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FUZZY IMPLICATIVE FILTERS OF LATTICE PSEUDO WAJSBERG ALGEBRAS

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ABSTRACT

In this paper, we introduce the notion of fuzzy implicative filter of lattice pseudo-Wajsberg algebra, and investigate some properties with illustrations. Further, we obtain some equivalent conditions of fuzzy implicative filter in lattice pseudo-Wajsberg algebra.

Keywords: *Wajsberg algebra; Pseudo-Wajsberg algebra; Implicative filter; Fuzzy implicative filter.*

Mathematical Subject classification 2010: *03E70, 03E72, 03G10.*

1. INTRODUCTION

Mordchaj Wajsbreg introduced the concept of Wajsberg algebras in 1935 and studied by Font, Rodriguez and Torrenns in [3]. In [3] defined lattice structure of Wajsberg algebras. Also, they [3] introduced the notion of an implicative filter of lattice Wajsberg algebras and discussed some of their properties. Basheer Ahamed and Ibrahim [1] introduced the definition of fuzzy implicative filters of lattice Wajsberg algebras and obtained some properties with illustrations. Pseudo-Wajsberg algebras are generalizations of Wajsberg algebras. Pseudo-Wajsberg algebras were introduced by Rodica Ceterchi [5] with the explicit purpose of providing a concept categorically equivalent to that of pseudo-MV algebras. Recently, the authors[4] introduced the notions of implicative filter of lattice pseudo Wajsberg algebra and discussed some of their properties. The concept of a fuzzy set (fuzzy subset) introduced by Zadeh [8] in 1965, provides a natural generalization for treating mathematically the fuzzy phenomena which exist pervasively in our real world and for building new branches of fuzzy Mathematics. In fuzzy set theory, the member of an element to a fuzzy set is a single value between 0 and 1.

The aim of this paper is to introduce the notion of fuzzy implicative filters of lattice pseudo -Wajsberg algebra, and we obtain some related properties and equivalent conditions. We show that characterization of fuzzy implicative filter of lattice pseudo-Wajsberg algebra.

2. PRELIMINARIES

In this section, we recall some basic definitions and their properties which are helpful to develop the main results.

Definition 2.1[2]: An algebra $(A, \to, ^{-}, 1)$ with a binary operation "$\to$"and a quasi complement "$^{-}$" is called a Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \to x = x$
(ii) $(x \to y) \to y = (y \to x) \to x$
(iii) $(x \to y) \to \big((y \to z) \to (x \to z)\big) = 1$
(iv) $(x^{-} \to y^{-}) \to (y \to x) = 1$.

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Definition 2.2[2]: A Wajsberg algebra A is called a lattice Wajsberg algebra if it satisfies the following axioms for all $x, y \in A$,

(i) The partial ordering "$\leq$" on a lattice Wajsberg algebra A, such that $x \leq y$ if and only if $x \to y = 1$
(ii) $(x \vee y) = (x \to y) \to y$
(iii) $(x \wedge y) = \left((x^- \to y^-) \to y^-\right)^-$.

Definition 2.3[5]: An algebra $(A, \to, \rightsquigarrow, ^-, ^\sim, 1)$ with a binary operations "$\to$", "$\rightsquigarrow$" and quasi complements "$^-$", "$^\sim$" is called a pseudo-Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) (a) $1 \to x = x$
    (b) $1 \rightsquigarrow x = x$
(ii) $(x \rightsquigarrow y) \to y = (y \rightsquigarrow x) \to x = (y \to x) \rightsquigarrow x = (x \to y) \rightsquigarrow y$
(iii) (a) $(x \to y) \to \left((y \to z) \rightsquigarrow (x \to z)\right) = 1$
    (b) $(x \rightsquigarrow y) \rightsquigarrow \left((y \rightsquigarrow z) \to (x \rightsquigarrow z)\right) = 1$
(iv) $1^- = 1^\sim = 0$
(v) (a) $(x^- \rightsquigarrow y^-) \to (y \to x) = 1$
    (b) $(x^\sim \to y^\sim) \rightsquigarrow (y \rightsquigarrow x) = 1$
(vi) $(x \to y^-)^\sim = (y \rightsquigarrow x^\sim)^-$.

