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TWO -S-THREE TRANSFORMATIONS AND ITS PROPERTIES

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ABSTRACT

This paper introduces a new type of discrete transformation function called 2S3 transformation using SM family of Graphs viz SM sum graphs and SM Balancing graphs. SM sum graphs are associated with the relationship between the powers of 2 and the positive integers. SM balancing graphs are formed by using the fact that all positive integers can be written as a linear combination of powers of 3. These graphs are systematically arranged graphs. Also an extended 2S3 transformation function is introduced. SM family of graphs are vertex labeled graphs. A new type of graph labeling is established exclusively for SM family of graphs. The extended 2S3 transformation function have been used for an edge labeling to the SM sum Graphs. The transformation function $T_s: N' \to N$, *where* $N' = \{1, 2, 3, \ . \ . \ . , n\}$, *is defined as follows. For each* $x = \sum_1^n x_i$ *with distinct* x_i, $x_i = 0$ *or* 2^m *for some* $0 \le m \le n-1$, $T_s(x) = \sum_1^n x_i^*$, *where each* x_i^* *is obtained by changing the base 2 of* x_i*s to base 3. This transformation is called 2S3 transformation function. Some properties of this function is also examined. Let P be the set* $P = \{2^m, 0 \le m \le n-1\}$ *for a fixed positive integer* $n \ge 2$. *Then for* $x \notin P$, $x = \sum_1^n x_i$, *with* $x_i = 0$ *or* 2^m *for some* $0 \le m \le n-1$ *and* x_i*s are distinct. Each* $x_i \ne 0$ *as an additive component of* x. *The extended transformation function is defined as* $T_s(v_x v_{2^j}) = \sum_1^n x_i^* + 3^j$, *where each* x_i^* *is obtained by changing the base 2 of* x_i*s to base 3 and* $v_x v_{2^j}$ *is an edge of the graph* $SM(\Sigma_n)$. *This extended transformation is used to label the edges of SM sum graphs. This labeling is named as 2S3 transformation labeling for SM graphs.*

AMS Subject Classification: *05C99, 54C99.*

Key words: *SM Balancing graphs,* n^{th} *SM sum graphs, 2S3 transformation, 2S3 transformation labeling.*

INTRODUCTION

Discrete transformations are very important and useful in many applications of daily life problems. Some graph theory related functions were used for graph labeling also. Graph labeling is an assignment of labels to the edges or vertices or both of a graph in Graph Theory. Many graph labeling have been introduced by different authors. Graph labeling is used in coding theory, X-ray crystallography, radar, astronomy, Circuit design, communication network addressing, database management etc. In automata theory and formal language, it is better to use labeled multi graphs. The purpose of this paper is to present a new type of discrete transformation function as well as a graph labeling particularly for SM sum graphs using an extended transformation function. Computers use different number systems like binary number system and balanced ternary number system. The combinatorial structure of these two number systems were studied in [4] and [5], using graph theoretical methods. For a fixed positive integer n, consider the set $P = \{2^m, 0 \le m \le n-1\}$. Any positive integer less than 2^n and not in P can be expressed as the sum of two or more distinct elements of P. If $p \notin P$ and $p = \sum_1^n x_i$, with distinct $x_i \in P$, then each x_i is called an additive component of p. The simple graph, $SM(\Sigma_n)$ is a graph with vertex set $\{v_1, v_2, v_3, \dots, v_{2^n-1}\}$ and adjacency of vertices defined by two distinct vertices v_i and v_j are adjacent if either i is an additive component of j or j is an additive component of i. This graph $SM(\Sigma_n)$ is related to combinatorial structure of the binary number system. More over this is related to the low weight polynomial form of integers which was used in elliptical curve cryptography. The Hamming weight of a string was defined as the number of 1s in a string of 0 and 1. Here the number of additive components gives the Hamming weight of string (binary) representation of all numbers in P^c . The Hamming weight of string (binary) representation of numbers in P is always 1. Also consider the set $T = \{3^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Any positive integer $\le \frac{1}{2}(3^n - 1)$, which is not a power of 3 can be expressed as a linear combination of two or more distinct elements of

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the set T with coefficients -1, 0 or 1. These numbers -1, 0 or 1 are the values of the digits in the balanced ternary system. The relation between x and the elements of T are used to form a new class of graphs called n^{th} SM Balancing graphs denoted by $SM(B_n)$. Some preliminaries are given below.

