International Journal of Mathematical Archive-9(5), 2018, 49-53
IJMA Available online through www.ijma.info ISSN 2229 – 5046

# STUDY OF THE THERMAL STRATIFICATION EFFECTS ON AN UNSTEADY NATURAL CONVECTION FLOW PAST AN INFINITE VERTICAL ACCELERATED PLATE

G. NARSIMLU*

Deptartent of Mathematics,
Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad, 500075, India.

*(Received On: 02-04-18; Revised & Accepted On: 01-05-18)*

***ABSTRACT***

***A** problem of one-dimensional unsteady natural convection flow past an infinite vertical accelerated plate, immersed in a thermally stratified fluid is examined. The governing equations are solved by Galerkin finite element procedures for the Prandtl number is unity. The velocity and temperature profiles are presented graphically and the skinfriction and the Nusselt number are studied for various parameters like Prandtl number, thermal Grashof number, stratification parameter and time.*

***Keywords:*** *Galerkin finite element, Prandtl number, stratification parameter, vertical accelerated plate.*

## INTRODUCTION

Combined heat and mass transfer problems play an important role in engineering sciences like chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear power plants, steel rolling, gas turbines etc. Free convection and mass transfer flow through a porous medium past an infinite vertical plate with time dependent temperature and concentration was studied by Sattar [1]. Das *et.al* [2] worked out on an unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical plate with suction. Magyari et.al [3] describes the unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium. Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium were studied by B.S. Jaiswal and V.M. Soundalgekar[4]. Muthucumaraswamy, R. and Kumar, G.S[5] discussed the heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. S. Mukhopadhyay, G. C. Layek [6] studied the effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface. Unsteady MHD free convection past an impulsively started isothermal vertical plate with radiation and viscous dissipation were studied by Hawa Singh *et.al* [7]. Hitesh Kumar[8] described the Radiative heat transfer with hydro-magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux.

In this paper, it is proposed to study the unsteady natural convection flow past an accelerated vertical plate in a thermally stratified fluid. The governing equations are solved by Galerkin finite element method and velocity and temperature profiles are presented graphically.

## MATHEMATICAL ANALYSIS

In this paper we consider viscous incompressible fluid and also coordinate system. The $x^*$-axis is taken along the plate in the upward direction and $y^* -$ axis is taken normal to the plate. In this system consider the motion is one-dimensional and non-zero velocity component $u^*$. Initially the fluid and plate are at the same temperature. At time $t^* > 0$, the temperature of the plate is raised from $t_\infty^*$ the constant wall temperature $T_w^*$ and the plate is given an impulsive constant acceleration $A > 0$.

***Corresponding Author: G. Narsimlu*, Deptartent of Mathematics,***
***Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad, 500075, India.***

The governing equation of the flow modal is given by

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_\infty^*\right) \quad \ldots \quad (1)$$

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial T^{*2}}{\partial y^{*2}} - \gamma u^* \quad \ldots \quad (2)$$

Where $\gamma = \frac{\partial T^*}{\partial z^*} + \frac{g}{c_p}$. Here $\frac{\partial T^*}{\partial z^*}$ is the thermal stratification, $\frac{g}{c_p}$ is the pressure wok term. If $\gamma >, =, or < 0$ then environment is statistically stable, neutral or unstable. We consider the cases of stable and neutral conditions only.

And the initial and boundary conditions have been assumed:

$$\left.\begin{array}{ll} t^* \le 0 : u^* = 0, T^* = T_\infty^* & \forall y^* \\ t^* > 0 : u^* = At^*, T^* = T_w^* & at \;\; y^* = 0 \\ u^* \to 0 \;\; T^* \to T_\infty^* & as \;\; y^* \to \infty \end{array}\right\} \quad (3)$$

Introducing the following non- dimensional quantities,

$$\left.\begin{array}{l} u = \dfrac{u^*}{(Av)^{1/3}}, \Pr = \dfrac{\mu C_p}{k}, y = \dfrac{y^* A^{1/3}}{v^{2/3}}, t = \dfrac{t^* A^{2/3}}{v^{1/3}}, v = \dfrac{\mu}{\rho} \\ \theta = \dfrac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, Gr = \dfrac{g\beta\left(T_w^* - T_\infty^*\right)}{A}, S = \dfrac{\gamma v^{2/3}}{A^{1/3}\left(T_w^* - T_\infty^*\right)} \end{array}\right] \quad (4)$$

where $Gr$, Pr, $S$ and $\xi$ are the thermal Grashof number, Prandtl number, on-dimensional stratification parameter and dimensionless coordinate normal to the plate respectively.

