International Journal of Mathematical Archive-9(5), 2018, 38-41
Available online through www.ijma.info ISSN 2229 – 5046

ON NANO gp^*-CLOSED SETS

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(Received On: 21-03-18; Revised & Accepted On: 02-05-18)

ABSTRACT

In this paper we introduce a new class of sets called nano gp^*-closed sets in nano topological spaces. Also we discuss some of their properties and investigate the relations between the associated nano topology.

Key words and phrases: *nano pre-closed sets, nano g^*-closed sets, nano gp-closed sets and nano gp^*-closed sets.*

2010 Mathematics Subject Classification: *5AC05, 5AC10, 54B05.*

1. INTRODUCTION

Lellis Thivagar *et al.* [3] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

The aim of this paper is to continue the study of nano gp^*-closed sets thereby contributing new innovations and concepts in the field of topology through analytical as well as research works.

Throughout this paper $(U, \tau_R(X))$ represent non empty nano topological spaces on which no separation axioms are assumed, unlesses otherwise mentioned.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ and (V, σ) (or X and Y) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, Ncl(H) and Nint(H) denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1: [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq G$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x\in U} \{R(x): R(x) \cap X \neq \varphi\}$.
(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

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Proposition 2.2: [3] If (U; R) is an approximation space and X; Y U; then
1. $L_R(X) \subseteq X \subseteq G_R(X)$;
2. $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq G_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq G_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_RU_R(X) = L_RU_R(X) = U_R(X)$;
10. $L_RL_R(X) = U_RL_R(X) = L_R(X)$.

Definition 2.3: [3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq G$. Then by the Property 2.2, R(X) satisfies the following axioms:
1. U and $\varphi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4: [3] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \varphi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5: [3] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq G$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by Nint(H).

That is, Nint(H) is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(H).

That is, Ncl(H) is the smallest nano closed set containing H.

Definition 2.6: [3] A subset H of a nano topological space $(U, \tau_R(X))$ is called;
1. nano pre open set if $H \subseteq N\,int(Ncl(H))$.
2. nano semi open set if $H \subseteq Ncl(Nint(H))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.7: A subset H of a nano topological space $(U, \tau_R(X))$ is called;
1. nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
2. nano gp-closed set [2] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
3. nano g^*-closed set [5] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is g-open.

The complements of the above mentioned sets is called their respective open sets.

3. ON NANO gp*-CLOSED SETS

Definition 3.1: A subset H of a space $(U, \tau_R(X))$ is nano gp*-closed set if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is gp-open.

The complement of nano gp*-open if $H^c = U - H$ is nano gp*-closed.

Example 3.2: Let U = {a, b, c, d} with U_R ={{a}, {b, c}, {d}} and X = {b, d}.

Then the nano topology $\tau_R(X) = \{\varphi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.
1. then {a} is nano gp*-closed set.
2. then {b, c} is nano gp*-open set.

Theorem 3.3: In a space $(U, \tau_R(X))$, every nano closed set is nano gp*-closed.

Proof: Let H be any nano closed set and G be any nano gp-open set containing $Ncl(H) \subseteq H = G$. Hence H is nano gp*-closed set.

Theorem 3.4: In a space $(U, \tau_R(X))$, every nano g*-closed set is nano gp*-closed set.

Proof: Let H be any nano g*-closed set in U and G be any nano g-open set containing H.

Since any nano g-open set is nano gp-open, Therefore Ncl(H) ⊆ G. Hence H is nano gp*-closed set.

Theorem 3.5: In a space $(U, \tau_R(X))$, every nano pg-closed set is nano gp*-closed set.

Proof: Let H be any nano pg-closed set in U and G be nano pre-open set containing H.

Since every nano pre-open set is nano gp-open set, we have Npcl(H) ⊆ Ncl(H) ⊆ G.

Therefore Ncl(H) ⊆G. Hence H is nano gp*-closed set.

Theorem 3.6: If H and K are nano gp*-closed sets in U then H ∪ K is nano gp*-closed set.

Proof: Let H and K are nano gp*-closed sets in U and G be any nano gp-open set containing H and K. Therefore Ncl(H) ⊆ G, Ncl(K) ⊆ G. Since H ⊆ G, K ⊆ G then H ∪ K ⊆ G. Hence Ncl(H ∪ K) = Ncl(H) ∪ Ncl(K) ⊆ K. Therefore H ∪ K is nano gp*-closed set.

Theorem 3.7: If a set H is nano gp*-closed set iff Ncl(H) − H contains no non empty nano gp-closed set.

