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CHROMATIC NUMBER TO THE TRANSFORMATION (G^{---}) OF P_n AND C_n

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ABSTRACT

Let $G = (V, E)$ be an undirected simple graph. The transformation graph G^{---} of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows: (a) two elements in $V(G)$ are adjacent if and only if they are non-adjacent in G, (b) two elements in $E(G)$ are adjacent if and only if they are non-adjacent in G, and (c) an element of $V(G)$ and an element of $E(G)$ are adjacent if and only if they are non-incident in G. In this paper, we determine the chromatic number of Transformation graph G^{---} for Path and Cycle graph.

Keywords: *Path, Cycle, Chromatic Number, Transformation Graph.*

1. INTRODUCTION

In this paper, we are concerned with finite, simple graph. Let $G = (V(G), E(G))$ be a graph, if there is an edge e joining any two vertices u and v of G, we say u and v are adjacent. An n-vertex colouring or an n-colouring of a graph $G = (V, E)$ is a mapping $f: V \to S$, where S is a set of n-colours.

Definition 1.1: A **graph** G is an ordered pair $(V(G), E(G))$ consisting of a non-empty set $V(G)$ of vertices and a set $E(G)$, disjoint from $V(G)$ of edges together with an incidence function ψ_G that associates with each edge of G is an unordered pair of vertices of G.

Definition 1.2: A **colouring** C of a simple graph G is proper if no two adjacent vertices are assigned the same colour.

A graph is **properly coloured** if it is coloured with the minimum possible number of colours.

Definition 1.3: The **chromatic number** of a graph G is the minimum number of colours required to colour the vertices of G and is denoted by $\chi(G)$.

Definition 1.4: The **total graph** $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent in T if and only if the elements are either adjacent or incident in G.

Definition 1.5: The **complement** $\bar{G}$ of a graph G, have the same vertex set $V(G)$ and those vertices which are adjacent in G are not adjacent in $\bar{G}$.

Definition 1.6: Walk is an alternating sequence of vertices and edges starting and ending with vertices. A walk in which all the vertices are distinct is called a **path**. A path containing *n*-vertices is denoted by P_n.

Definition 1.7: A closed path is called **cycle**. A cycle containing *n*-vertices is denoted by C_n, the length of a cycle is the number of edges occurring on it.

In [2] generalized the concept of total graphs to a transformation graph G^{xyz} with x, y, z; $\{-, +\}$, where G^{+++} is the total graph of G, and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complement of G^{++-}, G^{+-+} and G^{+--} respectively.

Here we investigate the transformation graph G^{---} of some graphs.

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Theorem 2.1: Let $G = P_n$ be any path graph, G^{---} is the transformation of G, then any particular colour c_i can be assigned to at most three vertices of G^{---}.

Proof:

Let $G = P_n$ be any path graph of length n and G^{---} is the transformation of G.
The vertex set of G^{---} is $V(G^{---}) = \{v_i, e_{i-1} \,/\, i = 1,2,\ldots\ldots,n\}$.
Let C be the colour class of G^{---} and $c_i \in C,\ i = 1,2,\ldots.$

We have to prove that the colour c_i can be assigned to at most three vertices of G^{---}.

Without loss of generality, we assume that the colour c_i can be given to at least four vertices of G^{---}. Let it be v, v_j, v_k, v_l.

Case-(i): Choose the vertex v of degree 2(middle vertex) in G and it is coloured by the colour c_i.

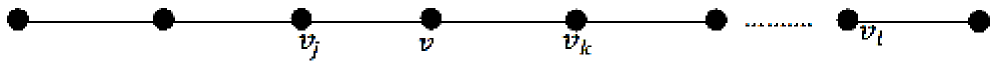

Figure.1

Let v_j and v_k be the neighbours of v in G, that is $N(v) = \{v_j, v_k\}$ in G and v_l be the vertex non-adjacent to $\{v, v_j, v_k\}$ in G.

Since v_j and v_k are independent in G, they are adjacent in G^{---}, so we can give the colour c_i either to the vertex v_j or v_k, but not to both.

Also, the vertex v_l is non-adjacent to v in G, so it is adjacent with v in G^{---}. Hence, we need another new colour to colour the remaining vertices.

Therefore, we need 2 −colours to colour the vertices v, v_j, v_k and v_l which is a contradiction.

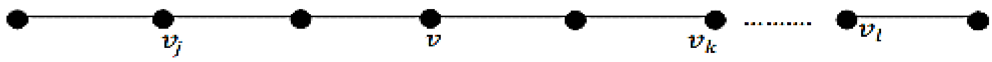

Figure.2

If the vertices v, v_j, v_k and v_l are independent in G; clearly, they are adjacent in G^{---}, so we cannot give the colour c_i to all the four vertices. Hence, we need more than one colour to colour these four vertices, which is again a contradiction.

Case-(ii): Suppose v is a pendent vertex in G. The vertex is coloured by the colour c_i in G^{---}.

