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A COMPARATIVE STUDY OF NIM WITH SOME EXISTING NUMERICAL METHODS FOR SIMULTANEOUS DIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper we have used New Iterative Method along with Runge Kutta, Picard and Taylor's method for solving simultaneous linear differential equations and have done a comparitative study of these methods.

Keywords: *Ordinary differential equation, Runge Kutta, NIM.*

1. INTRODUCTION

A differential equation is an equation involving dependent variable and independent variable and derivative of one or more dependent variables with respect to one or more independent variables. Such equations frequently arise when we wish to analyze mathematically many of the phenomena that arise in nature or in various aspects of human endeavors.

If in a differential equation there is only one independent variable then it is called an ordinary differential equation. An equation involving derivatives in which the dependent variables and all derivatives appearing in the equation are raised to the first power are known as linear differential equation.

The direct method (analytical method) gives the exact solution in which there is no error except the round off error due to the machine whereas iterative methods give the approximate solution in which there is some error however, iterative methods are suitable for solving linear differential equations when the no. of equations in a system is very large. Iterative methods are very effective concerning computer storage and time requirements. One of the advantages of using iterative methods is that they require fewer multiplications for large systems. Iterative methods are fast and simple to use.

2. NEW ITERATIVE METHOD (NIM)

Consider the following general functional equation:

$$u = N(u) + f, \tag{1}$$

where N is a nonlinear operator from a Banach space B→ B and f is a function. We are looking for a solution u of eq . (1) having the series form:

$$u=\sum_{i=0}^{\infty} u_i \tag{2}$$

The nonlinear operator N can be decomposed as $N(\sum_{i=0}^{\infty} u_i) = N(u_0)+\sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}$

From above equations eq (1) is equivalent to

$$\sum_{i=0}^{\infty} u_i= f+N(u_0)+\sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}$$

We define the recurrence relation:

$$G_0=u_0=f$$
$$G_1=u_1=N(u_0)$$
$$G_m=u_{m+1}=N(u_0+\text{----}+u_m)-N(u_0+\text{----}+u_{m-1}),\ m=1, 2,\text{---}$$

Then,

$$(u_1+\text{----}u_{m+1})=N(u_1+\text{----}u_m),\ m=1, 2,\text{----}$$

and $u(x)=f+\sum_{i=1}^{\infty} u_i$

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Runge Kutta Method
Runge Kutta is the one of the standard workhorses for solving ordinary differential equations. Runge Kutta method is particularly suitable in case when the computations are complicated.

For Runge Kutta following formulas used -

$k_1=hf_1(t_0,x_0,y_0)$, $l_1=hf_2(t_0,x_0,y_0)$

$k_2=hf_1(t_0+\frac{h}{2},x_0+\frac{k_1}{2},y_0+\frac{l_1}{2})$, $l_2=hf_2(t_0+\frac{h}{2},x_0+\frac{k_1}{2},y_0+\frac{l_1}{2})$

$k_3=hf_1(t_0+\frac{h}{2},x_0+\frac{k_2}{2},y_0+\frac{l_2}{2})$, $l_3= hf_2(t_0+\frac{h}{2},x_0+\frac{k_2}{2},y_0+\frac{l_2}{2})$

$k_4= hf_1(t_0+h,x_0+k_3,y_0+l_3)$, $l_4= hf_2(t_0+h,x_0+k_3,y_0+l_3)$

$x_1= x_0+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$

$y_1= y_0+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$

Picard's Method
Picard's iterates are given by-

$x(t) = x_0 +\int_0^t f_1(t, x, y)$ and

$y(t) = y_0+\int_0^t f_2(t, x, y)$

Taylor's Method
For Taylor's Series following formulae are used .

$$x(t) = x_0 + \frac{t}{1!}x_0' + \frac{t^2}{2!}x_0'' + \frac{t^3}{3!}x_0''' \ldots.$$

$$y(t) = y_0 + \frac{t}{1!}y_0' + \frac{t^2}{2!}y_0'' + \frac{t^3}{3!}y_0''' + \frac{t^4}{4!}y_0^{iv} + \cdots$$

