

**AN ANALYSIS OF HALL EFFECT ON HYDROMAGNETIC FREE CONVECTIVE FLOW PAST  
A VERTICAL PLATE WITH RAMPED WALL TEMPERATURE AND CONCENTRATION  
WITH RADIATION AND CHEMICAL REACTION**

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**INTRODUCTION**

In recent years, the study MHD flow problems has achieved remarkable interest due to its vast application in MHD pumps, MHD power generators and MHD flow meters. Consolidated effect of heat and mass transfer in the presence of magnetic field through a porous medium have attracted the attention of number of scholars. These problems are of great importance in many chemical formulations and reactive chemicals. Therefore, considerable attention had been paid in recent years to study the influence of the participating parameters on the velocity and temperature field. The phenomena of heat and mass transfer are also very common in theory of stellar structure and observable effects are detectable on the solar structure. Bejan and Khair [5] studied heat and mass transfer in porous medium. Natural convection with combined heat and mass transfer buoyancy effects in a porous medium has been studied by Trevisan and Bejan [36]. Yih [37] studied the effect of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium. The unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption has been investigated by Chamkha [7]. Choudhary and Jain [10] have studied the combined heat and mass Transfer Effects on MHD Free Convection Flow Past an Oscillating Plate Embedded in porous medium. The study of heat and mass transfer on free convective three dimensional unsteady flows over a porous vertical plate have been analysed by Ahmed [3]. The case of unsteady MHD convective heat and mass transfer in a boundary layer slip flow past a vertical permeable wall with thermal radiation and chemical reaction was examined by Pal *et.al*, [28]. Ahmed *et.al* [2] have studied the MHD mixed convection and mass transfer from an infinite vertical plate with chemical reaction in presence of heat source. Effect of heat and mass transfer on MHD unsteady convective flow along a vertical porous plate with constant suction and heat source has been investigated by Chand and Sharma [9].

All the above studies are free from rotation effect. Theoretical and applied research in MHD flow in rotating system through porous medium has received increased attention during the past three decades. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in cosmical and geophysical fluid dynamics. It can provide an explanation for the observed maintenance and secular variation of the geomagnetic field Hide and Robert [21]. It is also important in the solar Physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars [15]. Since we live on rotating earth, rotating flows are extremely important in geophysics in the ocean and atmosphere. In a rapidly rotating fluid, particles moves across the pressure gradient due to Coriolis forces. The Coriolis force exerts a strong influence on the hydromagnetic flow in the earth's liquid core which plays an important role in the mean geomagnetic field. The Coriolis force is an inertial force that acts on objects that are in motion relative to a rotating reference frame. In Physics and engineering, the MHD rotating flow of electrically conducting fluid has numerous applications. The study of positions and velocities with respect to a fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of magnetic field. Nanda and Mohanty [27] have discussed the hydromagnetic rotating channel flows.

Pop and Soundelgekar [29] analyzed unsteady boundary layers in rotating flow. Sarojamma and Krishan[32] has been studied the transient hydromagnetic convective flow in rotating fluid. Singh [34] studied the hydromagnetic free-convection flow past an impulsively started vertical plate in a rotating fluid. Unsteady flow over a stretching surface with a magnetic field in a rotating fluid has been discussed by Takhar and Nath [35]. Bestman and Adjepong [6] presented unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating channel. The heat and mass transfer of unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer was analyzed by Mbeledogu and Ogulu [25]. Das *et.al* [14] presented the Hall effects on MHD Couette flow in rotating system.

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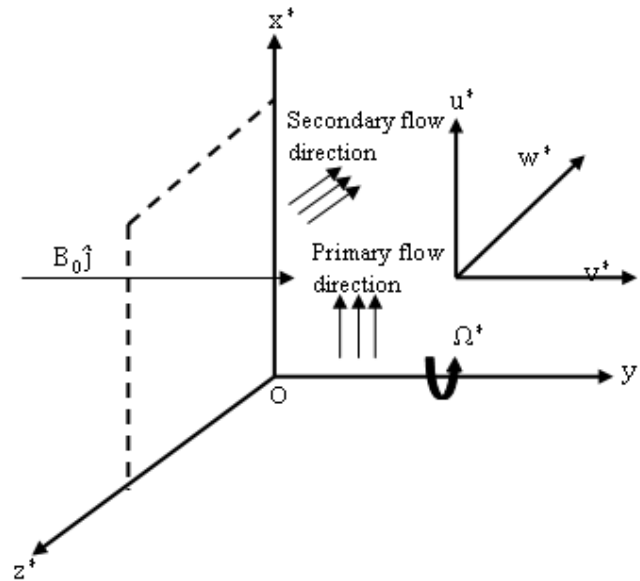
Magnetohydrodynamic flow in the present form is due to pioneer contribution of many famous authors like and Cowling [12]. Most of the time Hall current was ignored while applying Ohm's law because it has no significant effect for small and average values of the magnetic field. It was first observed by Cowling that when the strength of applied magnetic field is sufficiently large, Ohm's law needs to be modified to include Hall current. The Hall effect cannot be ignored completely, if the density of electrons is small and effect of the magnetic field is large as it is responsible for the change of the flow pattern of ionised gases. The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. It was discovered by Edwin Hall in 1879. This current is termed as Hall current. The effects of Hall currents and Coriolis force on Hartmann flow under general wall conditions has been studied by Nagy and Demandy [26]. The effects of Hall current on MHD Couette flow in a rotating system with arbitrary magnetic field have been analyzed by Ghosh [17]. Ghosh and Pop [16] investigated Hall effects on MHD plasma Couette flow in a rotating environment. The effects of Hall current on MHD flow in a rotating channel partially filled with a porous medium has been presented by Chauhan and Agrawal [11]. Guchhait *et.al* [18] discussed the combined effects of Hall current and rotation on unsteady Couette flow in a porous channel. Hall effects on unsteady rotating MHD flow through porous channel with variable pressure gradient has been analyzed by Das *et.al.* [13]. Chand and Sapna [8] has analyzed the Hall effect on MHD flow in a rotating channel through porous medium in the presence of an inclined magnetic field. Hayat *et.al* [19] investigated the heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid. Rao [31] *et.al* presented unsteady MHD free convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate with heat absorption, radiation, chemical reaction and Soret effects.

The conditions for velocity and temperature at the boundaries of plate are homogenous and well defined in all above studies either the solutions are obtained numerically or analytically. However there are many problems in which the arbitrary or non-uniform conditions exist. The various problems of MHD free convection with step discontinuities in wall temperature was investigated by Hayday *et.al* [20], Kao [22], Lee and Yovanovich [23]. Rajesh [30] studied the radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature. The MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature have been presented by Seth *et. al* [33]. Barik [4] analyzed chemical reaction and radiation effects of MHD free convective flow past an impulsively moving vertical plate with ramped wall temperature and concentration. So we intended to study Hall effect on hydromagnetic free convective flow past a vertical plate with ramped wall temperature and concentration with radiation and chemical reaction.

## FORMULATION OF THE PROBLEM

We consider an unsteady flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate embedded in a porous medium taking into account of Hall current in presence of a uniform magnetic field. We introduce the coordinate system  $(x^*, y^*, z^*)$ . The  $x^*$ - axis is taken along the plate in upward direction,  $y^*$ - axis normal to the plane of plate in the fluid and  $z^*$ - axis perpendicular to  $x^*y^*$  plane. The fluid as well as plate is assumed to rotate with uniform angular velocity  $\Omega^*$  about  $y^*$ -axis. Initially both the fluid and plate are at rest at uniform temperature  $T_\infty^*$ . At time  $t^* > 0$  the plate starts moving along  $x^*$  - axis with uniform velocity  $U_0$  and the temperature and concentration of the plate are supposed to be raised or lowered step wise accordance to the relations  $T^* = T_\infty^* + (T_w^* - T_\infty^*) t^*/t_0$  and  $C^* = C_\infty^* + (C_w^* - C_\infty^*) t^*/t_0$  when  $0 < t^* \leq t_0$ , and later on for  $t^* > t_0$ , they are supposed to be maintained at uniform temperature  $T_w^*$  and concentration  $C_w^*$ . Let  $r^* = \hat{i}u^* + \hat{j}v^* + \hat{k}w^*$  be the fluid velocity,  $J^* = J_x^*\hat{i} + J_y^*\hat{j} + J_z^*\hat{k}$  denote the current density at any point  $S(x^*, y^*, z^*, t^*)$  and  $B = B_0\hat{j}$  be the applied magnetic field, and  $(\hat{i}, \hat{j}, \hat{k})$  be the unit vectors. As the plate is infinite along  $x^*$  and  $z^*$  direction and is electrically non-conducting. Clearly all the physical quantities, except pressure, will be functions of  $y^*$  and  $t^*$  only. Further it is assumed that