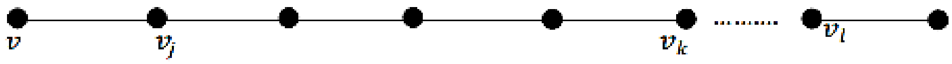

Figure.3

If $N(v) = \{v_j\}$ in G and v_k , v_l be the vertices non-adjacent with v in G. Since v and $N(v)$ are independent in G^{---}, we can give the colour c_i to v and $N(v)$. Also, the vertices v_k and v_l are adjacent to v in G^{---}, we need some new colours to colour the vertices v_k and v_l. Hence, we need more than one colour to colour these four vertices, which is again a contradiction.

Case-(iii): Choose the edge v of degree 2 in G and it is coloured by the colour c_i.

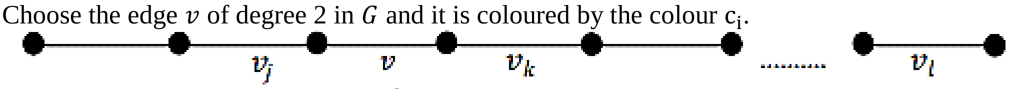

Figure.4

If $N(v) = \{v_j, v_k\}$ in G and v_l be the edge non-adjacent to v, v_j, v_k in G, then v_l is adjacent to v, v_j, v_k in G^{---}. It is a contradiction by case (i).

If the edges v, v_j, v_k and v_l are independent in G, then they are adjacent in G^{---}.

Figure.5

It is again a contradiction by case (i)

Case-(iv): Suppose v is a vertex in G of degree 2 and v_j, v_k are the edges incident with v and v_l be any other vertex or edge in G.

Figure.6

Clearly, v_j and v_k are non-adjacent with v in G^{---} and v_j and v_k are independent in G^{---}, so we can give the colour c_i to these three vertices v, v_j and v_k . But the vertex v_l is adjacent with v in G^{---}, so we need more than one colour to colour all these four vertices, which is a contradiction.

Hence, in G^{---}, any particular colour can be assigned to at most three vertices.

Hence proved.

Theorem 2.2: Let G be any simple graph and G^{---} is the transformation of G, then a colour can be given to three vertices of G^{---} if and only if either they formed a K_2 in G or a pair of edges are incident with a vertex in G.

Proof:

Let G be any simple graph with n-vertices.
Let $V(G^{---})$ be the vertex set of G^{---}, that is, $V(G^{---}) = \{v_i, e_j / i = 1,2, \dots, n; j = 1,2, \dots\}$.

Assume that, the vertices in G^{---} formed either a K_2 in G or a pair of edges are incident with a vertex in G.

Case-(i):

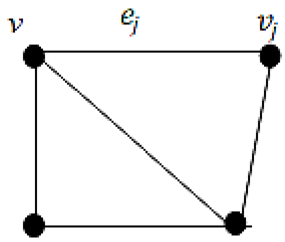

Figure: 7

Choose an arbitrary vertex v in G, $N(v) = \{v_j\}$ *and* e_j is an edge incident with v and v_j. Clearly, v, v_j and e_j are independent in G^{---}. Hence, we can give a single colour to v, v_j and e_j.

Case-(ii):

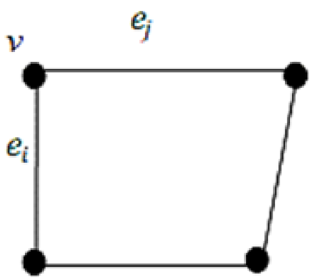

Figure: 8

Suppose v is a vertex in G and e_i, e_j are the edges incident with v in G. Clearly, v, e_i and e_j are independent in G^{---}. Hence, we can give a single colour to v, e_i and e_j.

Therefore, a single colour can be given to exactly three vertices.

Conversely, Assume that a single colour can be given to three vertices of G^{---} .

To prove that the vertices in G^{---} formed either a K_2 in G or a pair of edges are incident with a vertex in G.

Suppose the vertices in G^{---} formed neither a K_2 in G nor a pair of edges are incident with a vertex in G. Clearly, they are adjacent in G^{---}, so we need more than one colour to colour these three vertices in G^{---} which is a contradiction to our assumption.

Therefore, the vertices in G^{---} formed either a K_2 in G or a pair of edges are incident with a vertex in G.

Hence proved.

Theorem 2.3: Let G be any path or cycle graph. If its transformation G^{---} has $3k$ −vertices, then we need exactly k −colours.

Proof: Let G be any path or cycle graph and G^{---} be the transformation graph.

By theorem: 2.2,

We can give the same colour to exactly three vertices of G^{---}.

Therefore, we need k −colours to colour a graph with $3k$ −vertices.

Hence proved.

Theorem 2.4: Let $G = P_n$ be any path graph with n-vertices, then $\chi(G^{---}) = \left\lceil \frac{2n-1}{3} \right\rceil$.