3. PROBLEM

$\frac{dx}{dt}+5x-2y=t$

$\frac{dy}{dt}+2x+y=0$

$x_0=0$ and $y_0=0$

By taylor's series method-

$\frac{dx}{dt} = t-5x+2y$ $x_0=0$

$\frac{dy}{dt} = -2x-y$ $y_0=0$

$x'=t-5x+2y$ $x_0'=0$

$y'=-2x-y$ $y_0'=0$

$x''=1-5x'+2y'$ $x_0''=1$

$y''=-2x'-y'$ $y_0''=0$

$x'''=-5x''+2y''$ $x_0'''=-5$

$y'''=-2x''-y''$ $y_0'''=-2$

$x^{iv}=-5x'''+2y'''$ $x_0^{iv}=21$

$y^{iv}=-2x'''-y'''$ $y_0^{iv}=12$

………... …………….

By taylor's series expansion we have-

$$x(t) = x_0 + \frac{t}{1!}x_0' + \frac{t^2}{2!}x_0'' + \frac{t^3}{3!}x_0''' \ldots.$$

Substituting the values we get-

$$x(t) = \frac{1}{2}t^2 - \frac{5}{6}t^3 + \frac{7}{8}t^4 - \frac{27}{40}t^5 + \frac{33}{80}t^6 + \cdots$$

Again by taylor's series expansion we have-

$$y(t) = y_0 + \frac{t}{1!}y_0' + \frac{t^2}{2!}y_0'' + \frac{t^3}{3!}y_0''' + \frac{t^4}{4!}y_0^{iv} + \cdots$$

Substituting the values we get-

$$y(t) = \frac{-1}{3}t^3 + \frac{1}{2}t^4 - \frac{9}{20}t^5 + \frac{3}{10}t^6 + \cdots$$

By Picard's method-
Picard's iterates are given by-

$x(t) = x_0 +\int_0^t f_1(t, x, y)$ and

$y(t) = y_0+\int_0^t f_2(t,x,y)$

iterates are-

$x_1(t) = \frac{t^2}{2}$

$y_1(t) = 0$

$x_2(t) = \frac{t^2}{2} - \frac{5}{6}t^3$

$y_2(t) = \frac{-1}{3}t^3$

$x_3(t) = \frac{t^2}{2} - \frac{5}{6}t^3 + \frac{7}{8}t^4$

$y_3(t) = -\frac{t^3}{3}+\frac{t^4}{2}$

$x_4(t) = \frac{t^2}{2} - \frac{5}{6}t^3 + \frac{7}{8}t^4 - \frac{27}{40}t^5$

$y_4(t) = \frac{-t^3}{3} + \frac{t^4}{2} - \frac{9}{20}t^5$

$x_5(t) = \frac{t^2}{2} - \frac{5}{6}t^3 + \frac{7}{8}t^4 - \frac{27}{40}t^5 + \frac{33}{80}t^6$

$y_5(t) = \frac{-t^3}{3} + \frac{t^4}{2} - \frac{9}{20}t^5 + \frac{3}{10}t^6$

Therefore the solution is given as-

$x(t) = \frac{t^2}{2} - \frac{5}{6}t^3 + \frac{7}{8}t^4 - \frac{27}{40}t^5 + \frac{33}{80}t^6$

$y(t) = \frac{-t^3}{3} + \frac{t^4}{2} - \frac{9}{20}t^5 + \frac{3}{10}t^6$

Runge Kutta Method

$k_1=hf_1(t_0,x_0,y_0)$ $\quad l_1=hf_2(t_0,x_0,y_0)$

$k_1 = 0$ $\quad l_1=0$

$k_2=hf_1(t_0+\frac{h}{2},x_0+\frac{k_1}{2},y_0+\frac{l_1}{2})$ $\quad k_2=0.005$