1. PRELIMINARIES

Definition 1.1 [8]: A labeling is called a graceful labeling if there exists an injective function $f: V(G) \rightarrow \{1,2,3,\ldots,q\}$ such that for each edge uv, the labeling $|f(u) - f(v)|$ is distinct. A simple graph G is said to be a graceful graph if G has a graceful labeling.

Definition 1.1 [5]: Consider the set $T = \{3^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let $I = \{-1, 0, 1\}$. Any positive integer $x \le \frac{1}{2}(3^n - 1)$ which is not a power of 3 can be expressed as

$$x = \sum_{j=1}^{n} \alpha_j \, y_j \tag{1}$$

For some $\alpha_j \in I$ and $y_j \in T$, y_j are distinct. If $\alpha_j \neq 0$, then each y_j is called a balancing component of x.

Definition 1.3 [5]: Let T be the set $T = \{3^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Consider the simple graph G = (V, E), where the vertex set $V = \left\{v_1, v_2, v_3, \ldots, v_{\frac{1}{2}(3^n-1)}\right\}$ and adjacency of vertices defined by, two distinct vertices v_x is adjacent to v_{y_j} if (1) holds and $\alpha = -1$, and two distinct vertices v_{y_j} is adjacent to v_x if (1) holds and $\alpha = 1$. This directed graph G is called the n^{th} SMD Balancing graphs denoted by $SMD(B_n)$. The underlying undirected graph is called n^{th} SM Balancing graphs denoted by $SM(B_n)$.

Definition 1.4 [4]: If $p < 2^n$, is a positive integer which is not a power of 2, then $p = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Here we call each $x_i \neq 0$ as an additive component of p.

Definition 1.5 [4]: For a fixed integer $n \ge 2$, define a simple graph $SM(\Sigma_n)$, called the n^{th} SM Sum graph, with vertex set $\{v_1, v_2, v_3, \ldots, v_{2^n-1}\}$ and adjacency of vertices defined by, v_i and v_j are adjacent if either i is an additive component of j or j is an additive component of i.

2. TWO – S- THREE TRANSFORMATIONS AND ITS PROPERTIES

Definition 2.1: Let P be the set $P = \{2^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Each $x_i \neq 0$ and $x_i \in P$ is an additive component of x. Let N be the set of natural numbers and $N' = \{1, 2, 3, \ldots, n\}$. The transformation function $T_s: N' \rightarrow N$ is defined as $T_s(x) = \sum_1^n x_i^*$, where each x_i^* is obtained by changing the base 2 of x_i to base 3. This transformation function is called 2S3 transformations.

The following example illustrates the 2S3 transformation function.

Example 2.2: Let $x = 5 = 2^0 + 2^2$. Then $T_s(\mathrm{x}) = 3^0 + 3^2 = 10$.

Also when $x = 37 = 2^0 + 2^2 + 2^5$, then $T_s(\mathrm{x}) = 3^0 + 3^2 + 3^5 = 253$.

Definition 2.3: Let P be the set $P = \{2^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Each $x_i \neq 0$ and $x_i \in P$ is an additive component of x. Let $a = \sum_1^t a_i x_i$ and $b = \sum_1^r b_i x_i$, for some natural numbers t and r, be two positive integers with $a_i = 0 \; or \; 1 \; and \; b_i = 0 \; or \; 1$. Two numbers a and b are called component independent numbers if there is no term with coefficient $2a_j b_{k-j}$ in the multiplied form of $a \times b$. Otherwise they are called component dependent numbers.