Proof:
Necessity: Let F be a nano gp-closed set in U such that F ⊆ Ncl(H). Then H ⊆ U − F. Since H is nano gp*-closed set and U − F is nano gp-open then

Ncl(H) ⊆ U − F . (i.e.) F ⊆ U − Ncl(H). So F ⊆ (U − Ncl(H)) ∩ (Ncl(H) − H). Therefore F = φ.

Sufficiency: Let us assume that Ncl(H) − H contains no non empty nano semi-closed set. Let G ⊆ H, G is nano semi open. Suppose that Ncl(H) is not contained in G, Ncl(H)∩G^c is a nonempty nano gp-closed set of Ncl(H)−H which is contradiction. Therefore Ncl(H) ⊆ G. Hence H is nano gp*-closed.

Theorem 3.8: The intersection of any two subsets of nano gp*-closed sets in U is nano gp*-closed set in U.

Proof: Let H and K are any two sub sets of nano gp*-closed sets. H ⊆ G, G is any nano gp-open and K ⊆ G, G is nano gp-open. Then Ncl(H) ⊆ G, Ncl(K) ⊆ G, therefore Ncl(H ∩K) ⊆ G, G is nano gp-open in U. Since H and K are nano gp*-closed set, Hence H ∩ K is a nano gp*-closed set.

Theorem 3.9: If A is nano gp*-closed set in X and H ⊆ K ⊆ Ncl(H), Then K is nano gp*-closed set.

Proof: Since K ⊆ Ncl(H), we have Ncl(K) ⊆ Ncl(H) then Ncl(K)−K ⊆ Ncl(H)− H. By Theorem 4.2, Npcl(H) − H contains no non empty nano gp-closed set. Hence Ncl(K) − K contains no non empty nano gp-closed set. Therefore K is nano gp*-closed set.

Theorem 3.10: If H ⊆ V ⊆ U and suppose that H is nano gp*-closed set in U then H is nano gp*-closed set relative to V.

Proof: Given that H ⊆ V ⊆ U and H is nano gp*-closed set in X. To prove that H is nano gp*-closed set relative to V. Let us assume that H ⊆ V ∩ G, where G is nano gp-open in X. Since H is nano gp*-closed set, H ⊆ G implies Ncl(H) ⊆ G. It follows that V ∩ Ncl(A) ⊆ V ∩ G. That is H is nano gp*-closed set relative to V.

Theorem 3.11: If H is both nano gp-open and nano gp*-closed set in U, then H is nano gp-closed set.

Proof: Since H is nano gp-open and nano gp*-closed in U, Ncl(H) ⊆ G. But always H⊆ Ncl(H).
Therefore H = Ncl(H). Hence H is nano gp-closed set.

Theorem 3.12: If H and K are nano gp*-open sets in a space X. Then H ∩ K is also nano gp*-open set in X.

Proof: If H and K are nano gp*-open sets in a space X. Then H^c and K^c are nano gp*-closed sets in a space U. By Theorem 4.1 $H^c \cup K^c$ is also nano gp*-closed set in U. (i.e.) $H^c \cup K^c = (H \cap K)^c$ is a nano gp*-closed set in X. Therefore H ∩ K is nano gp*-open set in X.

Remark 3.13: The union of two nano gp*-open sets but not a nano gp*-open set in X.

Theorem 3.14: If Nint(K) ⊆ K ⊆ H and if H is nano gp*-open in U, then K is nano gp*-open in X.

Proof: Suppose that Nint(K) ⊆ K ⊆ H and H is nano gp*-open in U then $H^c \subseteq K^c \subseteq Ncl(H^c)$. Since H^c is nano gp*-closed in U, we have K is nano gp*-open in X.

Theorem 3.15: A set H is nano gp*-open if and only if F ⊆ Nint(H) where F is nano gp-closed and F ⊆ H.

Proof: If F ⊆ Nint(H) where F is nano gp-closed and F ⊆ H. Let $H^c \subseteq P$ where

$P= F^c$ is nano gp-open. Then $P^c \subseteq H$ and $P^c \subseteq Nint(H)$. Then we have H^c is nano gp*-closed. Hence H is nano gp*-open.

Conversely If H is nano gp*-open, F ⊆ H and F is nano gp-closed. Then F^c is nano gp-open and $H^c \subseteq F^c$. Therefore $Ncl(H^c) \subseteq (F^c)$. But $Ncl(H^c) = (Nint(H))^c$. Hence F ⊆ N int(H).

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Source of support: Nil, Conflict of interest: None Declared.