1. The magnetic Reynolds number is small, so that the induced magnetic field is negligible in comparison with the applied magnetic field.
2. Viscous dissipation, radiative effects, and Joule heating terms are neglected in the energy equation (usually in free convection flows the velocity has small values). However, according to Magyari and Pantokratoras [24], the radiative effects can be easily included by a simple rescaling of the Prandtl number.
3. No external electric field is applied and the effect of polarization of ionized fluid is negligible; therefore, electric field is assumed to be zero.
4. There exists a first-order chemical reaction between the fluid and species concentration. The level of species concentration is very low, so that the heat generated by chemical reaction can be neglected. Under usual Boussinesq's approximation, the flow is governed by the following system of equations



**Figure-1:** Schematic presentation of the problem.

**Equations of motion**

$$\frac{\partial u^*}{\partial t^*} + 2\Omega^* w^* = \nu \left( \frac{\partial^2 u^*}{\partial y^{*2}} \right) - \frac{\nu}{K^*} u^* + g\beta_T (T^* - T_\infty^*) + g\beta_C (C^* - C_\infty^*) - \frac{\sigma B_0^2 (u^* + mw^*)}{\rho(1+m^2)}, \tag{1}$$

$$\frac{\partial w^*}{\partial t^*} - 2\Omega^* u^* = \nu \left( \frac{\partial^2 w^*}{\partial y^{*2}} \right) - \frac{\nu}{K^*} w^* + \frac{\sigma B_0^2 (mu^* - w^*)}{\rho(1+m^2)}, \tag{2}$$

**Equation of energy**

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial y^{*2}} \right) - \frac{1}{\rho C_p} \frac{\partial q^*}{\partial y^*} - \frac{Q_0}{\rho C_p} (T^* - T_\infty^*), \tag{3}$$

**Equation of mass diffusion**

$$\frac{\partial C^*}{\partial t^*} = D \left( \frac{\partial^2 C^*}{\partial y^{*2}} \right) + D_1 \left( \frac{\partial^2 T^*}{\partial y^{*2}} \right) - K_r (C^* - C_\infty^*), \tag{4}$$

**The initial and boundary conditions for the problem are**

$$\left. \begin{aligned} u^* = 0, w^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ for } y^* \geq 0 \text{ and } t^* \leq 0, \\ u^* = U_0, w^* = 0 \text{ at } y^* = 0 \text{ for } t^* > 0, \\ T^* = T_\infty^* + (T_w^* - T_\infty^*) t^*/t_0, C^* = C_\infty^* + (C_w^* - C_\infty^*) t^*/t_0 \text{ at } y^* = 0 \text{ for } 0 < t^* \leq t_0, \\ T^* = T_w^*, C^* = C_w^* \text{ at } y^* = 0 \text{ for } t^* > t_0, \\ u^* \rightarrow 0, w^* \rightarrow 0, T^* \rightarrow T_\infty^*, C^* \rightarrow C_\infty^* \text{ as } y^* \rightarrow \infty \text{ for } t^* > 0. \end{aligned} \right\} \tag{5}$$

The radiative flux vector  $q^*$  under Rosseland approximation as in Brewster(1992), becomes

$$q^* = -\frac{4\sigma^*}{3k^*} \frac{\partial T^{*4}}{\partial y^*},$$

where  $\sigma^*$  and  $k^*$  are the Stefan-Boltzman constant and Rosseland means absorption coefficient, respectively. Assuming that the temperature differences within the flow are sufficiently small, as such  $T^{*4}$  in Taylor's series about  $T_\infty^*$  and neglecting higher order terms, we get,

$$T^{*4} \cong 4T_\infty^{*3} T^* - 3T_\infty^{*4}, \tag{6}$$

$$\frac{\partial q^*}{\partial y^*} = -\frac{4\sigma^*}{3k^*} \left( 4T_\infty^{*3} \frac{\partial T^*}{\partial y^*} \right) = -\frac{16\sigma^*}{3k^*} T_\infty^{*3} \frac{\partial T^*}{\partial y^*} \tag{7}$$

Using equation (7) in equation (3) it becomes,

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial y^{*2}} \right) + \frac{16\sigma^*}{3\rho C_p k^*} T_\infty^{*3} \frac{\partial T^*}{\partial y^*} - \frac{Q_0}{\rho C_p} (T^* - T_\infty^*). \tag{8}$$

Introducing the following non-dimensional quantities and parameters:

$$y = \frac{y^*}{U_0 t_0}, u = \frac{u^*}{U_0}, t = \frac{t^*}{t_0}, K_p = \frac{K^* U_0^2}{\nu^2}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, T = \frac{(T^* - T_\infty^*)}{(T_w^* - T_\infty^*)}, C = \frac{(C^* - C_\infty^*)}{(C_w^* - C_\infty^*)}, G_r = \frac{g\beta_T \nu (T_w^* - T_\infty^*)}{U_0^3}, G_m = \frac{g\beta_C \nu (C_w^* - C_\infty^*)}{U_0^3}, P_r = \frac{\rho \nu C_p}{k}, S_c = \frac{\nu}{D}, S_0 = \frac{D_1}{\nu} \left( \frac{T_w^* - T_\infty^*}{C_w^* - C_\infty^*} \right), N = \frac{16\sigma^* T_\infty^{*3}}{3kk^*}, K_r = \frac{K_r^* \nu}{U_0^2}, Q = \frac{\nu Q_0}{\rho C_p U_0^2}.$$

The reduced non-dimensional forms of equations are:

$$\frac{\partial u}{\partial t} + 2\Omega w = \frac{\partial^2 u}{\partial y^2} + G_r T + G_m C - \frac{u}{K_p} - \frac{M(u+mw)}{(1+m^2)} \quad (9)$$

$$\frac{\partial w}{\partial t} - 2\Omega u = \frac{\partial^2 w}{\partial y^2} - \frac{w}{K_p} - \frac{M(mu-w)}{(1+m^2)}, \quad (10)$$

$$\frac{\partial T}{\partial t} = \left(\frac{1+N}{P_r}\right) \frac{\partial^2 T}{\partial y^2} - QT, \quad (11)$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 T}{\partial y^2} - K_r S_c C. \quad (12)$$

The above non-dimensionalization process, the characteristic time  $t_0$  can be defined as

$$t_0 = \frac{v}{U_0^2} \quad (13)$$

Using the above non-dimensional parameters, the initial and boundary conditions become,

$$\left. \begin{aligned} u = 0, w = 0, T = 0, C = 0 \text{ for } y \geq 0 \text{ and } t \leq 0 \\ u = 1, w = 0 \text{ at } y = 0 \text{ for } t > 0 \\ T = t, C = t \text{ at } y = 0 \text{ for } 0 < t \leq 1 \\ T = 1, C = 1 \text{ at } y = 0 \text{ for } t > 1 \\ u \rightarrow 0, w \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \text{ for } t > 0 \end{aligned} \right\} \quad (14)$$

### Method of solution

Following Ahmed and Das [1], we introduce a new complex variable  $f = u + iv$ , where  $i = \sqrt{-1}$ . The non-dimensional form of the equation governing the flow can be written as

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial y^2} - \left\{ \frac{M(1-im)}{(1+m^2)} - 2i\Omega + \frac{1}{K_p} \right\} + G_r T + G_m C, \quad (15)$$

$$\frac{\partial T}{\partial t} = \left(\frac{1+N}{P_r}\right) \frac{\partial^2 T}{\partial y^2} - QT, \quad (16)$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 T}{\partial y^2} - K_r S_c C. \quad (17)$$

The transformed initial and boundary conditions became

$$\left. \begin{aligned} f = 0, T = 0, C = 0 \text{ for } y \geq 0 \text{ and } t \leq 0 \\ f = 1 \text{ at } y = 0 \text{ for } t > 0 \\ T = t, C = t \text{ at } y = 0 \text{ for } 0 < t \leq 1 \\ T = 1, C = 1 \text{ at } y = 0 \text{ for } t > 1 \\ f \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \text{ for } t > 0 \end{aligned} \right\}. \quad (18)$$

