example 4.2, $T_3 = \{e_1, e_5\}$is a restrained ev-weight m-dominating independent set of G^S.

Definition 4.6: Consider the S− valued graph $G^S = (V, E, \sigma, \psi)$. The restrained edge-vertex mixed domination number of G^S is defined by $\gamma_{REV}{}^S(G^S) = (|T|_S, |T|)$, where T is a minimal restrained ev-weight m-dominating set.

In example 4.2, restrained edge-vertex mixed domination number of G^S is
$\gamma_{REV}{}^S(G^S) = (|T_1|_S, |T_1|) = (|T_2|_S, |T_2|) = (b,1).$

Theorem 4.7: For a S-regular Star S_n^S, $\gamma_{REV}{}^S(S_n{}^S) = (\psi(e),1)$ where $\psi(e) \in S.$

Proof: Let S_n^S be an S-regular Star and let e be any edge of S_n^S.
Then all the vertices of S_n^S are m-dominated by the edge e. Also all the vertices of S_n^S are m-dominated by an edge in E – {e}. Hence {e} is a restrained ev-weight m-dominating set. And no proper subset of {e} is a restrained ev-weight m-dominating set. Therefore {e} is a minimal restrained ev-weight m-dominating set. Hence $\gamma_{REV}{}^S(S_n{}^S) = (\psi(e),1)$ where $\psi(e) \in S.$
Analogously, we can prove the following results,

Corollary3.9:

(1) For a S-regular Complete Graph K_n^S, $\gamma_{REV}{}^S(K_n{}^S) = (\psi(e),1)$ where $\psi(e) \in S.$
(2) For a S-regular Complete Bipartite Graph $K_{m,n}^S$, $\gamma_{REV}{}^S(K_{m,n}{}^S) = (\psi(e),1),$ where $\psi(e) \in S.$
(3) For a S-regular Wheel W_n^S, $\gamma_{REV}{}^S(W_n{}^S) = (\psi(e),1)$ where $\psi(e) \in S$, if e is a spoke.

Remark 4.9: For a S-regular Wheel W_n^S, with n >5, if e is not a spoke, then $\gamma_{REV}{}^S(W_n{}^S) \neq (\psi(e),1)$ where $\psi(e) \in S.$

In [3], itself we have proved with an example that, if e is not a spoke of a S-Wheel, then {e} will not be the minimal ev-weight m-dominating set. Hence {e} will not be a minimal restrained ev-weight m-dominating set. Therefore $\gamma_{REV}{}^S(W_n{}^S) \neq (\psi(e),1)$ where $\psi(e) \in S.$

Theorem 4.10: A restrained ev-weight m-dominating set T of a S-valued graph G^S is a minimal restrained ev-weight m-dominating set of G^S iff every edge e $\in$ T satisfies at least one of the following properties;

(1) There exists an edge f $\in$ E - T such that $N_S(f) \cap \{T \times S\} = \{(e, \psi(e))\}$
(2) e is adjacent to no edge of T.

Proof: Let e $\in$ T. Assume that e is adjacent to no edge of T. Then T – {e} cannot be a restrained ev-weight m-dominating set. $\Rightarrow$ T is a minimal restrained ev-weight m-dominating set. On the other hand, if for any e $\in$ T there exist an f $\in$ E – T such that $N_S(f) \cap \{T \times S\} = \{(e, \psi(e))\}$. Then f is adjacent to e $\in$ T and no other edge of T. In this case also, T – {e} cannot be a restrained ev-weight m-dominating set of G^S.

Conversely, assume that D is a minimal restrained ev-weight m-dominating set of G^S. Then for each e $\in$ T, T – {e} is not a minimal restrained ev-weight m-dominating set of G^S. Therefore there exist an edge f $\in$ E - (T – {e}) that is adjacent to no edge of (T – {e}).

If f = e, then e is adjacent to no edge of T.

If f $\neq$ e, then T is a restrained ev-weight m-dominating set and f $\notin$ T $\Rightarrow$ f is adjacent to at least one edge of T.

However f is not adjacent to any edge of T – {e} $\Rightarrow N_S(f) \cap \{T \times S\} = \{(e, \psi(e))\}$

Theorem 4.11: A subset T $\subseteq$ E of G^S is a restrained ev-weight m-dominating independent set iff T is a maximal independent edge set in G^S.

Proof: Clearly every maximal independent edge set T in G^S is a restrained ev-weight m-dominating independent set.

Conversely, assume that T is restrained ev-weight m-dominating independent set. Then T is independent and every edge not in T is adjacent to an edge of T and therefore T is a maximal independent edge set in G^S.

Theorem 4.12: Every maximal independent edge set of G^S is a minimal restrained ev-weight m-dominating set.

Proof: Let T be a maximal independent edge set of G^S. Then by theorem 4.11, T is a restrained ev-weight m-dominating independent set. Since T is independent, every edge of T is adjacent to no edge of T. Thus, every edge of T satisfies the second condition of theorem 4.10. Hence T is a minimal restrained ev-weight m-dominating set.
Combining the above two theorems, we obtain the following theorem,

Theorem 3.13: A subset T $\subseteq$ E of G^S is a restrained ev-weight m-dominating independent set iff T is a minimal restrained ev-weight m-dominating set.

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