International Journal of Mathematical Archive-9(5), 2018, 5-7
Available online through www.ijma.info ISSN 2229 – 5046

WEAK AND ALMOST SEMI REGULARITY IN A SPACE

Dr. Mamun Ar. Rashid*

Santipur College, Santipur, Nadia, (W.B.), India.

(Received On: 23-03-18; Revised & Accepted On: 17-04-18)

ABSTRACT

The concept of weak and almost regularity defined Singal and Arya in 1969[5] in a topological space. In this paper weak and almost semi regularity are introduced in a space defined by A.D. Alexandroff and some of their properties are investigated.

Mathematics Subject Classification 2010: *54XX.*

1. INTRODUCTION

Topological spaces have been generalized in several ways. For example Mashhour *et al.* [4] omitted the intersection condition and then Das and Samanta [3] investigated a space without any structural conditions. Perhaps the first to introduce such a generalization was Alexandroff [1], who weakened the union requirements of a topological space. Though every generalization has it's own impact, the generalization by Alexandroff [1] occupies a prominent role in the literature. In this paper weak and almost semi regularity are introduced in a space defined by A.D.Alexandroff and some of their properties are investigated

2. PRELIMINARIES

Definition 1 [1]: A set X is called a space if in it is chosen a system of subsets F satisfying the following axioms
(i) The intersection of a countable number of sets from F is a set in F.
(ii) The union of a finite number of sets from F is a set in F.
(iii) The void set is a set in F.
(iv) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open. It is clear that instead of closed sets in the definition of a space, one may put open sets with subject to the conditions of countable summability, finite intersectability and the condition that X and the void set should be open. The collection of such open sets will sometimes be denoted by τ and the space by (X, τ). In general τ is not a topology. By a space we shall always mean an Alexandroff space.

Definition 2 [1]: With every $M \subset X$ we associate its closure cl(M) the intersection of all closed sets containing M and scl(M) the intersection of all semi closed sets containing M.

Note that cl(M) and scl(M) is not necessarily closed and semi closed respectively.

Definition 3[7]: A set N, a subset of X is said to be a semi neighborhood of a point x of X if and only if there exist a semi open set O containing x such that $O \subset N$.

Definition 4[7]: The semi interior of a set A in a space X is define as the union of all semi open sets contained in A and is denoted by s-int (A).

3. WEEKLY SEMI REGULAR AND ALMOST SEMI REGULAR SPACE

Definition 5: Two sets A, B in X are said weakly semi separated if there are two semi open sets U,V such that $A \subset U, B \subset V$ and $A \cap V = B \cap U = \Phi$.

Corresponding Author: Dr. Mamun Ar. Rashid*

Definition 6: A subset A of a space X is called regularly semi open if it is the semi interior of it's own closure. A set A is said to be regularly semi closed if it is the semi closure of it's own interior.

It is evident that a set is regularly semi open iff it's complement is regularly semi closed.

Note 1: In a topological space a regular semi open set must be semi open. But this is not true in a space as shown by

Example 1: Let X=R-Q and τ ={X, ∅, G_i} where G_i runs over all countable subsets of R-Q. Then (X, τ) is a space but not a topological space .Clearly in this space for any α ∈X, cl{α}={α}.Let A be set of irrational numbers in [0,1]. Then A is uncountable and so A is not open that is not semi open. But s-int (cl(A))= s-in((A))= ∪ {{α}: α ∈ A}=A. So A is regularly semi open but not semi open.

Definition 7: A space X is said to be weakly semi regular if for any weakly separated pair consisting of a regularly semi closed set A and a singleton {x}, there are semi open sets U, V such that A⊂U, x∈V, U∩V=Φ.

Definition 8: A space X is said to be almost semi regular if for any x ϵX and any a regularly closed set A not containing x, there are semi open sets U, V such that A⊂U, x∈V, U∩V=Φ.

Theorem 1: A topological space (X ,σ) is weakly semi regular if and only if for any point x ϵX and any regularly semi open set U such that σ –cl({x}) ⊂U, there is a semi open set V such that x϶V⊂σ –cl(V)⊂ U. Since the semi closure of a set in a space is not necessarily semi closed set, the characterization of weakly semi regularity in a space is somewhat different.

Theorem 2: A space X is weakly semi regular if and only if for each x ϵX and any regularly semi open set U such that x ϵ F⊂U, where F is semi closed set ,there is a semi open set V and a semi closed set F_1, such that x϶V ⊂F_1⊂U.

Proof: .Let X be weakly semi regular. Let x ϵX and U be a regularly semi open set such that x ϵF⊂U for some semi closed set F. Since U is the semi interior of its closure. U is the union of some semi open sets. So there is a semi open set V such that x ϵV ⊂U. Also X-U⊂X-F, where X-F is semi open .Hence {x} and the regularly semi closed set X-U are weakly semi separated .Then there are semi open sets U_1,V_1 such that xϵU_1, X-U⊂V_1 , U_1∩V_1=Φ.
Therefore x ϵU_1 X-V_1= F_1 U, where F_1 is semi closed.

Conversely let the given condition hold. Let x ϵX and F be a regularly semi closed set such that {x} and F are weakly semi separated. So there is a semi open set V_1 such that F⊂V_1 and x does not belongs to V_1, Therefore x ϵX-V1⊂X-F, where X-V_1 is semi closed and X-F is regularly semi open. Now by the given condition there is a semi open set U and a semi closed set F_1 such that x ϵU⊂F_1⊂X-F. Hence x ϵ U, F⊂X-F_1=V where U, V are semi open and U∩V= U∩(X-F_1) = Φ.