$l_2=hf_2(t_0+\frac{h}{2},x_0+\frac{k_1}{2},y_0+\frac{l_1}{2})$ $\quad l_2=0$

$k_3=hf_1(t_0+\frac{h}{2},x_0+\frac{k_2}{2},y_0+\frac{l_2}{2})$ $\quad k_3=0.00375$

$l_3= hf_2(t_0+\frac{h}{2},x_0+\frac{k_2}{2},y_0+\frac{l_2}{2})$ $\quad l_3= -0.00050$

$k_4= hf_1(t_0+h,x_0+k_3,y_0+l_3)$ $\quad k_4=0.008025$

$l_4= hf_2(t_0+h,x_0+k_3,y_0+l_3)$ $\quad l_4= -0.0007$

Therefore,

$x_1=x(0.1)=x_0+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$

$\mathbf{x_1=x(0.1)=0.004254166}$

$y_1=y(0.1)= y_0+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$

$\mathbf{y_1=y(0.1)= -0.00028333}$

Again applying Runge Kutta for values at 0.2

$k_1=hf_1(t_1,x_1,y_1)$ $\quad l_1=hf_2(t_1,x_1,y_1)$

$k_1=0.007816254$ $\quad l_1= -0.000822499$

$k_2=hf_1(t_1+\frac{h}{2},x_1+\frac{k_1}{2},y_1+\frac{l_1}{2})$ $\quad k_2=0.0107799376$

$l_2=hf_2(t_1+\frac{h}{2},x_1+\frac{k_1}{2},y_1+\frac{l_1}{2})$ $\quad l_2= -0.0015630006$

$k_3=hf_1(t_1+\frac{h}{2},x_1+\frac{k_2}{2},y_1+\frac{l_2}{2})$ $\quad k_3=0.0099649665$

$l_3= hf_2(t_1+\frac{h}{2},x_1+\frac{k_2}{2},y_1+\frac{l_2}{2})$ $\quad l_3= -0.0018223439$

$k_4= hf_1(t_1+h,x_1+k_3,y_1+l_3)$ $\quad k_4=0.0124692992$

$l_4= hf_2(t_1+h,x_1+k_3,y_1+l_3)$ $\quad l_4= -0.002633259$

Therefore, $x_2=x(0.2)=x_1+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$

$\mathbf{x_2=x(0.2)=0.014550059}$

$y_2=y(0.2)= y_1+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$

$\mathbf{y_2=y(0.2)= -0.001987737833}$

Again applying Runge Kutta for values at 0.3

$t_2=0.2$, $x_2=0.014550059$, $\quad y_2= -0.001987737833$

$k_1=hf_1(t_2,x_2,y_2)$ $\quad l_1=hf_2(t_2,x_2,y_2)$

$k_1=0.0123274229$ $\quad l_1= -0.002711238$

$k_2=hf_1(t_2+\frac{h}{2},x_2+\frac{k_1}{2},y_2+\frac{l_1}{2})$ $\quad k_2=0.0139744436$

$l_2=hf_2(t_2+\frac{h}{2},x_2+\frac{k_1}{2},y_2+\frac{l_1}{2})$ $\quad l_2= -0.0038084183$

$k_3=hf_1(t_2+\frac{h}{2},x_2+\frac{k_2}{2},y_2+\frac{l_2}{2})$ $\quad k_3=0.0134529706$

$l_3= hf_2(t_2+\frac{h}{2},x_2+\frac{k_2}{2},y_2+\frac{l_2}{2})$ $\quad l_3= -0.0039182613$

$k_4= hf_1(t_2+h,x_2+k_3,y_2+l_3)$ $\quad k_4=0.0148172856$

$l_4= hf_2(t_2+h,x_2+k_3,y_2+l_3)$ $l_4= -0.0050100058$

Therefore, $x_3=x(0.3)=x_2+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$