Definition 2.4[5]: An algebra $(A, \to, \rightsquigarrow, ^-, ^\sim, 1)$ is called a lattice pseudo-Wajsberg algebra if it satisfies the following axioms for all $x, y \in A$,

(i) A partial ordering "≤" on a lattice pseudo-Wajsberg algebra A, such that x≤ y if and only if $x \to y = 1 \Leftrightarrow x \rightsquigarrow y = 1$.
(ii) $x \vee y = (x \to y) \rightsquigarrow y = (y \to x) \rightsquigarrow x = (x \rightsquigarrow y) \to y = (y \rightsquigarrow x) \to x$
(iii) $x \wedge y = (x \rightsquigarrow (x \to y)^\sim)^- = \left((x \to y) \to x^-\right)^\sim$
$= (y \rightsquigarrow (y \to x)^\sim)^- = \left((y \to x) \to y^-\right)^\sim$
$= (y \to (y \rightsquigarrow x)^-)^\sim = \left((y \rightsquigarrow x) \rightsquigarrow y^\sim\right)^-$
$= (x \to (x \rightsquigarrow y)^-)^\sim = \left((x \rightsquigarrow y) \rightsquigarrow x^\sim\right)^-$

Proposition 2.5[2]: A Wajsberg algebra $(A, \to, ^-, 1)$ satisfies the following axioms for all $x, y \in A$,

(i) $x \to x = 1$
(ii) If $x \to y = y \to x = 1$, then $x = y$
(iii) $x \to 1 = 1$
(iv) $x \to (y \to x) = 1$
(v) If $x \to y = y \to z = 1$, then $x \to z = 1$
(vi) $(x \to y) \to \left((z \to x) \to (z \to y)\right) = 1$
(vii) $x \to (y \to z) = y \to (x \to z)$.

Proposition 2.6[5]: A lattice pseudo-Wajsberg algebra $(A, \to, \rightsquigarrow, ^-, ^\sim, 1)$ satisfies the following axioms for all $x, y \in A$,

(i) $x \to x = 1, x \rightsquigarrow x = 1$
(ii) $x \to (y \rightsquigarrow x) = 1,\ x \rightsquigarrow (y \to x) = 1$
(iii) $x \leq y \Rightarrow z \to x \leq z \to y\,;\ z \rightsquigarrow x \leq z \rightsquigarrow y$
(iv) $x \leq y \Rightarrow y \to z \leq x \to z\,;\ y \rightsquigarrow z \leq x \rightsquigarrow z$
(v) $x \leq y \to x\,;\ x \leq y \rightsquigarrow x$
(vi) $x \to y \leq (y \to z) \rightsquigarrow (x \to z)\,;\ x \rightsquigarrow y \leq (y \rightsquigarrow z) \to (x \rightsquigarrow z)$
(vii) $x \to y \leq (z \to x) \to (z \to y)\,;\ x \rightsquigarrow y \leq (z \rightsquigarrow x) \rightsquigarrow (z \rightsquigarrow y)$
(viii) $x \to (y \rightsquigarrow z) = y \rightsquigarrow (x \to z)$.

Definition 2.7[2]: Let A be Wajsberg algebra. A non-empty subset F of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $1 \in F$
(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$.

Definition 2.8[4]: Let A be latticepseudo-Wajsbergalgebra. A non empty subset F of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $1 \in F$
(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$
(iii) $x \in F$ and $x \rightsquigarrow y \in F$ imply $y \in F$.

Proposition 2.9[4]: Let F be any implicative filter of lattice pseudo-Wajsbergalgebra of A satisfying the axiom
$x \rightsquigarrow (y \rightsquigarrow z) = y \rightsquigarrow (x \rightsquigarrow z)$for all $x, y \in A$.

Definition 2.10[1]: Let A be a set. A function μ: $A \rightarrow [0, 1]$ is called a fuzzy subset on A, for each $x \in A$, the value of $\mu(x)$ describes a degree of membership of x in μ.

Definition 2.11[1]: Let A be lattice Wajsberg algebra. A non-empty fuzzy subset μ of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $\mu(1) \geq \mu(x)$
(ii) $\mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}$.

3. FUZZY IMPLICATIVE FILTER OF LATTICE PSEUDO WAJSBERG ALGEBRA

In this section, we introduce fuzzy implicative filter of lattice pseudo-Wajsberg algebra, and investigate some properties with illustrations.

Definition 3.1: Let A be lattice pseudo-Wajsberg algebra. A non empty fuzzy subset μ of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $\mu(1) \geq \mu(x)$
(ii) $\mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}$
(iii) $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$.