Example 2.4: Let us take two numbers 5 and 10 where $5 = 2^0 + 2^2$ and $10 = 2^1 + 2^3$. When we multiply component wise, $10 \times 5 = (2^0 + 2^2)(2^1 + 2^3) = 2^3 + 2^1 + 2^5 + 2^3 = 2.2^3 + 2^1 + 2^5$. Since there is 2.2^3, 5 and 10 are component dependent numbers.

Now consider another two numbers 6 and 10. We have $6 = 2^1 + 2^2$ and $10 = 2^1 + 2^3$. When we multiply component wise, $10 \times 6 = (2^1 + 2^2)(2^1 + 2^3) = 2^2 + 2^4 + 2^5 + 2^3$. Since there is no term of the form $2a_j b_{k-j} 2^i$, 6 and 10 are component independent numbers.

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Lemma 2.5 [4]: If $= SM(\Sigma_n)$, $P = \{2^m, 0 \leq m \leq n-1\}$, then

$$d(v_i, v_j) = \begin{cases} 1, \ if \ i \ is \ an \ additive \ \text{component} \ of \ j \ or \ j \ is \ an \ additive \ \text{component of i}. \\ 2, if \ i,j \in P \ or \ i,j \notin P, \ i \ and \ j \ have \ atleast \ one \ common \ additive \ component. \\ 3, neither \ i \ nor \ j \ is \ an \ additive \ component \ but \ exactly \ one \ of \ them \ belongs \ to \ P \\ 4, i, j \notin P, i \ and \ j \ have \ no \ common \ additive \ component. \end{cases}$$

Proposition 2.6 [4]: Let $G = SM(\Sigma_n)$ be an n^{th} SM sum graph. Let $d_r(v_i, v_j)$ denotes the number of unordered pairs of vertices for which $d(v_i, v_j) = r$. Then

$$d_r(v_i, v_j) = \begin{cases} n.(2^{n-1} - 1) & , if \ r = 1 \\ \frac{n(n-1)}{2} + \left[\frac{(2^n - n - 2)(2^n - n - 1)}{2} - \delta\right] & , if \ r = 2. \\ (n+1).2^n - (n+2)2^{n-1} - n^2 & , if \ r = 3 \\ \delta & , if \ r = 4 \end{cases}$$

where $\delta = \frac{1}{2}\sum_{r=2}^{n-2}\left[\binom{n}{r}\sum_{k=2}^{n-2}\binom{n-r}{k}\right]$ and $\delta = 0 \ when \ n = 2 \ or \ n = 3$.

Now let us discuss some properties of the 2S3 transformation function. The following theorem illustrates some of the properties of this function.

Theorem 2.7: Let $T_s(x)$ be a 2S3 transformation function. Then the following holds.

i) $T_s(1) = 1$

ii) T_s is one to one.

iii) (Product rule).

If x and y are component independent numbers, then $T_s(xy) = T_s(x)\,T_s(y)$.

iv) In general, $T_s(x+y) \neq T_s(x) + T_s(y)$.

Proof: Let P be the set $P = \{2^m, 0 \leq m \leq n-1\}$ for a fixed positive integer $n \geq 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \leq m \leq n-1$ and x_is are distinct. Each $x_i \neq 0$ and $x_i \in P$ is an additive component of x.

Given that $T_s : N' \to N$ such that $T_s(x) = \sum_1^n x_i^*$, for all $x \in N'$, where $N' = \{1, 2, 3, \ldots, n\}$ and each x_i^* is obtained by changing the base 2 of x_i to base 3.

From the definition it is clear that $T_s(1) = 1$. This proves (i).

Now let $x > y \ and \ x, y \in N'$. By definition, we get $T_s(x) > T_s(y)$ for all $x, y \in N'$.