$\mathbf{x_3=x(0.3)=0.028216648}$

$y_3=y(0.3)= y_2+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$

$\mathbf{y_3=y(0.3)= -0.00585017166}$

Again applying Runge Kutta for values at 0.4

$t_3=0.3,\ x_3=0.028216648,$ $y_2= -0.00585017166$

$k_1=hf_1(t_3,x_3,y_3)$ $l_1=hf_2(t_3,x_3,y_3)$

$k_1=0.014721641$ $l_1= -0.005058312434$

$k_2=hf_1(t_3+\frac{h}{2},x_3+\frac{k_1}{2},y_3+\frac{l_1}{2})$ $k_2=0.0155354004$

$l_2=hf_2(t_3+\frac{h}{2},x_3+\frac{k_1}{2},y_3+\frac{l_1}{2})$ $l_2= -0.00627756081$

$k_3=hf_1(t_3+\frac{h}{2},x_3+\frac{k_2}{2},y_3+\frac{l_2}{2})$ $k_3=0.015210035$

$l_3= hf_2(t_3+\frac{h}{2},x_3+\frac{k_2}{2},y_3+\frac{l_2}{2})$ $l_3= -0.0062979743$

$k_4= hf_1(t_3+h,x_3+k_3,y_3+l_3)$ $k_4=0.015857029$

$l_4= hf_2(t_3+h,x_3+k_3,y_3+l_3)$ $l_4= -0.0074705221$

Therefore, $x_4=x(0.4)=x_3+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$

$\mathbf{x_4=x(0.4)=0.043561571}$

$y_4=y(0.4)= y_3+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$

$\mathbf{y_4=y(0.4)= -0.012130155}$

New Iterative Method (NIM)-

The corresponding integral equations are

$$x(t) =0+\frac{t^2}{2}+\int_0^t(-5x + 2y)dt$$

$$y(t)=0+\int_0^t(-2x - y)dt$$

Setting $x_{10}=\frac{t^2}{2}$, $y_{20}=0$ and

$N1(x10, y20) =\int_0^t(-5x + 2y)\,dt,$ $N_2(x_{10},y_{20})=\int_0^t(-2x - y)dt$

Following the NIM we obtain following approximations:

$x_{11} = N_1(x_{10},y_{20})=-\frac{5t^3}{6}$ $y_{21}=N_2(x_{10},y_{20})=-\frac{t^3}{3}$

$x_{12} = \frac{7}{8}t^4$ $y_{22}= \frac{t^4}{2}$

$x_{13} =-\frac{27}{40}t^5$ $y_{23}=-\frac{9}{20}t^5$

$x_{14} = \frac{33}{80}t^6$ $y_{24}=\frac{3}{10}t^6$

Therefore, the solution in series form is

$$x(t) = \frac{1}{2}t^2 - \frac{5}{6}t^3 + \frac{7}{8}t^4 - \frac{27}{40}t^5 + \frac{33}{80}t^6 + \cdots$$

$$y(t) = \frac{-1}{3}t^3 + \frac{1}{2}t^4 - \frac{9}{20}t^5 + \frac{3}{10}t^6 + \cdots$$

COMPARISON OF RESULTS

Table-1A

	Taylor's	Picard's	Runge-Kutta	NIM	Analytical
x(0.1)	0.004247829	0.004247829	0.004254166	0.004247829	0.004247809
x(0.2)	0.014543733	0.014543733	0.014550059	0.014543733	0.014541274
x(0.3)	0.028247962	0.028247962	0.028216648	0.028247962	0.028207590
x(0.4)	0.043844266	0.043844266	0.043561571	0.043844266	0.043553321

Table-1B

	Taylor's	Picard's	Runge-Kutta	NIM	Analytical
y(0.1)	-0.000287533	-0.000287533	-0.000283330	-0.000287533	-0.000287548
y(0.2)	-0.001991466	-0.001991466	-0.001987737	-0.001991466	-0.001993352
y(0.3)	-0.005824800	-0.005824800	-0.005850171	-0.005824800	-0.005855704
y(0.4)	-0.011912533	-0.011912533	-0.012130155	-0.011912533	-0.012134924