Proof:

Figure: 9 (G)

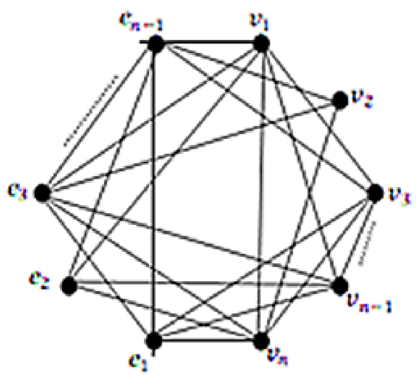

Figure: 10 $(G = P_n^{---})$

Let $G = P_n$ be any path graph with n-vertices, whose vertices $\{v_i / i = 1,2,\ldots,n\}$ are linear. Its transformation G^{---} has $(2n-1)$-vertices.

Let $V(G^{---}) = \{v_i, e_{i-1} / i = 1,2,\ldots,n\}$ be the vertex set of G^{---}.

Now, we divide the vertex set of G^{---} into three sets,

- $i)$ $V_1 = \{v_n / n \equiv 1(mod\ 3)$
- $ii)$ $V_2 = \{v_n / n \equiv 2(mod\ 3)$
- $iii)$ $V_3 = \{v_n / n \equiv 0(mod\ 3)$

Case-(i): If $n \equiv 1(mod\ 3)$, that is $n = 3k+1$, we have $(6k+1)$-vertices in G^{---}.

By theorem: 2.3, To colour $6k$-vertices, we need $2k$-colours, that is $\left\lceil \frac{6k}{3} \right\rceil$-colours.

The $(6k+1)^{th}$-vertex of G^{---} is a pendent vertex in G is of degree $2n-3$. It is adjacent with all the vertices which are coloured by the $(2k)$-colours, so we need a new colour to colour the pendent vertex. Hence, we need $(2k+1)$-colours to colour the $(6k+1)$-vertices of G^{---}.

$$\Rightarrow \left\lceil \frac{6k+1}{3} \right\rceil = \left\lceil \frac{2(3k+1)-1}{3} \right\rceil$$

Therefore, we need $\left\lceil \frac{2(3k+1)-1}{3} \right\rceil$-colours to colour the $(6k+1)$-vertices of G^{---}.

Case-(ii): If $n \equiv 2(mod\ 3)$, then $|V(G^{---})| = 6k+3$.

By theorem: 2.3,

$$\chi(G^{---}) = 2k+1 = \left\lceil \frac{6k+3}{3} \right\rceil = \left\lceil \frac{2(3k+2)-1}{3} \right\rceil.$$

Therefore, we need $\left\lceil \frac{2(3k+2)-1}{3} \right\rceil$-colours to colour the $(6k+3)$-vertices of G^{---}.

Case-(iii): If $n \equiv 0(mod\ 3)$, then $|V(G^{---})| = 6k-1$.

By theorem: 2.3, To colour $(6k-3)$-vertices, we need $(2k-1)$-colours.

The $(6k-2)^{th}\ and\ (6k-1)^{th}$ vertices of G^{---} is a leaf in G. It is adjacent with all the vertices which are coloured by the $(2k-1)$-colours, so we need a new colour to colour the leaf. Hence, we need $(2k)$-colours to colour the $(6k-1)$-vertices of G^{---}.

Therefore, we need $\left\lceil \frac{2(3k)-1}{3} \right\rceil$-colours to colour the $(6k-1)$-vertices of G^{---}.

Hence, in all three cases we need $\left\lceil \frac{2n-1}{3} \right\rceil$-colours to colour the $(2n-1)$-vertices of G^{---}.

Therefore, $\chi(G^{---}) = \left\lceil \frac{2n-1}{3} \right\rceil$.

Corollary 2.5: Let $G = C_n$ be any cycle graph with n-vertices, then $\chi(G^{---}) = \left\lceil \frac{2n}{3} \right\rceil$.

Proof: Let $G = C_n$ be any cycle graph with n-vertices, whose vertices $\{v_i / i = 1,2, \dots \dots, n\}$ are linear. Its transformation G^{---} has $(2n)$-vertices.

Let $V(G^{---}) = \{v_i, e_i / i = 1,2, \dots, n\}$ be the vertex set of G^{---}.

By theorem: 2.2 and by theorem: 2.3,

$$\chi(G^{---}) = \left\lceil \frac{2n}{3} \right\rceil.$$

Hence the proof.

REFERENCES

1. H. Abdollahzadeh Ahangar, L. Pushpalatha, "On the chromatic number of some Harary graphs", International Mathematical Forum, 4,2009, No.31, pp. 1511-1514.
2. B. Wu, J. Meng, "Basic properties of total transformation graphs", J. math study 34(2) (2001), pp.109-116.
3. Nikhilesh Sil, A. Datta, S. Samanta, S. Bhattacharya, Priti Kumar Roy, Effect of Migration of Susceptible Prey; Mathematical Sciences International Research Journal ISSN 2278 – 8697. Vol 3 Issue 1 (2014), Pg. 447-451
4. B. Basavanagoud, Keerthi Mirajkarandshripurnamalghan, "Transversability and Planarity of the Transformation graph G^{xyz} Proceedings of International conference on graph theory and Applications, Amritha school, 2009, pp. 153-165.
5. Douglas B. West, "Introduction to graph theory", Second edition, Prentice-Hall of India Private Limited, New Delhi, 2006.

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