This implies that $T_s(x)$ is a strictly increasing function. This proves (ii). Therefore T_s is a one to one function.

For proving (iii), consider two positive integers x and y from set N'. Let $x = \sum_{k=0}^{r} a_k 2^k$ and $y = \sum_{k=0}^{m} b_k 2^k$ be two positive integers with $a_i = 0 \ or \ 1 \ and \ b_i = 0 \ or \ 1$.

Then we get the product as $x \times y = \sum_{k=0}^{m+r}\sum_{j=0}^{k} a_j b_{k-j} 2^k$.

Case-1: When x and y are component independent numbers. In this case

$$x \times y = \sum_{k=0}^{r} a_k 2^k . \sum_{k=0}^{m} b_k 2^k = \sum_{k=0}^{m+r}\sum_{j=0}^{k} a_j b_{k-j} 2^k \quad (2)$$

Since x and y are component independent numbers, there will not be any term with coefficient $2a_j b_{k-j}$ in the multiplied form of equation (2). So we can apply the 2S3 transformation function. By using the definition of $T_s(x)$, we get

$$T_s(xy) = \sum_{k=0}^{m+r}\sum_{j=0}^{k} a_j b_{k-j} 3^k. \quad (3)$$

Also $$T_s(x).T_s(y) = \sum_{k=0}^{r} a_k 3^k . \sum_{k=0}^{m} b_k 3^k = \sum_{k=0}^{m+r}\sum_{j=0}^{k} a_j b_{k-j} 3^k \quad (4)$$

From equations (3) and (4), we get $T_s(xy) = T_s(x)\,T_s(y)$ for all integers x and y that are component independent numbers.

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Case-2: When x and y are component dependent numbers, then there will be terms with coefficient $2a_j b_{k-j}$ in the multiplied form of equation (2). So we can not apply the 2S3 transformation as the additive components are not distinct. This completes the proof.

Definition 2.8: Let $T_s : N' \to S$ where $S \subset N$, $N' = \{1, 2, 3, \ . \ . \ ., n\}$ and S is the range of T_s. Then the inverse of T_s from S to N exists and defined as follows. For each $x = \sum_1^n {y_i}^*$, with ${y_i}^* = 0$ or 3^m, for some $0 \leq m \leq n-1$ and ${y_i}^*$s are distinct, the inverse is defined as ${T_s}^{-1}(x) = \sum_1^n y_i$, where each y_i is obtained by changing the base 3 of ${y_i}^*$ to base 2. This ${T_s}^{-1}$ function is named as 3S2 transformation.

Theorem 2.9: Suppose $G = SM(\Sigma_{\text{n}})$ be an n^{th} SM sum graph. Let T_s be the 2S3 transformation. The number of cases in which $T_s(xy) \neq T_s(x)\,T_s(y)$ is denoted by $N(T')$. Then $N(T') < \frac{(2^n-n-2)(2^n-n-1)}{2}$, where x and y are distinct.

Proof: By lemma 2.5, when $d(v_x, v_y) = 1 \ and \ 3$, we can see that x and y are component independent numbers. In these cases we get $T_s(xy) = T_s(x)\,T_s(y)$.

Let $P = \{2^m, 0 \leq m \leq n-1\}$ for a fixed positive integer $n \geq 2$.

When $d(v_x, v_y) = 2$, then two different cases arise.

Case-1: When $x, y \in P$. in this situation it is clear that x and y are component independent numbers. Therefore we get $T_s(xy) = T_s(x)\,T_s(y)$.

Case-2: When $x\,, y \notin P$. Then x and y may or may not be component independent numbers. So $T_s(xy)$ may or may not be equal to $T_s(x)\,T_s(y\,)$ depending on the nature of x and y as given in definition 2.4.