TABLES FOR ERRORS

Table-2A: Error using Taylor's Method

	Taylor's Method	Analytical Method	Absolute Error	Relative Error	Percentage Error
x(0.1)	0.004247829	0.004247809	.000000019	.000004684	.0004684
x(0.2)	0.014543733	0.014541274	.000002459	.000169104	.0169104
x(0.3)	0.028247962	0.028207590	.000040372	.001431245	.1431245
x(0.4)	0.043844266	0.043553321	.000290945	.006680202	.6680202

	Taylor's Method	Analytical Method	Absolute Error	Relative Error	Percentage Error
y(0.1)	-0.000287533	-0.000287548	.000000015	.000052165	.0052165
y(0.2)	-0.001991466	-0.001993352	.000001886	.000946144	.0946144
y(0.3)	-0.005824800	-0.005855704	.000030904	.005277589	.5277589
y(0.4)	-0.011912533	-0.012134924	.000222391	.018326525	1.832652

Table-2B: Error using Picard's Method

	Picard's	Analytical	Absolute Error	Relative Error	Percentage Error
x(0.1)	0.004247829	0.004247809	.000000019	.000004684	.0004684
x(0.2)	0.014543733	0.014541274	.000002459	.000169104	.0169104
x(0.3)	0.028247962	0.028207590	.000040372	.001431245	.1431245
x(0.4)	0.043844266	0.043553321	.000290945	.006680202	.6680202

	Picard's Method	Analytical Method	Absolute Error	Relative Error	Percentage Error
y(0.1)	-0.000287533	-0.000287548	.000000015	.000052165	.0052165
y(0.2)	-0.001991466	-0.001993352	.000001885	.000945843	.0945843
y(0.3)	-0.005824800	-0.005855704	.000030904	.005277589	.5277589
y(0.4)	-0.011912533	-0.012134924	.000222391	.018326525	1.832652

Table-2C: Error using Runge-Kutta Method

	Runge Kutta Method	Analytical Method	Absolute Error	Relative Error	Percentage Error
x(0.1)	0.004254166	0.004247809	.000006356	.001496512	.1496512
x(0.2)	0.014550059	0.014541274	.000008785	.000604142	.0604142
x(0.3)	0.028216648	0.028207590	.000009058	.000321119	.0321119
x(0.4)	0.043561571	0.043553321	.000008250	.000189422	.0189422

	Runge Kutta Method	Analytical Method	Absolute Error	Relative Error	Percentage Error
y(0.1)	-0.000283330	-0.000287548	.000004218	.014668855	1.4668855
y(0.2)	-0.001987737	-0.001993352	.000005614	.002816461	.28164610
y(0.3)	-0.005850171	-0.005855704	.000005532	.000944788	.09447881
y(0.4)	-0.012130155	-0.012134924	.000004769	.000392997	.03929979

Table-2D: Error using NIM

	NIM	Analytical Method	Absolute Error	Relative Error	Percentage Error
x(0.1)	0.004247829	0.004247809	.000000019	.000004684	.0004684
x(0.2)	0.014543733	0.014541274	.000002459	.000169104	.0169104
x(0.3)	0.028247962	0.028207590	.000040372	.001431245	.1431245
x(0.4)	0.043844266	0.043553321	.000290945	.006680202	.6680202

	NIM	Analytical Method	Absolute Error	Relative Error	Percentage Error
y(0.1)	-0.000287533	-0.000287548	.000000015	.000052165	.0052165
y(0.2)	-0.001991466	-0.001993352	.000001885	.000945843	.0945843
y(0.3)	-0.005824800	-0.005855704	.000030904	.005277589	.5277589
y(0.4)	-0.011912533	-0.012134924	.000222391	.018326525	1.8326525

4. CONCLUSION

In this study, we have done a comparative study of Taylor's Picard's Runge Kutta and New Iterative method (NIM). The values obtained in Taylor's, Picard's, Runge-Kutta and New Iterative Method are closer to Analytical method. But New Iterative Method has advantage over all other methods as calculation size is reduced. Therefore, NIM is efficient and convenient.

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