Example 3.2: Consider a set $A = \{0, a, b, c, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 < a < b,\ c < 1$ and the binary operations $" \rightarrow "," \rightsquigarrow "$ and quasi complements $" ^{-} ", "\ \ "$ given by the following tables (1), (2), (3) and (4).

x	x^{-}
0	1
a	c
b	c
c	0
1	0

Table-(1)

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	c	c	1	c	1
c	0	b	b	1	1
1	0	a	b	c	1

Table-(2)

x	$x^{\sim}$
0	1
a	b
b	0
c	b
1	0

Table-(3)

$\rightsquigarrow$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	1	1	1
b	0	c	1	c	1
c	b	b	b	1	1
1	0	a	b	c	1

Table-(4)

Consider the fuzzy subset μ on A as,$\mu(x) = \begin{cases} 0.5 & if\ \ x = 1 \\ 0.2 & Otherwise \end{cases}$ for all$x \in A$. Then, we have μ is fuzzy implicative filter of lattice pseudo-Wajsberg algebra A.

Example 3.3: Consider a set $A = \{0, a, b, c, d, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 < a,\ b < c < d < 1$ and the binary operations $" \rightarrow "," \rightsquigarrow "$ and quasi complements $" ^{-} ", "\ \ "$ given by the following tables (5), (6), (7) and (8).

x	x^{-}
0	1
a	d
b	d
c	d
d	0
1	0

Table -(5)

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	d	1	1	1
b	d	d	1	1	1	1
c	d	d	d	1	1	1
d	0	a	b	c	1	1
1	0	a	b	c	d	1

Table-(6)

x	$x^{\sim}$
0	1
a	c
b	c
c	c
d	c
1	0

Table-(7)

$\rightsquigarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	c	1	1	1
b	c	c	1	1	1	1
c	c	c	c	1	1	1
d	c	c	c	c	1	1
1	0	a	b	c	d	1

Table-(8)

Consider the fuzzy subset μ on A as, $\mu(x) = \begin{cases} 0.6 & if \ \ x \in \{1, a, b, c\} \\ 0.4 & if \ \ x \in \{0, d\} \end{cases}$ for all $x \in A$. Then, we have μ is not fuzzy implicative filter of lattice pseudo-Wajsberg algebra A.

Since, we have $\mu(d) = 0.4$, but $\min\{\mu(a \to d), \mu(a)\} \geq \min\{\mu(1), \mu(a)\} = 0.6$.

Thus, we have $\mu(d) \ngeq \min\{\mu(a \to d), \mu(a)\}$.

Proposition 3.4: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra, then for all $x, y \in A$, $x \leq y$ implies $\mu(x) \leq \mu(y)$.

Proof: Let $x, y \in A$ be such that $x \leq y$ if and only if $x \to y = 1$ and $x \rightsquigarrow y = 1$ for all $x, y \in A$,

We have $\mu(y) = \mu(1 \to y) = \mu(1 \rightsquigarrow y)$
$\geq \mu\big((x \to y) \rightsquigarrow y\big) = \mu\big((y \rightsquigarrow x) \to x\big) \geq \mu(x)$ [From (ii) of definition 2.4]
Thus, we have $x \leq y$ implies $\mu(x) \leq \mu(y)$.

Proposition 3.5: A fuzzy subset μ of A is an implicative filter of lattice pseudo-Wajsberg algebra A if and only if for all $x, y, z \in A, x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$. (1)

Proof: Let μ be fuzzy subset of A and satisfying the condition (1). Since, $x \leq x \to 1$ and $x \leq x \rightsquigarrow 1$ for all $x \in A$, $\mu(1) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$.

Thus, we have $\mu(1) \geq \mu(x)$.

Since, $x \to y \leq x \to y$ and $x \rightsquigarrow y \leq x \rightsquigarrow y$

Hence, we have $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$ and $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$.

Therefore, μ is a fuzzy implicative filter of lattice pseudo-Wajsberg algebra.

Conversely, if fuzzy subset μ is an implicative filter of lattice pseudo-Wajsberg algebra A.

Let $x, y, z \in A$, if $x \leq y \to z$ and $x \leq y \rightsquigarrow z$, then $\mu(x) \leq \mu(y \to z)$ and $\mu(x) \leq \mu(y \rightsquigarrow z)$ [From Proposition 3.4.]

We have $\mu(z) \geq \min\{\mu(y \to z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$ and
$\mu(z) \geq \min\{\mu(y \rightsquigarrow z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$.

Hence, $x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y, z \in A$.