With the help of non- dimensional quantities, equations (1) and (2) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta \quad (5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - Su \quad (6)$$

The corresponding dimensions less boundary condition are

$$\left.\begin{array}{ll} t \le 0 : u = 0, \theta = 0 & \forall y \\ t > 0 : u = t, \theta = 1 & at \;\; y = 0 \\ \quad u \to 0 \;\; \theta \to 0 & as \;\; y \to \infty \end{array}\right\} \quad (7)$$

**METHOD OF SOLUTION**

On solving the equations (5) and (6). By applying Galerkin finite element method for equation (5) over the element $(e)$, $\left(y_j \le y \le y_k\right)$ is:

$$\int_{y_j}^{y_k} \left\{ N^{(e)T}\left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} + P\right]\right\} dy = 0 \quad (8)$$

Where $P = (Gr)\theta$ integrating the first term in equation (8) by parts and neglecting the first term we get

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} - P\right)\right\} dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ = be the linear piecewise approximation solution over the element $(e)$ ($y_j \le y \le y_k$) where $N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix}, \phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T$ and $N_j = \dfrac{y_k - y}{y_k - y_j}, N_k = \dfrac{y - y_j}{y_k - y_j}$ are the basis functions. On simplifying we get

$$\frac{1}{l^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} = \frac{Pl^{(e)}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where dot are denotes differentiation with respect to $y$. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} = \frac{P}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{9}$$

Now put row corresponding to the node $i$ to zero, from equation (10) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{6}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] + \frac{1}{h^2}\left[-u_{i-1} + 2u_i - u_{i+1}\right] = P$$

Applying Crank – Nicholson method to the above equation, we get

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^j + A_5 u_i^j + A_6 u_{i+1}^j + P^* \tag{10}$$

where $A_1 = A_3 = 1 - 3r \quad A_4 = A_6 = 1 + 3r$
$A_2 = 4 + 6r \quad A_5 = 4 + 6r \quad P^* = 6Pk = 6kGr\theta_i^j$

Now from equation (6) following equation is obtained:

$$B_1\theta_{i-1}^{j+1} + B_2\theta_i^{j+1} + B_3\theta_{i+1}^{j+1} = B_4\theta_{i-1}^j + B_5\theta_i^j + B_6\theta_{i+1}^j + P^{**} \tag{11}$$

Where $B_1 = B_3 = \Pr - 3r,\ B_4 = B_6 = \Pr - 3r, P^{**} = 6P_1k = -6kSu\Pr$
$B_2 = 4\Pr + 6r, \quad B_5 = 4\Pr - 6r,$

Here, $r = \dfrac{k}{h^2}$ and $h, k$ are mesh size along the $y$ direction and the time direction respectively. Index $i$ refers to the space, and $j$ refers to the time. In the Equations (10) – (11), taking $i = 1,...,n$ and using boundary conditions (7), the following system of equations are obtained:

$$A_i X_i = B_i, \qquad i = 1,...,n \tag{12}$$

Where $A_i$ 's are matrix of order $n$ and $X_i, B_i$ 's column matrices having $n$ components. The solutions of above systems of equations are obtained by using the Thomas algorithm for velocity, temperature and concentration. Also the numerical solutions are obtained by executing the MATLAB-program with the smaller values of $h$ and $k$. No significant change was observed in $u$, and $\theta$ then the Galerkin finite element method is stable and convergent.