Also when $d(v_x, v_y) = 4$, there may be some cases in which x and y are component dependent numbers. Since G is a simple graph,
$N(T') < d_2(v_x, v_y) + \delta - \frac{n(n-1)}{2} = \frac{(2^n-n-2)(2^n-n-1)}{2}$, where $v_x v_y$ is an edge of G and x and y are distinct. Hence the theorem.

Corollary 2.10: Suppose $G = SM(\Sigma_{\text{n}})$ be an n^{th} SM sum graph and $G^l = SM(\Sigma_{\text{n}})$ be a SM sum graph with single loop on each vertex. Let T_s be the 2S3 transformation. The number of cases in which $T_s(xy) \neq T_s(x)\,T_s(y)$ is denoted by $N(T')$. Then $N(T') < \frac{(2^n-n-2)(2^n-n-1)}{2} + \alpha(G)$, where $\alpha(G)$ is the vertex independence number of G.

Proof: The vertex independence number of G is $2^n - n - 1$. This proves the corollary.

Observation 2.11: Suppose $G = SM(\Sigma_{\text{n}})$ be an n^{th} SM sum graph and $G^l = SM(\Sigma_{\text{n}})$ be a SM sum graph with single loop on each vertex. Let T_s be the 2S3 transformation. Then $\sum_{x \in V(G^l)} T_s\,(x)$ is the sum of all first n natural numbers whose balanced ternary system expression is having digit values only 0 or 1 or 0 and 1.

Theorem 2.12: If ${T_s}^{-1}(x)$ be the 3S2 transformation and S is range of T_s, then the following holds.
(i) ${T_s}^{-1}(1) = 1$
(ii) ${T_s}^{-1}(xy) = {T_s}^{-1}(x).{T_s}^{-1}(y)$ for all integers x, y and $xy \in S$.

Proof: Consider the set $S = \{T_s(x), 1 \leq x \leq n\}$ for a fixed positive integer $n \geq 2$. Then for each $x = \sum_1^n {y_i}^*$, with ${y_i}^* = 0$ or 3^m, for some $0 \leq m \leq n-1$ and ${y_i}^*$s are distinct, the inverse is defined as ${T_s}^{-1}(x) = \sum_1^n y_i$, where each y_i is obtained by changing the base 3 of ${y_i}^*$ to base 2.
It is obvious that ${T_s}^{-1}(1) = 1$.

To prove (ii), when x, y and $xy \in S$, consider $x = \sum_{k=0}^r a_k\, 3^k$ and $y = \sum_{k=0}^m b_k 3^k$ be two positive integers with $a_i = 0 \ or \ 1 \ \ and \ b_i = 0 \ or \ 1$.

$$x \times y = \sum_{k=0}^r a_k\, 3^k . \sum_{k=0}^m b_k 3^k = \sum_{k=0}^{m+r} \sum_{j=0}^k a_j b_{k-j}\, 3^k \quad (5)$$

Since x, y and $xy \in S$, there will not be any term with coefficient $2a_j b_{k-j}$ in the multiplied form of equation (5). So we can apply the 3S2 transformation.

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Using the definition of T_s^{-1}, we get $T_s^{-1}(xy) = \sum_{k=0}^{m+r} \sum_{j=0}^{k} a_j b_{k-j}\, 2^k$ (6)

Also $$T_s^{-1}(x).T_s^{-1}(y) = \sum_{k=0}^{r} a_k\, 2^k . \sum_{k=0}^{m} b_k 2^k = \sum_{k=0}^{m+r} \sum_{j=0}^{k} a_j b_{k-j}\, 2^k \quad (7)$$

From equations (6) and (7) we get

$$T_s^{-1}(xy) = T_s^{-1}(x)T_s^{-1}(y)$$

This completes the proof.

Definition 2.13: Let P be the set $P = \{2^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Each $x_i \ne 0$ and $x_i \in P$ is an additive component of x. The extended transformation function is defined as $T_s(v_x v_{2^j}) = \sum_1^n x_i^* + 3^j$, where the positive integer $x \notin P$, $j \in P$, each x_i^* is obtained by changing the base 2 to 3 and $v_x v_{2^j}$ is an edge. This extended transformation is used to label SM sum graphs. This labeling is called 2S3 transformation labeling of graphs.