### Case-1: Solution in case of ramped temperature and concentration

Equations (15) to (17), subject to the boundary conditions prescribed in equation (18) and solved analytically using Laplace transform technique, and the solution for the fluid temperature, fluid concentration and fluid velocity are presented in following form

$$T(y, t) = P_1(y, t) - H(t-1)P_1(y, t-1), \quad (19)$$

$$C(y, t) = P_2(y, t) + d\{P_3(y, t) - P_4(y, t)\} - H(t-1)[P_2(y, t-1) + d\{P_3(y, t-1) - P_4(y, t-1)\}] \quad (20)$$

$$f(y, t) = P_5(y, t) + \alpha P_6(y, t) + d\alpha_1 P_7(y, t) - d\alpha_2 P_8(y, t) - \alpha P_9(y, t) - \alpha_1 E_1(y, t) - d\alpha_1 E_2(y, t) + d\alpha_2 E_3(y, t) - H(t-1)\{\alpha P_6(y, t-1) + d\alpha_1 P_7(y, t-1) - d\alpha_2 P_8(y, t-1) - \alpha P_9(y, t-1) - \alpha_1 E_1(y, t-1) - d\alpha_1 E_2(y, t-1) + d\alpha_2 E_3(y, t-1)\} \quad (21)$$

where,

$$P_1(y, t) = \left(\frac{t}{2} - \frac{y}{4\sqrt{Q}}\right) e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + \left(\frac{t}{2} + \frac{y}{4\sqrt{Q}}\right) e^{y\sqrt{aQ}} \operatorname{erfc}(t_2),$$

$$P_2(y, t) = \left(\frac{t}{2} - \frac{y}{4\sqrt{K_r}}\right) e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) + \left(\frac{t}{2} + \frac{y}{4\sqrt{K_r}}\right) e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4),$$

$$P_3(y, t) = \left(\frac{Q+b}{2b^2}\right) e^{bt} \left\{ e^{-y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_5) + e^{y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_6) \right\} - \left\{ \frac{Q(tb+1)+b}{2b^2} - \frac{Qy}{4b\sqrt{K_r}} \right\} e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) -$$

$$\left\{ \frac{Q(tb+1)+b}{2b^2} + \frac{Qy}{4b\sqrt{K_r}} \right\} e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4),$$

$$P_4(y, t) = \left(\frac{Q+b}{2b^2}\right) e^{bt} \left\{ e^{-y\sqrt{a(b+Q)}} \operatorname{erfc}(t_7) + e^{y\sqrt{a(b+Q)}} \operatorname{erfc}(t_8) \right\} - \left\{ \frac{Q(tb+1)+b}{2b^2} - \frac{y\sqrt{Q}}{4b} \right\} e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) - \left\{ \frac{Q(tb+1)+b}{2b^2} +$$

$$yQ4beyaQerfct2e\beta t2\beta 2,$$

$$P_5(y, t) = \frac{1}{2} \left\{ e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) + e^{y\sqrt{M_1}} \operatorname{erfc}(t_{10}) \right\},$$

$$P_6(y, t) = \frac{e^{\beta t}}{2\beta^2} \left\{ e^{-y\sqrt{M_1+\beta}} \operatorname{erfc}(q_1) + e^{y\sqrt{M_1+\beta}} \operatorname{erfc}(q_2) \right\} - \left( \frac{\beta t+1}{2\beta^2} - \frac{y}{4\beta\sqrt{M_1}} \right) e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) - \left( \frac{\beta t+1}{2\beta^2} + \frac{y}{4\beta\sqrt{M_1}} \right) e^{-y\sqrt{M_1}} \operatorname{erfc}(t_{10}),$$

$$P_7(y, t) = \left\{ \frac{A_1}{2} + B_1 \left( \frac{t}{2} - \frac{y}{4\sqrt{M_1}} \right) \right\} e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) + \left\{ \frac{A_1}{2} + B_1 \left( \frac{t}{2} - \frac{y}{4\sqrt{M_1}} \right) \right\} e^{y\sqrt{M_1}} \operatorname{erfc}(t_{10}) + \frac{C_1 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_3) + e^{y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_4) + D_1 e^{b t} 2e^{-yM_1+\beta_1} \operatorname{erfc} q_5 + e^{yM_1+\beta_1} \operatorname{erfc} q_6 \right\},$$

$$P_8(y, t) = \left\{ \frac{A_2}{2} + B_2 \left( \frac{t}{2} - \frac{y}{4\sqrt{M_1}} \right) \right\} e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) + \left\{ \frac{A_2}{2} + B_2 \left( \frac{t}{2} - \frac{y}{4\sqrt{M_1}} \right) \right\} e^{y\sqrt{M_1}} \operatorname{erfc}(t_{10}) + \frac{C_2 e^{\beta t}}{2} \left\{ e^{-y\sqrt{M_1+\beta}} \operatorname{erfc}(q_1) + e^{y\sqrt{M_1+\beta}} \operatorname{erfc}(q_2) + D_2 e^{b t} 2e^{-yM_1+\beta} \operatorname{erfc} q_5 + e^{yM_1+\beta} \operatorname{erfc} q_6 \right\},$$

$$P_9(y, t) = \frac{e^{\beta t}}{2\beta^2} \left\{ e^{-y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_7) + e^{y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_8) \right\} - \left( \frac{\beta t+1}{2\beta^2} - \frac{y}{4\beta\sqrt{Q}} \right) e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) - \left( \frac{\beta t+1}{2\beta^2} + \frac{y}{4\beta\sqrt{Q}} \right) e^{y\sqrt{aQ}} \operatorname{erfc}(t_2),$$

$$E_1(y, t) = \frac{e^{\beta_1 t}}{2\beta_1^2} \left\{ e^{-y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_9) + e^{y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_{10}) \right\} - \left( \frac{\beta t+1}{2\beta^2} - \frac{y}{4\beta\sqrt{K_r}} \right) e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) - \left( \frac{\beta t+1}{2\beta^2} + \frac{y}{4\beta\sqrt{K_r}} \right) e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4),$$

$$E_2(y, t) = \left\{ \frac{A_1}{2} + B_1 \left( \frac{t}{2} - \frac{y}{4\sqrt{K_r}} \right) \right\} e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) + \left\{ \frac{A_1}{2} + B_1 \left( \frac{t}{2} - \frac{y}{4\sqrt{K_r}} \right) \right\} e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4) + \frac{C_1 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_9) + e^{y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_{10}) + D_1 e^{b t} 2e^{-yS_c b + K_r} \operatorname{erfc} X_1 + e^{yS_c b + K_r} \operatorname{erfc} X_2 \right\},$$

$$E_3(y, t) = \left\{ \frac{A_2}{2} + B_2 \left( \frac{t}{2} - \frac{y}{4\sqrt{Q}} \right) \right\} e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + \left\{ \frac{A_2}{2} + B_2 \left( \frac{t}{2} + \frac{y}{4\sqrt{Q}} \right) \right\} e^{y\sqrt{aQ}} \operatorname{erfc}(t_2) + \frac{C_2 e^{\beta t}}{2} \left\{ e^{-y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_7) + e^{y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_8) + D_2 e^{b t} 2e^{-y a b + Q} \operatorname{erfc} X_3 + e^{y a b + Q} \operatorname{erfc} X_4 \right\},$$

Here  $\operatorname{erfc}$  and  $H(t-1)$  are, respectively, complimentary error function and Heaviside unit step function.

### Case-2: Solution in case of isothermal plate

Equations (19), (20) and (21) present analytic solution for the fluid temperature, fluid concentration and fluid velocity of flow of viscous incompressible electrically conducting fluid near a vertical moving plate with ramped temperature and ramped concentration. In order to highlight the effects of ramped temperature and ramped concentration of the plate on the fluid flow, it is worthwhile to compare such a flow with the one near a moving plate with uniform temperature and uniform concentration. Taking into consideration the assumptions made above the solution for the fluid temperature, fluid concentration and fluid velocity for natural convection flow near an isothermal moving plate is obtained and is presented in the following form

$$T(y, t) = E_4(y, t), \tag{22}$$

$$C(y, t) = E_5(y, t), \tag{23}$$

$$f(y, t) = P_5(y, t) + \alpha E_6(y, t) + \alpha_1 E_7(y, t) - \alpha_2 E_8(y, t) - \alpha E_9(y, t) - \alpha_1 E_{10}(y, t) - \alpha_2 E_{12}(y, t) + \alpha_2 E_{13}(y, t), \tag{24}$$

where,

$$E_4(y, t) = \frac{1}{2} \left\{ e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + e^{y\sqrt{aQ}} \operatorname{erfc}(t_2) \right\},$$

$$E_5(y, t) = \left( \frac{1}{2} - \frac{Qd}{2b} \right) e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) + \left( \frac{1}{2} - \frac{Qd}{2b} \right) e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4) + \frac{d(Q+b)e^{bt}}{2b} \left\{ e^{-y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_5) + e^{y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_6) \right\} + \frac{dQ}{2b} \left\{ e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + e^{y\sqrt{aQ}} \operatorname{erfc}(t_2) \right\} - \frac{d(Q+b)e^{bt}}{2b} \left\{ e^{-y\sqrt{a(b+Q)}} \operatorname{erfc}(t_7) + e^{y\sqrt{a(b+Q)}} \operatorname{erfc}(t_8) \right\},$$

$$E_6(y, t) = \frac{1}{\beta} P_5(y, t) + \frac{e^{\beta t}}{2\beta} \left\{ e^{-y\sqrt{M_1+\beta}} \operatorname{erfc}(q_1) + e^{y\sqrt{M_1+\beta}} \operatorname{erfc}(q_2) \right\},$$

$$E_7(y, t) = \frac{A_3}{2} \left\{ e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) + e^{y\sqrt{M_1}} \operatorname{erfc}(t_{10}) \right\} + \frac{B_3 e^{bt}}{2} \left\{ e^{-y\sqrt{M_1+b}} \operatorname{erfc}(q_5) + e^{y\sqrt{M_1+b}} \operatorname{erfc}(q_6) \right\} + \frac{C_3 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_3) + e^{y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_4) + D_1 e^{b t} 2e^{-yM_1+\beta_1} \operatorname{erfc} q_5 + e^{yM_1+\beta_1} \operatorname{erfc} q_6 \right\},$$