**SKIN FRICTION AND NUSSELT NUMBER**

The dimension less skin friction and Nusselt numbers are obtained as $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$ , $Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$

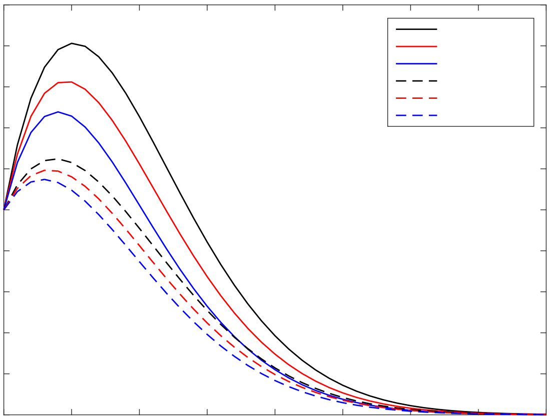

**Fig.1:** Velocity profile for various values of $Gr, S$

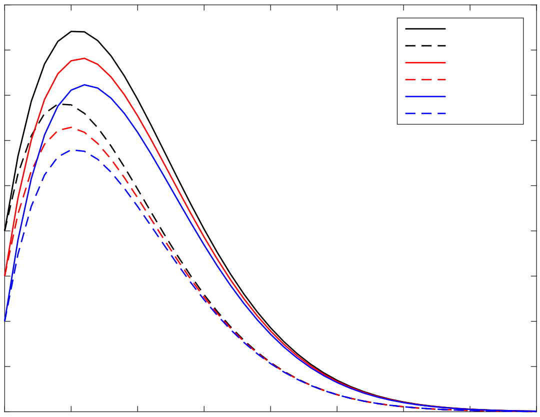

**Fig.2:** Velocity profile for various values of $Gr, S$

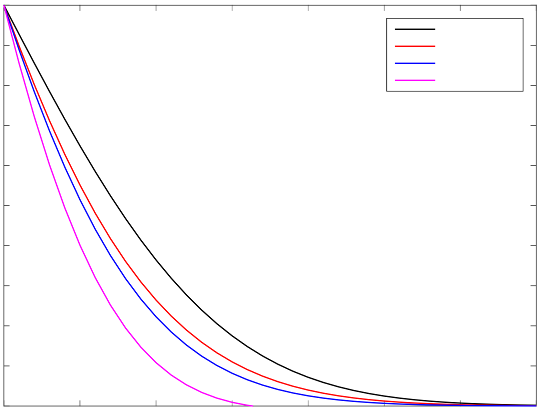

**Fig.3:** Temperature profile for various values of $Gr, S$

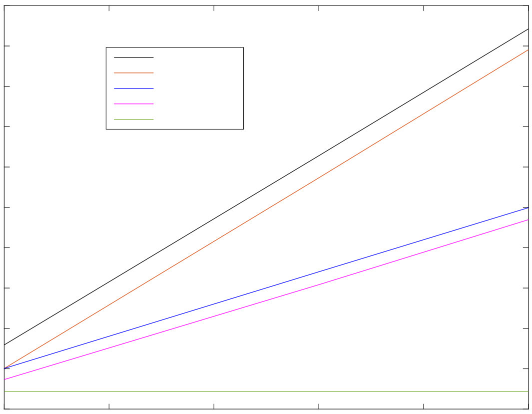

**Fig.4:** Skin-friction for various values of $Gr, S$

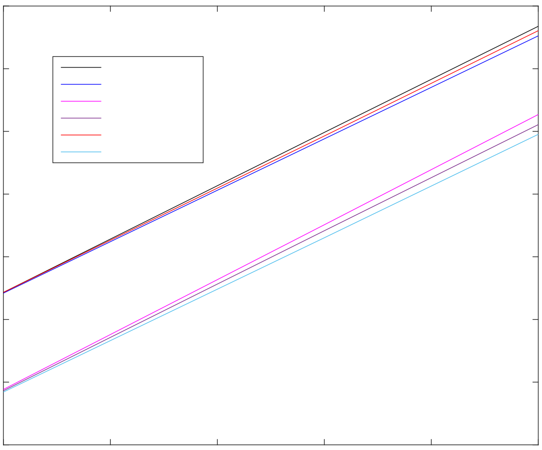

**Fig.5:** Nusselt for various values of $Gr,\ S$

| Table-1: Skin friction and Nusselt number for different parameters | | | | |
|---|---|---|---|---|
| $Gr$ | $S$ | $t$ | $\tau$ | $Nu$ |
| 10 | 0.5 | 0.4 | -2.45903 | 1.771171 |
| 10 | 0.5 | 0.8 | -1.77899 | 1.824546 |
| 10 | 1 | 0.4 | -2.41778 | 1.888546 |
| 10 | 1 | 0.8 | -1.71811 | 2.008796 |
| 5 | 0.5 | 0.4 | -0.90403 | 1.741421 |
| 5 | 0.5 | 0.8 | -0.23144 | 1.797046 |
| 5 | 1 | 0.4 | -0.89028 | 1.824546 |
| 5 | 1 | 0.8 | -0.20394 | 1.956296 |

**RESULTS AND DISCUSSION**

Figures 1 and 2 represents to the cross-sections of the vertical velocity profile $u$ against $y$ for various estimations of$S$, $Gr$ and $t$. From figures it is observed that thermal stratification has huge effect on the vertical velocity field. Increment of Grashof number $Gr$ and time t extends to an expansion of velocity which is obvious because more $Gr$ implies additionally heat and less density. When the value of stratification parameter S increases velocity is decreases. Which is because of the layering effect of thermal stratification as it acts like a resistive power and also the velocity increases with time t. Figure 3 illustrate the temperature profile $\theta$ against $y$ for different estimations of $Gr$ and $S$. It shows that the temperature decreases with the increase of stratification parameter $S$ and Grashof number$Gr$. Additionally the fall of temperature is quicker when S is made bigger alongside Gr. In figure 4 skinfriction is introduced for various values of Gr and S. From the figure clear distinction can be seen between the skinfrinction with and without thermal stratification. Without thermal stratification, skinfriction decreases continuously against time but in stratified fluid however initially decreases but as time progresses it gradually increases. Also it decreases when Gr increases and increase when S increases.