Theorem 2.14: The graph $G = SM(\Sigma_n)$ admits 2S3 transformation labeling.

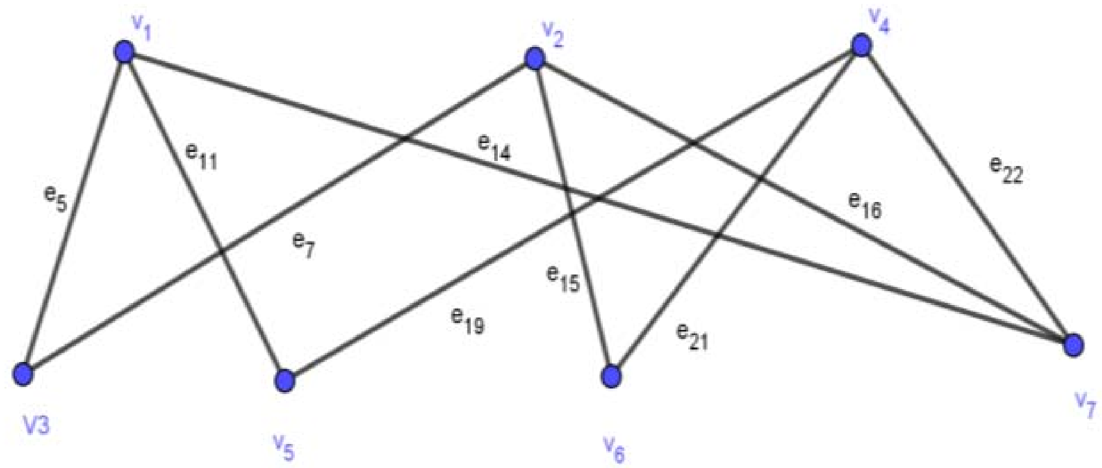
Proof: Consider the graph $G = SM(\Sigma_n)$. Let P be the set $P = \{2^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Each $x_i \ne 0$ and $x_i \in P$ is an additive component of x.

We have $T_s(x) = \sum_1^n x_i^*$, each x_i^* is obtained by changing the base 2 of x_i to base 3.

Now consider the extended 2S3 transformation function $T_s(v_x v_{2^j}) = \sum_1^n x_i^* + 3^j$, where each x_i^* is obtained by changing the base 2 to 3. This is a one to one function. The graph $G = SM(\Sigma_n)$ is a vertex labeled graph. For the edge labeling, we use this extended 2S3 transformation function. This completes the proof.

Example 2.15: Here an example of this 2S3 transformation function labeling is given below.

This figure is an example of 2S3 extended transformation labeling for the graph $G = SM(\Sigma_n)$ for $n = 3$.



Theorem 2.16: There exists an isomorphism between the SM sum graph $SM(\Sigma_n)$ and an edge induced sub graph of the SM balancing graph $SM(B_n)$ for each $n \ge 2$.

Proof: Suppose graph $G = SM(\Sigma_n)$ and $G_1 = SM(B_n)$. We make use of the 2S3 transformation function to show that there exists an isomorphism between the graph G and an edge induced sub graph of $SM(B_n)$.

Let P be the set $P = \{2^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let x be a positive integer. Then $x = \sum_1^n x_i$, with $x_i = 0$ or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Each $x_i \ne 0$ and $x_i \in P$ is an additive component of x.

Given that $T_s : N' \to N$ such that $T_s(x) = \sum_1^n x_i^*$, where $N' = \{1, 2, 3, \ldots, n\}$ and each x_i^* is obtained by changing the base 2 of x_i to base 3.