$$E_8(y, t) = \frac{B_2}{2} \left\{ e^{-y\sqrt{M_1}} \operatorname{erfc}(t_9) + e^{y\sqrt{M_1}} \operatorname{erfc}(t_{10}) \right\} + \frac{B_4 e^{bt}}{2} \left\{ e^{-y\sqrt{M_1+b}} \operatorname{erfc}(q_5) + e^{y\sqrt{M_1+b}} \operatorname{erfc}(q_6) \right\} + \frac{C_4 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_3) + e^{y\sqrt{M_1+\beta_1}} \operatorname{erfc}(q_4) + D_2 e^{b t} 2e^{-yM_1+\beta_1} \operatorname{erfc} q_5 + e^{yM_1+\beta_1} \operatorname{erfc} q_6 \right\},$$

$$E_9(y, t) = -\frac{1}{2\beta} \left\{ e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + e^{y\sqrt{aQ}} \operatorname{erfc}(t_2) \right\} + \frac{e^{\beta t}}{2\beta} \left\{ e^{-y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_7) + e^{y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_8) \right\},$$

$$E_{10}(y, t) = -\frac{1}{2\beta_1} \left\{ e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) + e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4) \right\} + \frac{e^{\beta_1 t}}{2\beta_1} \left\{ e^{-y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_9) + e^{y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_{10}) \right\},$$

$$E_{12}(y, t) = \frac{A_3}{2} \left\{ e^{-y\sqrt{S_c K_r}} \operatorname{erfc}(t_3) + e^{y\sqrt{S_c K_r}} \operatorname{erfc}(t_4) \right\} + \frac{B_3 e^{bt}}{2} \left\{ e^{-y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_5) + e^{y\sqrt{S_c(b+K_r)}} \operatorname{erfc}(t_6) \right\} + \frac{C_3 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_9) + e^{y\sqrt{S_c(\beta_1+K_r)}} \operatorname{erfc}(q_{10}) \right\},$$

$$E_{13}(y, t) = \frac{B_2}{2} \left\{ e^{-y\sqrt{aQ}} \operatorname{erfc}(t_1) + e^{y\sqrt{aQ}} \operatorname{erfc}(t_2) \right\} + \frac{B_4 e^{bt}}{2} \left\{ e^{-y\sqrt{a(b+Q)}} \operatorname{erfc}(t_7) + e^{y\sqrt{a(b+Q)}} \operatorname{erfc}(t_8) \right\} + \frac{C_4 e^{\beta_1 t}}{2} \left\{ e^{-y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_7) + e^{y\sqrt{a(\beta+Q)}} \operatorname{erfc}(q_8) \right\},$$

$$t_1, t_2 = \frac{y}{2} \sqrt{\frac{a}{t}} \mp \sqrt{Qt}; \quad t_3, t_4 = \frac{y}{2} \sqrt{\frac{S_c}{t}} \mp \sqrt{K_r t}; \quad t_5, t_6 = \frac{y}{2} \sqrt{\frac{S_c}{t}} \mp \sqrt{(K_r + b)t}; \quad t_7, t_8 = \frac{y}{2} \sqrt{\frac{a}{t}} \mp \sqrt{(Q + b)t}; \quad t_9, t_{10} = \frac{y}{2} \sqrt{\frac{a}{t}} \mp \sqrt{M_1 t};$$

$$q_1, q_2 = \frac{y}{2\sqrt{t}} \mp \sqrt{(M_1 + \beta)t}; \quad q_3, q_4 = \frac{y}{2\sqrt{t}} \mp \sqrt{(M_1 + \beta_1)t}; \quad q_5, q_6 = \frac{y}{2\sqrt{t}} \mp \sqrt{(M_1 + b)t}; \quad q_7, q_8 = \frac{y}{2} \sqrt{\frac{a}{t}} \mp \sqrt{(Q + \beta)t};$$

$$q_9, q_{10} = \frac{y}{2} \sqrt{\frac{S_c}{t}} \mp \sqrt{(K_r + \beta_1)t};$$

$$X_1, X_2 = \frac{y}{2} \sqrt{\frac{S_c}{t}} \mp \sqrt{(K_r + b)t}; \quad X_3, X_4 = \frac{y}{2} \sqrt{\frac{a}{t}} \mp \sqrt{(Q + b)t};$$

$$a = \frac{P_r}{1+N}, \quad b = \frac{S_c K_r - Qa}{a - S_c}, \quad d = \frac{aS_c S_0}{a - S_c}, \quad \alpha = \frac{G_r}{a-1}, \quad \beta = \frac{M_1 - aQ}{a-1}, \quad \alpha_1 = \frac{G_m}{S_c - 1}, \quad \beta_1 = \frac{M_1 - S_c K_r}{S_c - 1}, \quad M_1 = \frac{M(1-im)}{1+m^2} - 2i\Omega + \frac{1}{K_p}, \quad A_1 = \frac{1}{\beta_1 b} \left\{ 1 + \frac{Q}{\beta_1 b} (\beta_1 + b) \right\}, \quad B_1 = \frac{Q}{\beta_1 b}, \quad C_1 = \frac{-(Q+\beta_1)}{\beta_1^2(b-\beta_1)}, \quad D_1 = \frac{(Q+b)}{b^2(b-\beta_1)}, \quad A_2 = \frac{1}{\beta b} \left\{ 1 + \frac{Q}{\beta b} (\beta + b) \right\}, \quad B_2 = \frac{Q}{\beta b}, \quad C_2 = \frac{-(Q+\beta)}{\beta^2(b-\beta)}, \quad D_2 = \frac{(Q+b)}{b^2(b-\beta)}, \quad A_3 = \frac{Q}{\beta_1 b}, \quad B_3 = \frac{-(Q+b)}{b(\beta_1-b)}, \quad C_3 = \frac{(\beta_1+Q)}{\beta_1(\beta_1-b)}, \quad A_4 = \frac{Q}{\beta b}, \quad B_4 = \frac{-(Q+b)}{b(\beta-b)}, \quad C_4 = \frac{(\beta+Q)}{\beta(\beta-b)}.$$

**Some important characteristic parameters of flow field**

Skin friction, Nusselt number and Sherwood number are the measures of rate of shear stress, rate of heat transfer and rate of mass transfer respectively. These parameters are presented for ramped wall temperature by the expansion of following form

$$\text{Skin friction}(\tau) = \left( \frac{\partial f}{\partial y} \right)_{y=0}$$

$$\tau = f_5(t) + \alpha f_6(t) + d\alpha_1 f_7(t) - d\alpha_2 f_8(t) - \alpha f_9(t) - \alpha_1 e_1(t) - d\alpha_1 e_2(t) + d\alpha_2 e_3(t) - H(t-1) \{ \alpha f_6(t-1) + d\alpha_1 f_7(t-1) - d\alpha_2 f_8(t-1) - \alpha f_9(t-1) - \alpha_1 e_1(t-1) - d\alpha_1 e_2(t-1) + d\alpha_2 e_3(t-1) \}$$

$$\text{Nusselt number} (Nu) = - \left( \frac{\partial T}{\partial y} \right)_{y=0},$$

$$Nu = f_1(t) - H(t-1) f_1(t-1),$$

$$\text{Sherwood number} (Sh) = - \left( \frac{\partial C}{\partial y} \right)_{y=0},$$

$$Sh = f_2(t) + d \{ f_3(t) - f_4(t) \} - H(t-1) [ f_2(t-1) + d \{ f_3(t-1) - f_4(t-1) \} ].$$

Skin friction, Nusselt number and Sherwood number for isothermal plate are given by the following expressions

$$\text{Skin friction}(\tau) = \left( \frac{\partial f}{\partial y} \right)_{y=0}$$

$$\tau = f_5(t) + \alpha e_6(t) + d\alpha_1 e_7(t) - d\alpha_2 e_8(t) - \alpha e_9(t) - \alpha_1 e_{10}(t) - d\alpha_1 e_{12}(t) + d\alpha_2 e_{13}(t)$$

$$\text{Nusselt number} (Nu) = - \left( \frac{\partial T}{\partial y} \right)_{y=0},$$

$$Nu = e_4(t),$$

$$\text{Sherwood number} (Sh) = - \left( \frac{\partial C}{\partial y} \right)_{y=0},$$

$$Sh = e_5(t).$$

where

$$f_1(t) = \left( \frac{1}{2\sqrt{Q}} + t(\sqrt{aQ}) \right) \operatorname{erf}(\sqrt{Qt}) + \sqrt{\frac{at}{\pi}} e^{-Qt},$$

$$f_2(t) = \left( \frac{1}{2\sqrt{K_r}} + t(\sqrt{S_c K_r}) \right) \operatorname{erf}(\sqrt{K_r t}) + \sqrt{\frac{S_c t}{\pi}} e^{-K_r t},$$

$$f_3(t) = \frac{(Q+b)e^{bt}}{b^2} (\sqrt{S_c(b+K_r)}) \operatorname{erf}(\sqrt{(b+K_r)t}) - \frac{Q}{2\sqrt{K_r}} \operatorname{erf}(\sqrt{K_r t}) - \left\{ \frac{Q(tb+1)+b}{b^2} \right\} (\sqrt{S_c K_r}) \operatorname{erf}(\sqrt{K_r t}) - \left\{ \frac{Q(tb+1)+b}{b^2} - (Q+b)b^2 S_c t \pi e^{-K_r t} \right\}$$