Proposition 3.6: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Then the following are equivalent for all $x, y, z \in A$.
(i) (a) $\mu(x \to z) \geq \min\{\mu\big(x \to (y \to z)\big), \mu(x \to y)\}$
(b) $\mu(x \rightsquigarrow z) \geq \min\{\mu\big(x \rightsquigarrow (y \rightsquigarrow z)\big), \mu(x \rightsquigarrow y)\}$
(ii) (a) $\mu(x \to y) \geq \mu\big(x \to (x \to y)\big)$
(b) $\mu(x \rightsquigarrow y) \geq \mu\big(x \rightsquigarrow (x \rightsquigarrow y)\big)$
(iii) (a) $\mu\big((x \to y) \to (x \to z)\big) \geq \mu\big(x \to (y \to z)\big)$
(b) $\mu\big((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z)\big) \geq \mu\big(x \rightsquigarrow (y \rightsquigarrow z)\big)$.

Proof:

(i)$(a) \Rightarrow$ (ii)(a)

If (i) (a) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*. Let $z = y$ and $y = x$ in (i)(a).

We have $\mu(x \to y) \geq \min\{\mu(x \to (x \to y)), \mu(x \to x)\}$
$= \min\{\mu(x \to (x \to y)), \mu(1)\}$
Therefore, $\mu(x \to y) \geq \mu(x \to (x \to y))$.

(i)(b) $\Rightarrow$ (ii)(b)

We have $\mu(x \rightsquigarrow y) \geq \min\{\mu(x \rightsquigarrow (x \rightsquigarrow y)), \mu(x \rightsquigarrow x)\} = \min\{\mu(x \rightsquigarrow (x \rightsquigarrow y)), \mu(1)\}$
Therefore, $\mu(x \rightsquigarrow y) \geq \mu(x \rightsquigarrow (x \rightsquigarrow y))$.

(ii)(a) $\Rightarrow$ (iii)(a)

If (ii) (a) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*.
Let $x \to (y \to z) \leq x \to ((x \to y) \to (x \to z))$ [From (vii) of proposition 2.6]

It follows that $\mu((x \to y) \to (x \to z)) = \mu(x \to ((x \to (y \to z)))$ [From (vii) of proposition 2.5]

$$\geq \mu\left(x \to \left(x \to ((x \to y) \to z)\right)\right)$$
$$= \mu\left(x \to ((x \to y) \to (x \to z))\right) \geq \mu(x \to (y \to z))$$

(ii)(b) $\Rightarrow$ (iii)(b)

If (ii) (b) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*.
Let $x \rightsquigarrow (y \rightsquigarrow z) \leq x \rightsquigarrow ((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))$ [From (vii) of proposition 2.6]

It follows that $\mu((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z)) = \mu(x \rightsquigarrow ((x \rightsquigarrow (y \rightsquigarrow z)))$ [From proposition 2.9]

$$\geq \mu\left(x \rightsquigarrow \left(x \rightsquigarrow ((x \rightsquigarrow y) \rightsquigarrow z)\right)\right)$$
$$= \mu\left(x \rightsquigarrow ((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))\right) \geq \mu(x \rightsquigarrow (y \rightsquigarrow z))$$

(iii)(a) $\Rightarrow$ (i)(a)

If (iii) (a) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*.
Let $\mu(x \to z) \geq \min\{\mu(x \to y) \to (x \to z), \mu(x \to y)\}$
Therefore, $\mu(x \to z) \geq \min\{\mu(x \to (y \to z)), \mu(x \to y)\}$

(iii)(b) $\Rightarrow$ (i)(b)

If (iii) (b) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*.
Let $\mu(x \rightsquigarrow z) \geq \min\{\mu(x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z), \mu(x \rightsquigarrow y)\}$
Therefore, $\mu(x \rightsquigarrow z) \geq \min\{\mu(x \rightsquigarrow (y \rightsquigarrow z)), \mu(x \rightsquigarrow y)\}$.

Proposition 3.7: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A* if and only if satisfies the following axioms for all $x, y \in A$.
(i) $\mu(x) \geq min\{\mu(y), \mu(x \to y)\}$; (ii) $\mu(x) \geq min\{\mu(y), \mu(x \rightsquigarrow y)\}$.