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Let S be the range of T_s . We define an edge between $T_s(x)$ and each $x_i{}^*$. Let these edge set be E'. Let $G_2 = (S, E')$ be the edge induced sub graph of $SM(\Sigma_n)$ induced by E'.

Since T_s is one to one, $V(G) = V(G_2)$. Also the degrees of corresponding vertices are same. We can clearly observe that T_s preserves the adjacency.
Therefore $G \cong G_2$. This completes the proof.

Corollary 2.17: Let the graph $G = SM(\Sigma_n)$, $P = \{2^m, 0 \le m \le n-1\}$. Let G' be a SM sum graph with a single loop on every vertex v_x such that $x \in P$. Then G' is isomorphic to a sub graph of SM balancing graph with single loop on every vertex where $x \in T = \{3^m, 0 \le m \le n-1\}$

Proof: The proof follows from the above theorem and the 2S3 transformation function.

3. SOME OTHER LABELING FOR SM FAMILY OF GRAPHS

Theorem 3.1: The graph $G = SM(\Sigma_n)$ admits graceful labeling .

Proof: Let the graph $G = SM(\Sigma_n)$ be an SM sum graph. $P = \{2^m, 0 \le m \le n-1\}$. Let $V_1 = P$ and $V_2 = V(G) - P$ be two vertex partitions of G such that $|V_1| = n$ and $|V_2| = 2^n - n - 1$.

Let us assign the values $0,1,2,3, \ldots, n-1$ to the vertices of V_1 *and* $n, 2n, 3n, \ldots, n(2^n - n - 1)$ to the vertices of V_2. Then it is easy to see that the edge labeling for G is a graceful labeling. This completes the proof.

Corollary 3.2: The n^{th} SM balancing graph $G = SM(B_n)$ admits graceful labeling.

Proof: Same as Theorem 3.1.

4. CONCLUSION

Here we have explored the relation between the binary number system and ternary number system by means of the 2S3 transformation. The product rule of this function is related with the distances in the $SM(\Sigma_n)$ graphs. And also as a result, we can not have an isomorphism between $SM(\Sigma_n)$ and $SM(B_n)$. But it has been proved here that there exists an isomorphism between $SM(\Sigma_n)$ and an edge induced sub graph of (B_n), for each $n \ge 2$. The discrete transformation given in this paper may have some more properties and applications in related fields. The newly introduced transformation function is studied through graph theoretical way. Some of the combinatorics and topological indices of these two graphs can be studied and analyzed through this 2S3 transformation function.

5. CONFLICT OF INTEREST

I hereby declare that I have no potential conflict of interest.

6. REFERENCES

1. Acharya B D, “Construction of certain infinite families of graceful graphs from a given graceful graph", Defense Science Journal, Vol. 32, No.3, (July 1982), 231 - 236.
2. Harary F, “Graph theory", Addison-Wesly Publishing Company Inc(1994).
3. Kishori P Narayankar, Lokesh S B, Venna Mathad and Ivan Gutman, “Hosoya Polynomial of Hanoi Graphs", Kragujevac Journal of Mathematics, Vol 36, No 1 (2012), 51 - 57.
4. K.G.Sreekumar, K Manilal, "n^{th} SM Sum graphs and Some parameters", International Journal of Mathematical Analysis,Vol-11, No.3 (2017), 105 - 113.
5. K.G.Sreekumar and Dr.Manilal.K, "n^{th} SM balancing graphs and some of its parameters", International Journal of Computer and Mathematical Science, Vol. 5, Issue 10, October 2016, 56 - 65.
6. Manilal .K , et.al , “The relation between Wiener indices and Nandu Sequence of SM family of Graphs ", Global Journal of Pure and Applied Mathematics, Vol.13, No.2, (2017), 215 - 224.
7. Parthasarathy K R, “Basic Graph theory", Tata Mc Graw Hill New York, 1994.
8. Suresh Singh, “Graph theory", PHI learning Private limited, Publishing Company (2010).

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