$$f_4(t) = \frac{(Q+b)e^{bt}}{b^2} (\sqrt{a(b+Q)}) \operatorname{erf}(\sqrt{(b+Q)t}) - \frac{\sqrt{Q}}{2} \operatorname{erf}(\sqrt{Qt}) - \left\{ \frac{Q(tb+1)+b}{b^2} \right\} (\sqrt{aQ}) \operatorname{erf}(\sqrt{Qt}) - \left\{ \frac{Q(tb+1)+b}{b^2} - (Q+b) \right\} \frac{1}{2\pi t} e^{-Qt},$$

$$f_5(t) = -\sqrt{M_1} \operatorname{erf}(\sqrt{M_1 t}) - \frac{1}{\sqrt{\pi t}} e^{-(M_1 t)},$$

$$f_6(t) = \left\{ \left( \frac{\beta t + 1}{\beta^2} \right) \sqrt{M_1} + \frac{1}{2\beta\sqrt{M_1}} \right\} \operatorname{erf}(\sqrt{M_1 t}) - \left( \frac{\sqrt{M_1 + \beta}}{\beta^2} \right) e^{\beta t} \operatorname{erf}(\sqrt{(M_1 + \beta)t}) + \frac{1}{\beta} \sqrt{\frac{t}{\pi}} e^{-(M_1 t)},$$

$$f_7(t) = -\left\{ A_1 + B_1 \left( \frac{1}{2\sqrt{M_1}} + t \right) \right\} \operatorname{erf}(\sqrt{M_1 t}) - \frac{e^{-(M_1 t)}}{\sqrt{\pi t}} (A_1 + B_1 t + C_1 + D_1) - C_1 e^{\beta_1 t} (\sqrt{M_1 + \beta_1}) \operatorname{erf}(\sqrt{(M_1 + \beta_1)t}) - D_1 e^{\beta t} (\sqrt{M_1 + b}) \operatorname{erf}(\sqrt{(M_1 + b)t}),$$

$$f_8(t) = -\left\{ A_1 + B_1 \left( \frac{1}{2\sqrt{M_1}} + t \right) \right\} \operatorname{erf}(\sqrt{M_1 t}) - \frac{e^{-(M_1 t)}}{\sqrt{\pi t}} (A_1 + B_1 t + C_1 + D_1) - C_1 e^{\beta_1 t} (\sqrt{M_1 + \beta_1}) \operatorname{erf}(\sqrt{(M_1 + \beta_1)t}) - D_1 e^{\beta t} (\sqrt{M_1 + b}) \operatorname{erf}(\sqrt{(M_1 + b)t}),$$

$$f_9(t) = -\frac{e^{\beta t}}{\beta^2} (\sqrt{(\beta+Q)a}) \operatorname{erf}(\sqrt{(\beta+Q)t}) + \frac{1}{2\beta\sqrt{Q}} \operatorname{erf}(\sqrt{Qt}) + \left( \frac{t\beta+1}{\beta^2} \right) (\sqrt{aQ}) \operatorname{erf}(\sqrt{Qt}) + \frac{1}{2\beta} \sqrt{\frac{at}{\pi}} e^{-Qt},$$

$$e_1(t) = -\frac{e^{\beta_1 t}}{\beta_1^2} (\sqrt{(\beta_1 + K_r)S_c}) \operatorname{erf}(\sqrt{(\beta_1 + K_r)t}) + \frac{1}{2\beta\sqrt{K_r}} \operatorname{erf}(\sqrt{K_r t}) + \left( \frac{t\beta_1+1}{\beta_1^2} \right) (\sqrt{S_c K_r}) \operatorname{erf}(\sqrt{K_r t}) + \frac{1}{2\beta_1} \sqrt{\frac{S_c t}{\pi}} e^{-(K_r t)},$$

$$e_2(t) =$$

$$-\frac{B_1}{2\sqrt{K_r}} \operatorname{erf}(\sqrt{K_r t}) - (A_1 + B_1 t) (\sqrt{S_c K_r}) \operatorname{erf}(\sqrt{K_r t}) - (A_1 + B_1 t + C_1 + D_1) \sqrt{\frac{S_c}{\pi t}} e^{-(K_r t)} -$$

$$C_1 e^{\beta_1 t} (\sqrt{(\beta_1 + K_r)S_c}) \operatorname{erf}(\sqrt{(\beta_1 + K_r)t}) - D_1 e^{\beta t} (\sqrt{(b + K_r)S_c}) \operatorname{erf}(\sqrt{(b + K_r)t}),$$

$$e_3(t) =$$

$$-\frac{B_2}{2\sqrt{Q}} \operatorname{erf}(\sqrt{Qt}) - (A_2 + B_2 t) (\sqrt{Qa}) \operatorname{erf}(\sqrt{Qt}) - (A_2 + B_2 t + C_2 + D_2) \sqrt{\frac{a}{\pi t}} e^{-Qt} -$$

$$C_2 e^{\beta t} (\sqrt{(\beta+Q)a}) \operatorname{erf}(\sqrt{(\beta+Q)t}) - D_2 e^{\beta t} (\sqrt{(b+Q)a}) \operatorname{erf}(\sqrt{(b+Q)t}),$$

$$e_4(t) = \sqrt{Qa} \operatorname{erf}(\sqrt{Qt}) + \sqrt{\frac{a}{\pi t}} e^{-Qt},$$

$$e_5(t) = \frac{(b-Qd)}{b} (\sqrt{S_c K_r}) \operatorname{erf}(\sqrt{K_r t}) + \frac{d(Q+b)}{b} e^{\beta t} \left\{ (\sqrt{(b+K_r)S_c}) \operatorname{erf}(\sqrt{(b+K_r)t}) + (\sqrt{(b+Q)a}) \operatorname{erf}(\sqrt{(b+Q)t}) \right\} +$$

$$(1+d) \sqrt{\frac{S_c}{\pi t}} e^{-(K_r t)} + \frac{(2dQ+b)}{b} \sqrt{\frac{a}{\pi t}} e^{-Qt},$$

$$e_6(t) = \frac{1}{\beta} f_5(t) - \frac{1}{\beta} \left\{ (\sqrt{M_1 + \beta}) \operatorname{erf}(\sqrt{(M_1 + \beta)t}) + \frac{1}{\sqrt{\pi t}} e^{-(M_1 t)} \right\},$$

$$e_7(t) = A_3 f_5(t) - B_3 e^{\beta t} (\sqrt{M_1 + b}) \operatorname{erf}(\sqrt{(M_1 + b)t}) - C_3 e^{\beta_1 t} (\sqrt{M_1 + \beta_1}) \operatorname{erf}(\sqrt{(M_1 + \beta_1)t}) - (B_3 + C_3) \frac{1}{\sqrt{\pi t}} e^{-(M_1 t)},$$

$$e_8(t) = A_4 f_5(t) - B_4 e^{\beta t} (\sqrt{M_1 + b}) \operatorname{erf}(\sqrt{(M_1 + b)t}) - C_4 e^{\beta t} (\sqrt{M_1 + \beta}) \operatorname{erf}(\sqrt{(M_1 + \beta)t}) - (B_4 + C_4) \frac{1}{\sqrt{\pi t}} e^{-(M_1 t)},$$

$$e_9(t) = \frac{\sqrt{aQ}}{\beta} \operatorname{erf}(\sqrt{Qt}) - \frac{e^{\beta t}}{\beta} (\sqrt{a(Q+\beta)}) \operatorname{erf}(\sqrt{(\beta+Q)t}),$$

$$e_{10}(t) = \frac{(\sqrt{S_c K_r})}{\beta} \operatorname{erf}(\sqrt{K_r t}) - \frac{e^{\beta_1 t}}{\beta_1} (\sqrt{S_c (K_r + \beta_1)}) \operatorname{erf}(\sqrt{(\beta_1 + K_r)t}),$$

$$e_{12}(t) =$$

$$-A_3 (\sqrt{S_c K_r}) \operatorname{erf}(\sqrt{K_r t}) - B_3 e^{\beta t} (\sqrt{(b+K_r)S_c}) \operatorname{erf}(\sqrt{(b+K_r)t}) - C_3 e^{\beta_1 t} (\sqrt{(\beta_1 + K_r)S_c}) \operatorname{erf}(\sqrt{(\beta_1 + K_r)t}) -$$

$$(A_3 + B_3 + C_3) \sqrt{\frac{S_c}{\pi t}} e^{-(K_r t)},$$

$$e_{13}(t) = -A_4 (\sqrt{aQ}) \operatorname{erf}(\sqrt{Qt}) - B_4 e^{\beta t} (\sqrt{(b+Q)a}) \operatorname{erf}(\sqrt{(b+Q)t}) - C_4 e^{\beta t} (\sqrt{(\beta+Q)a}) \operatorname{erf}(\sqrt{(\beta+Q)t}) -$$

$$(A_3 + B_3 + C_3) \sqrt{\frac{a}{\pi t}} e^{-Qt}.$$

## RESULTS AND DISCUSSION

In order to get physical insight into the problem we have carried out numerical calculations for non-dimensional velocity, temperature and concentration fields for different values of physical parameters involved and these values have been demonstrated in graphs. Magnetic field parameter ( $M$ ), Hall parameter ( $m$ ), radiation parameter ( $N$ ), permeability parameter ( $K_p$ ), Prandtl number ( $P_r$ ), Grashof number ( $G_r$ ), Modified Grashof number ( $G_m$ ), Chemical reaction parameter ( $K_r$ ), Schmidt number ( $S_c$ ), Soret parameter ( $S_0$ ), heat absorption coefficient ( $Q$ ) and rotation parameter ( $\Omega$ ) have been considered arbitrarily. To study the effects of various parameters on the fluid velocity, for both ramped temperature and isothermal plates, the numerical values of fluid velocity, computed from the analytical solutions reported above, are displayed graphically.