Proof: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra *A*. For any $x, y \in A$,
Let $y \to 1 \leq (x \to y) \to (x \to 1)$ [From (vii) of proposition 2.6]
Thus, $\mu(y \to 1) \leq \mu((x \to y) \to (x \to 1))$ [From proposition 3.4]

Consider $\mu(x \to 1) \geq \min\{\mu((x \to y) \to (x \to 1)), \mu(x \to y)\}$ [From (ii) of definition 3.1]

Thus, $\mu(x \to 1) \geq \min\{\mu(y \to 1), \mu(x \to y)\}$ [From (vii) of proposition 2.6]

We have, $\mu(x \to 1) = \mu(x)$and $\mu(y \to 1) = \mu(y)$
Therefore, $\mu(x) \geq \min\{\mu(y), \mu(x \to y)\}$

Similarly, to prove $\mu(x) \geq \min\{\mu(y), \mu(x \rightsquigarrow y)\}$ for all $x, y \in A$.

The converse part is straight forward.

Proposition 3.8: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Then the following are equivalent for all $x, y, z \in A$.

(i) (a) $\mu(x) \geq \min\left\{\mu\left(z \to \left((x \to y) \to x\right)\right),\ \mu(z)\right\}$

(b) $\mu(x) \geq \min\left\{\mu\left(z \rightsquigarrow \left((x \rightsquigarrow y) \rightsquigarrow x\right)\right), \mu(z)\right\}$

(ii) (a) $\mu(x) \geq \mu\left((x \to y) \to x\right)$

(b) $\mu(x) \geq \mu\left((x \rightsquigarrow y) \rightsquigarrow x\right)$

(iii) (a) $\mu(x) = \mu\left((x \to y) \to x\right)$

(b) $\mu(x) = \mu\left((x \rightsquigarrow y) \rightsquigarrow x\right)$.

Proof:

(i)(a) $\Rightarrow$ **(ii)(a)**

If (i)(a) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Let z =1 in (i)(a)

We have, $\mu(x) \geq \min\left\{\mu\left(1 \to \left((x \to y) \to x\right)\right), \mu(1)\right\}$
$=\mu\left((x \to y) \to x\right)$

(i)(b) $\Rightarrow$ **(ii)(b)**

If (i)(b) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Let z =1 in (i)(b)

We have $\mu(x) \geq \min\left\{\mu\left(1 \rightsquigarrow \left((x \rightsquigarrow y) \rightsquigarrow x\right)\right), \mu(1)\right\}$
$= \mu\left((x \rightsquigarrow y) \rightsquigarrow x\right)$

(ii)(a) $\Rightarrow$ **(iii)(a)**

If (ii)(a) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A.

Let $x \leq (x \to y) \to x$. We have $\mu(x) \leq \mu\left((x \to y) \to x\right)$ [From proposition 3.4]

It follows from (ii) (a) that $\mu(x) = \mu\left((x \to y) \to x\right)$

(ii)(b) $\Rightarrow$ **(iii)(b)**

If (ii)(b) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A.

Let $x \leq (x \rightsquigarrow y) \rightsquigarrow x$. We have $\mu(x) \leq \mu\left((x \rightsquigarrow y) \rightsquigarrow x\right)$ [From proposition 3.4]

It follows from (ii)(b) that $\mu(x) = \mu\left((x \rightsquigarrow y) \rightsquigarrow x\right)$

(iii) (a) $\Rightarrow$ **(i)(a)**

If (iii)(a) holds and let μ be a fuzzy implicative filter of lattice pseudo Wajsberg algebra A.

We have, $\mu((x \to y) \to x) \geq \min\left\{\mu\left(z \to \left((x \to y) \to x\right)\right), \mu(z)\right\}$ [From (ii) of definition 3.1]

Combining (iii)(a), we have $\mu(x) \geq \min\left\{\mu\left(z \to \left((x \to y) \to x\right)\right), \mu(z)\right\}$

Hence, $\mu(x) \geq \min\left\{\mu\left(z \to \left((x \to y) \to x\right)\right),\ \mu(z)\right\}$

Similarly, we show **(iii) (b)** $\Rightarrow$ **(i)(b).**

Proposition 3.9: Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Then μ satisfies the following axioms for all $x, y, z \in A$, (i) $\mu\left((y \to x) \rightsquigarrow x\right) \geq \mu\left((x \to y) \to y\right)$; (ii) $\mu\left((y \rightsquigarrow x) \to x\right) \geq \mu\left((x \rightsquigarrow y) \rightsquigarrow y\right)$.