Theorem 3: A weakly semi regular T_1 space is almost semi regular.

Proof is simple and so omitted.

Theorem 4: For a space (X, τ) the following are equivalent.

(a) (X ,τ) is almost semi regular.
(b) For each point x ϵX and each regularly semi open set V containing x ,there is a regularly semi open set U and a semi closed set F such that x ϵU⊂F⊂V,
(c) For each point x ϵX and each semi neighbourhood M of x, there is a regularly semi open neighbourhood V of x and a semi closed set F such that x ϵV⊂F⊂s-int(cl(M)).
(d) For each point x ϵX and each semi neighbourhood M of x, there is a semi open neighbourhood V of x and a semi closed set F such that x ϵV⊂F⊂s- int (cl (M))..
(e) For every regularly semi closed set F and each point x not belong to F, there exist semi open sets U, V and semi closed sets F_1, F_2 such that x ϵU⊂F_1, F⊂V⊂F_2 and F_1∩F_2=Φ.

Proof.
(a)=>(b): Let x ϵ X and U be regularly semi open set containing x. Then U^C is regularly semi closed set not containing x. Therefore there exist semi open sets U_1, U_2 such that xϵU_1, U^c ⊂U_2, U_1 U_2 = Φ. Then x ϵU_1 s-int (cl(U_1)). cl(U_1) U_2^c U. Take s-int(cl(U_1)).=V and U_2^c=F. Then V is regularly open, F is semi closed that x ϵV F U.

(b)=>(c): The proof is obvious.

(c) =>(d): Since every regularly semi open set is the union of some semi open sets, the result follows.

(d) =>(e): Let F be regularly semi closed set and x does not belongs to F. Then F^c is a semi neighborhood of x. Therefore there is a semi open set V_1 and a semi closed set $F_1^{/}$ such that $x\in V_1\subset F_1\subset F^c$. Again since V_1 is also a semi neighborhood of x, there is a semi open set U and a semi closed set F_1 such that $x\in U\subset F_1\subset V_1$. Take $V=(F_1^{/})^c$ and $F_2=V_1^c$. Then $F\subset V\subset F_2$ where V is semi open and F_2 is semi closed and $F_1\cap F_2\subset V_1\cap V_2=\Phi$.

(e) =>(a): The proof is obvious.

Theorem 5: Every regularly semi open subspace of an almost semi regular space is almost semi regular.

Proof is simple.

Definition 9: A set A in a space X is said to be almost semi bi compact if every semi open cover of A has a finite sub collection whose closures cover A.

Definition 10: Two sets A, B in X are said strongly semi separated if there are two semi open sets U, V such that $A \subset U$, $B \subset V$ and $V\cap U=\Phi$.

Theorem 6: In an almost semi regular space, every pair consisting of an almost semi bicompact set and a disjoined regularly semi closed set can be strongly semi separated.

Proof: Let (X, τ) be an almost semi regular space. Let A be an almost semi bi compact subset of X and B be a regularly semi closed set with $A\cap B=\Phi$. Since X is almost semi regular, for each $x\in A$, there are semi open sets U_x, V_x and semi closed sets E_x, F_x such that $x \in U_x\subset E_x$, $B\subset V_x \subset F_x$, $E_x \cap F_x = \Phi$. Now $\{U_x \cap A: x \in A\}$ is a relatively semi open cover of the almost semi bi compact set A and so there is a finite subfamily $\{U_{xi} \cap A: I = 1, 2,\ldots.n\}$ whose closures cover A. Since the closures of $U_{xi} \cap A$ in A.
$cl(U_{xi} \cap A) \cap A\subset cl(U_{xi})\cap A\subset cl(U_{xi})\subset E_{xi}$. Hence $A\subset U\{E_{xi}: i = 1, 2,..n\}$. Let $U=\{\cap V_{xi} :i=1,2,..n\}$, $V=X-\{\cap F_{xi} :i= 1,2,..n\}$. Then $A \subset \{\cup E_{xi} :i=1,2,..n\} \subset \{\cup (F_{xi})^c: i = 1, 2,..n\}= X-\{\cap F_{xi}: i= 1, 2,..n\} = V$. and $B \subset U$. Also U and V are semi open sets and $U\cap V=\Phi$.

REFERENCES

1. Alexandroff, A.D., Additive set functions in abstract spaces, Mat. Sb. (N.S.) 8(50) (1940) 307–348 (English, Russian summary).
2. Alexandroff, A.D., Additive set functions in abstract spaces, Mat. Sb. (N.S.) 9(51) (1941), 563–628 (English, Russian summary).
3. Das, P. and Samanta, S.K., Pseudo-topological spaces, Sains Malaysiana, 21(4) (1992), 101–107.
4. Mashhour, A. S., Allam, A.A., Mahmoud, A.A. and Khedr, F.H., On supratopological spaces, Indian J. Pure Appl. Math. 14(4) (1983), 502–510.
5. Singal .M.K. and Arya .S.P., On almost regular spaces, Glasnik Mathematicki, Tom 4(24), No. 1(1969), 89-99.
6. Singal .M.K. and Arya .S.P. Almost normal and Almost completely regular spaces, Glasnik Mathematicki, Tom 5(25), No. 1(1979), 140-151..
7. Das Phullendu, Note on some applications of semi open sets (visva Bharati).

Source of support: Nil, Conflict of interest: None Declared.