### **Temperature Profile:**

It is evident from figure 2 that temperature profile increases with an increase in radiation parameter ( $N$ ) for both ramped and isothermal surface which implies that, for both ramped and isothermal plate, thermal radiations tends to enhance the fluid temperature. From figure 3, it can be seen that with the increase in Prandtl number ( $P_r$ ), temperature profile decreases for both ramped and isothermal surfaces.

### **Concentration Profile:**

To study the effects of various parameters on the fluid concentration, for both ramped temperature and isothermal plates, the numerical values of fluid concentration, computed from the analytical solutions reported above, are displayed graphically in figure 4,5 & 6 for different values of chemical reaction parameter ( $K_r$ ), Schmidt number ( $S_c$ ) and Soret number ( $S_o$ ) by taking  $t = 0.3$  (arbitrarily) respectively. Figure 4, 5 & 6 depicts that with an increase in chemical reaction parameter ( $K_r$ ) and Schmidt number ( $S_c$ ), the fluid concentration decreases for both ramped temperature and isothermal plate and reverse effect is obtained for Soret number ( $S_o$ ).

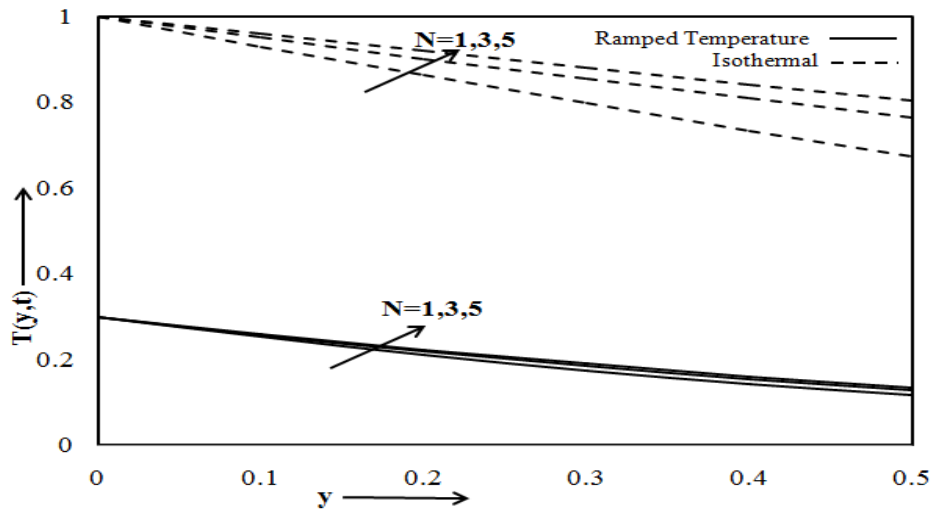
### **Velocity Profile:**

It is noticed that for both ramped and isothermal surfaces the primary and secondary velocities attain a distinctive maximum value in the domain of plate and then decrease with increase in boundary layer co-ordinate to approach the free stream. Figure 7 & 10 display the effect of magnetic field parameter ( $M$ ) on primary and secondary velocities and it is seen from these figures that the primary as well as secondary velocity decreases with the increase in magnetic field parameter ( $M$ ) for both ramped and isothermal surface of the plate. Figure 8 & 11 demonstrate the effect of Hall parameter ( $m$ ) for primary and secondary velocity. It is seen that the primary and secondary motion are accelerated due to Hall parameter ( $m$ ) for both ramped and isothermal surface of the plate. Figures 9 & 12 exhibit the variation of primary and secondary velocity under the influence of rotation parameter ( $\Omega$ ) and it is obtained from this that velocity profile enhances for secondary velocity whereas reverse effect is observed for primary velocity with the increase in rotation parameter ( $\Omega$ ) for both ramped and isothermal surface of plate.

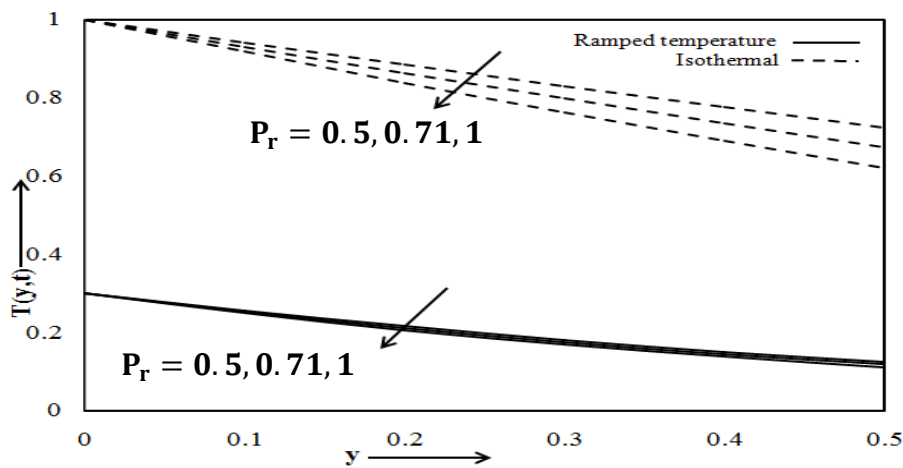
Numerical results are presented in tables 1-4 for skin friction ( $-\tau$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ), table 1 & 2 expresses the behaviour of primary and secondary skin friction ( $-\tau$ ) and table 3&4 represents the rate of heat transfer i.e., Nusselt number and rate of mass transfer ( $Nu$ ) i.e., Sherwood number ( $Sh$ ) respectively for both ramped and isothermal surfaces of the plate. It is obtained from table 1 that with the increase in radiation parameter ( $N$ ), Prandtl number ( $P_r$ ), Hall parameter ( $m$ ) and permeability parameter ( $K_p$ ), primary skin friction ( $-\tau$ ) decreases, while increases for Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ), Magnetic field parameter ( $M$ ) and rotation parameter ( $\Omega$ ) for ramped wall temperature. For isothermal surface it depicts that with the increase in radiation parameter ( $N$ ), permeability parameter ( $K_p$ ), Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ), magnetic field parameter ( $M$ ) and rotation parameter ( $\Omega$ ), primary skin friction ( $-\tau$ ) increases while decreases for Hall parameter ( $m$ ) and permeability parameter ( $K_p$ ), primary skin friction ( $-\tau$ ) almost same for Prandtl number ( $P_r$ ). Table 2 shows the secondary skin friction ( $-\tau$ ) values with various parameters. From table 2 it is obtained for ramped wall temperature that secondary skin friction ( $-\tau$ ) increases with an increase in radiation parameter ( $N$ ), Prandtl number ( $P_r$ ), magnetic field parameter ( $M$ ), Hall parameter ( $m$ ), rotation parameter ( $\Omega$ ) and permeability parameter ( $K_p$ ), while decreases for Grashof number ( $G_r$ ) and modified Grashof number ( $G_m$ ). Table 2 also depicts that secondary skin friction ( $-\tau$ ) increases with radiation parameter ( $N$ ), magnetic field parameter ( $M$ ), Hall parameter ( $m$ ) rotation parameter ( $\Omega$ ) and permeability parameter ( $K_p$ ) while decreases for rest of the parameters in case of isothermal surface.

Table 3 shows that rate of heat transfer i.e, Nusselt number ( $Nu$ ) increases with increase in Prandtl number ( $P_r$ ) and heat absorption coefficient ( $Q$ ) while reverse effect is obtained for radiation parameter ( $N$ ) for both ramped wall temperature and isothermal surface. It is analyzed from table 4 that rate of mass transfer decreases with the increase in Soret parameter ( $S_o$ ) and chemical reaction parameter ( $K_r$ ) while it increases with Schmidt number ( $S_c$ ) ramped wall temperature. For isothermal surface the Sherwood number ( $Sh$ ) tend to reduce with an increase in Soret parameter ( $S_o$ ) while reverse pattern is followed for chemical reaction parameter ( $K_r$ ) and Schmidt number ( $S_c$ ).