Nusselt number $Nu$ is exhibited in figure 5. Near the plate the heat transfer coefficient is infinite and without stratification parameter, it decreases consistently as time increases and it approaches to zero as $t \to \infty$. However, in stratified fluid it diminishes for smaller values $t$ and thereafter increases as time increases. Also Nusselt number increments when S and Gr increment. The values of skin friction and the Nusselt number are tabulated in table-1. When the values of $S$ are increased the values of skin friction and the Nusselt number are also increased. But if values of $Gr$and $t$ are increased the values of skin friction and Nusselt number are decreased.

**REFERENCES**


1. Satter, M.A., "Free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration'', *Ind.J.Pure Apple. Math., 23, pp. 759-766, 1994.*
2. Das, S.S. Shoo., S.K .and Dash. G.C, "Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction'', *Bull. Malays Math Science Soc. (2), 29, pp.33-42, 2006.*
3. Magyari E., Pop I., and Keller B.,"Unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium",*Transport in Porous Media 62, pp.233-249, 2006.*
4. B.S. Jaiswal and V.M. Soundalgekar, "Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium", *Heat and Mass Transfer, 37, pp.125-131, 2001.*
5. Muthucumaraswamy, R. and Kumar, G.S., "Heat and mass transfer effects on moving vertical plate in the presence of thermal radiation", *Theoret. Appl. Mech*., Vol.31, pp.35-46, 2004.
6. S. Mukhopadhyay, G. C. Layek "Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface", Int. J. Heat Mass Transfer 51, 2167–2178.2008.
7. Unsteady MHD Free Convection Past an Impulsively Started Isothermal Vertical Plate with Radiation and Viscous Dissipation were studied by Hawa Singh et.al [7]. FDMP, vol.10, no.4, pp.521-550, 2014.
8. Hitesh Kumar, Radiative heat transfer with hydro-magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux, Thermal Sci, 13(2)(2009), 163-169.


**Source of support: Nil, Conflict of interest: None Declared.**