Proof: If (i) holds and let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. From (vii) and (viii) of proposition 2.6, we have,$(x \to y) \to y \leq (y \to x) \rightsquigarrow \left((x \to y) \to x\right) = (x \to y) \to \left((y \to x) \rightsquigarrow x\right)$

On other hand, from (v) and (iv) of proposition 2.6 we have, $x \leq (y \to x) \rightsquigarrow x$,$\left((y \to x) \rightsquigarrow x\right) \to y \leq x \to y$,
$(x \to y) \to \left((y \to x) \rightsquigarrow x\right) \leq \left(\left((y \to x) \rightsquigarrow x\right) \to y\right) \to \left((y \to x) \rightsquigarrow x\right)$
Hence,$(x \to y) \to y \leq \left(\left((y \to x) \rightsquigarrow x\right) \to y\right) \to \left((y \to x) \rightsquigarrow x\right)$.

We have, $\mu\big((x \to y) \to y\big) \le \mu\Big(\Big(\big((y \to x) \rightsquigarrow x\big) \to y\Big) \to \big((y \to x) \rightsquigarrow x\big)\Big)$[From proposition 3.4]

From (ii) (a) of proposition (3.8), we have $\mu\Big(\Big(\big((y \to x) \rightsquigarrow x\big) \to y\Big) \to \big((y \to x) \rightsquigarrow x\big)\Big) \le \mu\big((y \to x) \rightsquigarrow x\big)$.
Hence, $\mu((y \to x) \rightsquigarrow x) \ge \mu\big((x \to y) \to y\big)$.
Similarly, we prove $\mu\big((y \rightsquigarrow x) \to x\big) \ge \mu\big((x \rightsquigarrow y) \rightsquigarrow y\big)$.

Proposition 3.10: A fuzzy subset μ of A is an implicative filter of lattice pseudo-Wajsberg algebra A if and only if for all $x, y \in A$, (i) $\mu(x) = \mu\big((x \to y) \to x\big)$; (ii) $\mu(x) = \mu\big((x \rightsquigarrow y) \rightsquigarrow x\big)$.

Proof:
(i) Let μ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Let $x, y \in A$.
Consider $x \le (x \to y) \to x$ and $\mu$is a fuzzy implicative filter,
We have, $\mu(x) \le \mu\big((x \to y) \to x\big)$ [From proposition 3.4]
Since μ is a fuzzy implicative filter,

We have $\mu(x) \ge \min\Big\{\mu\Big(1 \to \big((x \to y) \to x\big)\Big), \mu(1)\Big\} = \mu\big((x \to y) \to x\big)$. Hence, $\mu(x) = \mu\big((x \to y) \to x\big)$.

(ii) Letμ be a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A. Let $x, y \in A$. Consider $x \le (x \rightsquigarrow y) \rightsquigarrow x$ and μ is a fuzzy implicative filter, from proposition 3.4, we have $\mu(x) \le \mu\big((x \rightsquigarrow y) \rightsquigarrow x\big)$. Since μ is a fuzzy implicative filter, $\mu(x) \ge \min\Big\{\mu\Big(1 \to \big((x \rightsquigarrow y) \rightsquigarrow x\big)\Big), \mu(1)\Big\} = \mu\big((x \rightsquigarrow y) \rightsquigarrow x\big)$.
Hence, $\mu(x) = \mu\big((x \rightsquigarrow y) \rightsquigarrow x\big)$.

Conversely, (i) and (ii) holds. Clearly, $\mu(1) \ge \mu(x)$ for all $x \in A$.

We have $\mu(y) = \mu\big((y \to x) \to y\big) \ge \min\{\mu\big((x \to y) \to x\big), \mu(x)\}$
$\mu(y) \ge \min\{\big(\mu(x \to y), \mu(x)\big)\}$ for all $x, y \in A$.
$\mu(y) = \mu\big((y \rightsquigarrow x) \rightsquigarrow y\big) \ge \min\Big\{\Big(\mu\big((x \rightsquigarrow y) \rightsquigarrow x\big), \mu(x)\Big)\Big\}$
$\mu(y) \ge \min\{\big(\mu(x \rightsquigarrow y), \mu(x)\big)\}$ for all $x, y \in A$.
Hence, μ is a fuzzy implicative filter of lattice pseudo-Wajsberg algebra A.

4. CONCLUSION

In this paper, we have introduced the notion of fuzzy implicative filter of lattice pseudo-Wajsberg algebra. The properties and equivalent conditions of fuzzy implicative filter of lattice pseudo-Wajsberg algebra are discussed. Further, we extend this idea as intuitionistic fuzzy implicative filter of lattice pseudo-Wajsberg algebra.

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