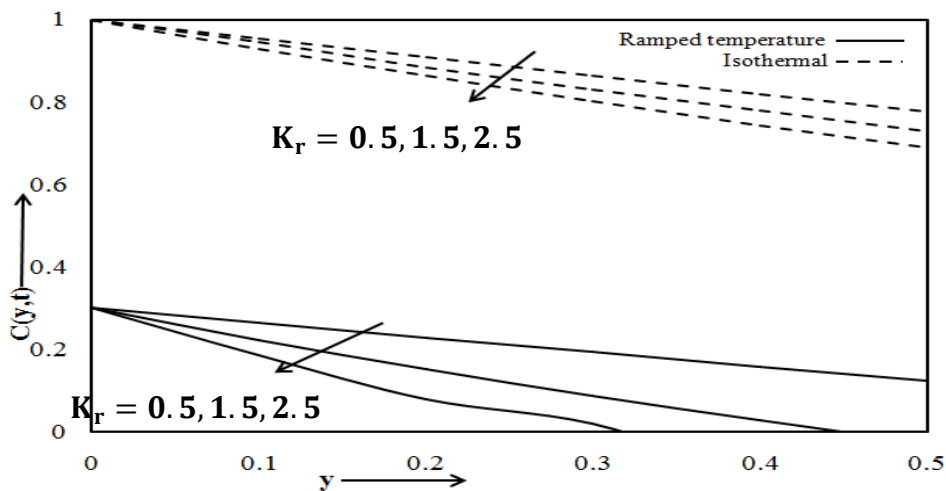




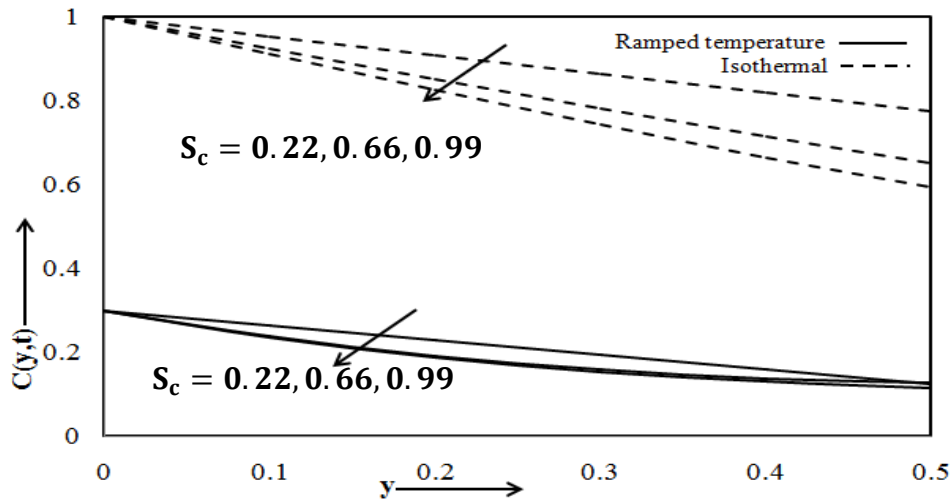
**Figure-2:** Temperature profile for different values of  $N$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $P_r = 0.71$ ,  $K_p = 0.2$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



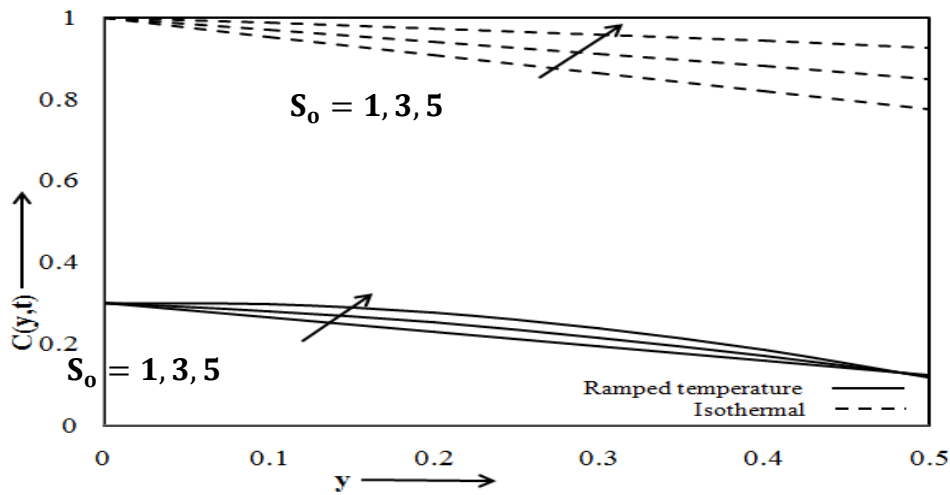
**Figure-3:** Temperature profile for different values of  $P_r$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



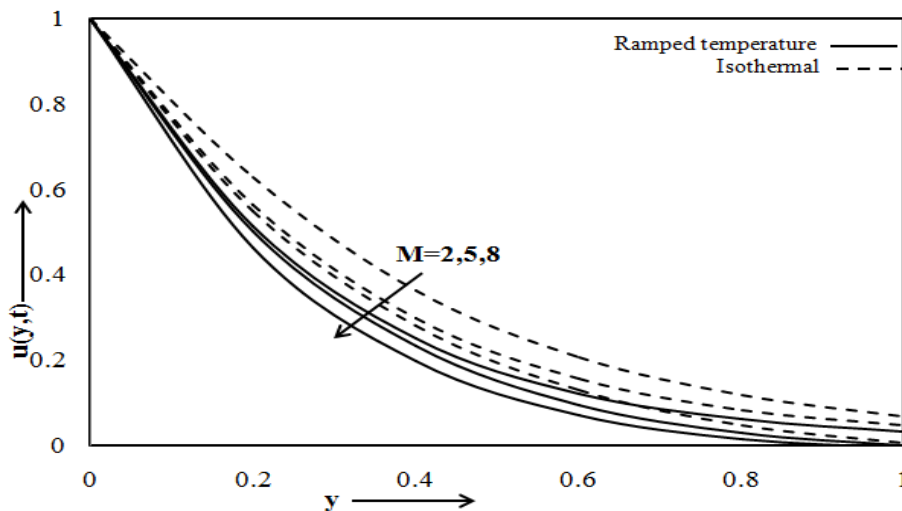
**Figure-4:** Concentration profile for different values of  $K_r$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



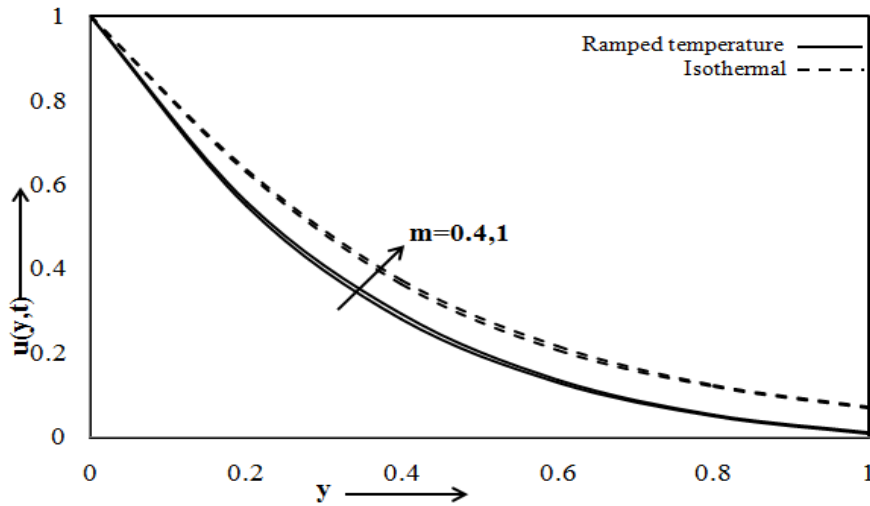
**Figure-5:** Concentration profile for different values of  $S_c$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



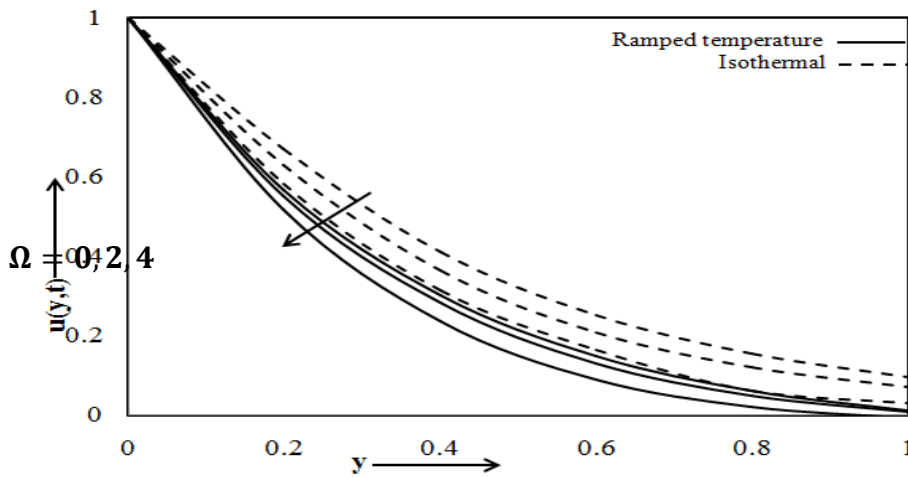
**Figure-6:** Concentration profile for different values of  $S_0$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $\Omega = 2$  and  $t = 0.3$ .



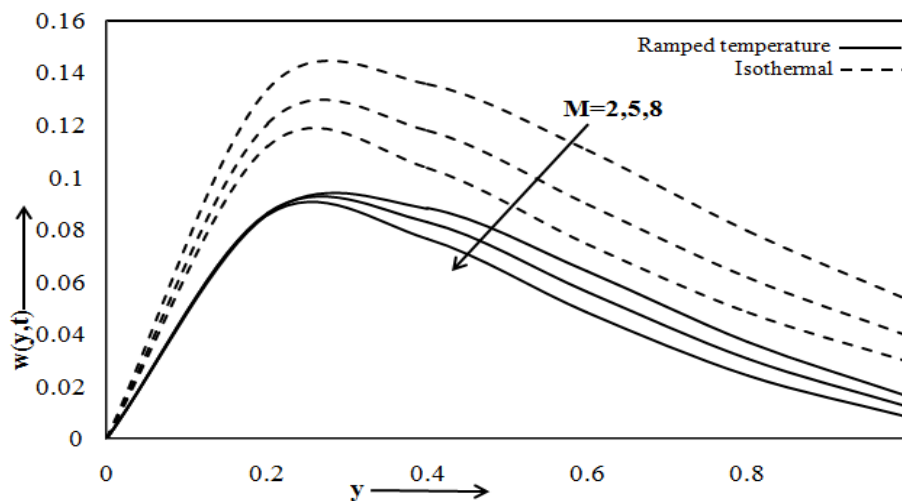
**Figure-7:** Primary velocity profile for different values of  $M$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



**Figure-8:** Primary velocity profile for different values of  $m$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .



**Figure-9:** Primary velocity profile for different values of  $\Omega$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $M = 2$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$  and  $t = 0.3$ .



**Figure-10:** Secondary velocity profile for different values of  $M$  when  $G_r = 0.2$ ,  $G_m = 0.3$ ,  $Q = 0.5$ ,  $m = 0.4$ ,  $N = 1$ ,  $K_p = 0.2$ ,  $P_r = 0.71$ ,  $K_r = 0.5$ ,  $S_c = 0.22$ ,  $S_0 = 1$ ,  $\Omega = 2$  and  $t = 0.3$ .

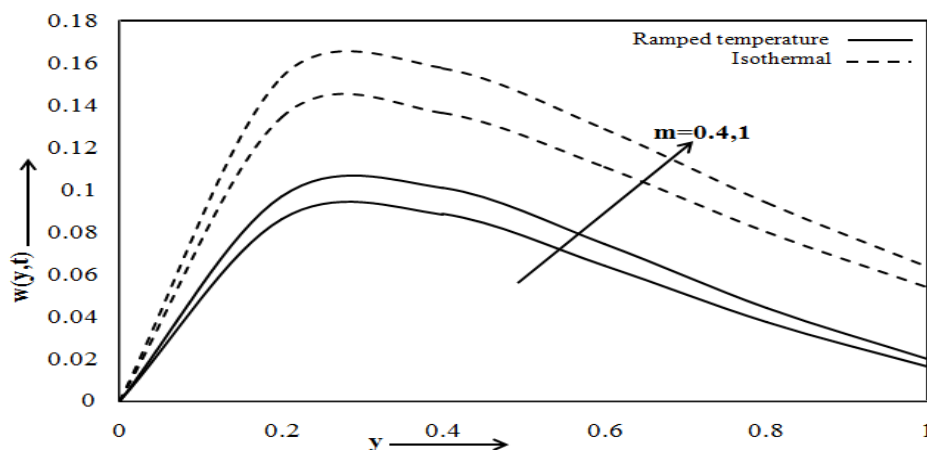


Figure-11: Primary velocity profile for different values of  $m$  when  $G_r = 0.2, G_m = 0.3, Q = 0.5, M = 2, N = 1, K_p = 0.2, P_r = 0.71, K_r = 0.5, S_c = 0.22, S_0 = 1, \Omega = 2$  and  $t = 0.3$ .

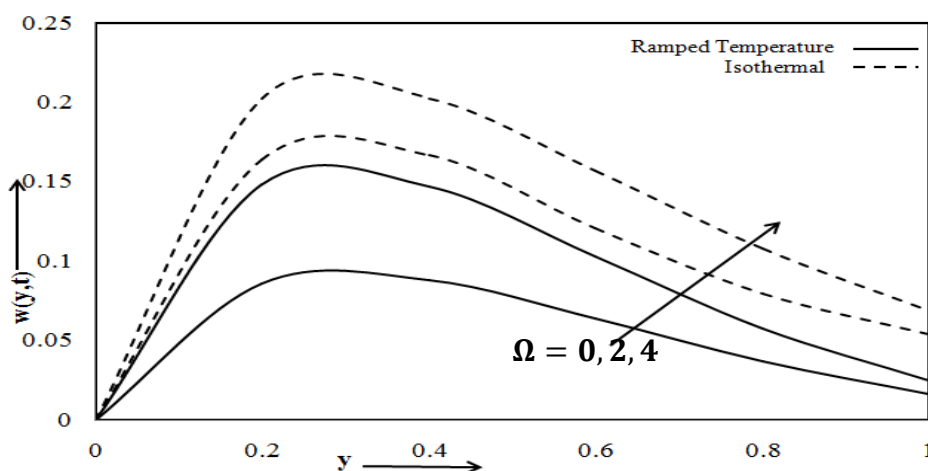


Figure-12: Secondary velocity profile for different values of  $\Omega$  when  $G_r = 0.2, G_m = 0.3, Q = 0.5, M = 2, m = 0.4, N = 1, K_p = 0.2, P_r = 0.71, K_r = 0.5, S_c = 0.22, S_0 = 1$  and  $t = 0$ .

Table-1: Primary skin friction( $-\tau$ ) for ramped wall isothermal temperature.

N	$P_r$	$G_r$	$G_m$	M	m	$\Omega$	$K_p$	Ramped wall temperature( $-\tau$ )	isothermal( $-\tau$ )
1	0.71	0.2	0.3	2	0.4	2	.2	3.8316	3.829
<b>3</b>	0.71	0.2	0.3	2	0.4	2	0.2	3.6596	3.857
1	<b>7.0</b>	0.2	0.3	2	0.4	2	0.2	3.6744	3.823
1	0.71	<b>1</b>	0.3	2	0.4	2	0.2	3.8647	3.876
1	0.71	0.2	<b>0.5</b>	2	0.4	2	0.2	3.8517	3.868
1	0.71	0.2	0.3	<b>5</b>	0.4	2	0.2	4.2404	4.224
1	0.71	0.2	0.3	2	<b>1</b>	2	0.2	3.7229	3.725
1	0.71	0.2	0.3	2	0.4	<b>1</b>	0.2	3.8062	3.8124
1	0.71	0.2	0.3	2	0.4	2	<b>0.4</b>	3.4318	3.441

Table-2: Secondary skin friction( $-\tau$ ) for ramped wall isothermal temperature.

N	$P_r$	$G_r$	$G_m$	M	m	$\Omega$	$K_p$	Ramped temperature( $-\tau$ )	isothermal( $-\tau$ )
1	0.71	0.2	0.3	2	0.4	2	.2	.7247	.68392
<b>3</b>	0.71	0.2	0.3	2	0.4	2	0.2	.8195	.69862
1	<b>7.0</b>	0.2	0.3	2	0.4	2	0.2	.8451	.62149
1	0.71	<b>1</b>	0.3	2	0.4	2	0.2	.7024	.32149
1	0.71	0.2	<b>0.5</b>	2	0.4	2	0.2	.7006	.62943
1	0.71	0.2	0.3	<b>5</b>	0.4	2	0.2	.8237	.79150
1	0.71	0.2	0.3	2	<b>1</b>	2	0.2	.7813	.73935
1	0.71	0.2	0.3	2	0.4	<b>1</b>	0.2	.4117	.38758
1	0.71	0.2	0.3	2	0.4	2	<b>0.4</b>	.7555	.70038

**Table-3:** Nusselt number (Nu) for ramped and isothermal wall temperature.

N	P <sub>r</sub>	Q	Ramped temperature(Nu)	Isothermal (Nu)
1	0.71	0.5	.50531	.70356
<b>3</b>	0.71	0.5	.44349	.43749
1	<b>7.0</b>	0.5	.95698	2.2091
1	0.71	<b>1</b>	.51746	.78917

**Table-4:** Sherwood number (Sh) for ramped and isothermal wall temperature.

S <sub>c</sub>	S <sub>0</sub>	K <sub>r</sub>	Ramped temperature(Sh)	Isothermal (Sh)
0.22	1	0.5	.43441	.79132
<b>0.66</b>	1	0.5	.52309	1.5622
0.22	<b>2</b>	0.5	.40843	.50054
0.22	1	<b>1</b>	.39279	3.0953

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