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ON FSC, FSS AND FSPS DOMINATION

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ABSTRACT

Let $G_{A,V}$ be a fuzzy soft graph and $S \subseteq V$ is a fuzzy soft dominating set in $G_{A,V}$, then S is said to be a ***fuzzy soft connected (FSC) dominating set*** *if the fuzzy soft graph induced by S $\left(\langle S \rangle\right)$ is connected for each parameter $e \in A$. S is said to be a* ***fuzzy soft point set (FSPS) dominating set*** *if for every $V_1 \subseteq V - S$ and for each parameter $e \in A$, there exist a vertex $x_i \in S$ such that $\langle V_1 \cup \{x_i\} \rangle$ is a connected fuzzy soft graph.*

Key words: *Fuzzy soft connected domination , Fuzzy soft connected domination number, fuzzy soft set domination, fuzzy soft set domination number, fuzzy soft point set domination, fuzzy soft point set domination number.*

1. INTRODUCTION

The concept of domination in fuzzy graphs was first introduced by A Somasundaram and S Somasundaram [8]. The same concept in terms of strong arcs was described by A Nagoorgani [5]. The notion of fuzzy soft graphs was introduced by Sumit Mohinta and Samanta[7] and later on Muhammed Akram and Saira Nawas[6] introduced different types of fuzzy soft graphs and their properties.S. Vimala and J S Sathya[4] introduced the concept of connected and point set domination in fuzzy graphs. In 2017 T K Mathew varkey and Rani Rajeevan[9] introduced the concept of domination in fuzzy soft graphs.

In this paper we introduce three domination parameters such as Fuzzy soft connected (FSC) domination, fuzzy soft set (FSS) domination and fuzzy soft point set (FSPS) domination.

2. PRELIMINARIES

Definition 2.1 [3]: Assume X be a universal set, S be the set of parameters and $P(X)$ denote the power set of X. If there is a mapping $F : S \to P(X)$, then we call the pair (F, S), a ***soft set*** over X.

Definition 2.2 [3]: Let X be a universal set, S be the set of parameters and $E \subset S$. If there is a mapping $F : E \to I^X$, I^X be the set of all fuzzy subsets of S, then we say that (F, E) is a ***fuzzy soft set*** over X.

Definition 2.3 [7]: Let $V = \{x_1, x_2, x_3, \cdots x_n\}$ (non empty set) E (parameters set) and $A \subseteq E$. Also let

(i) $\rho : A \to F(V)$, collection of all fuzzy subsets in V and each element e of A is mapped to $\rho(e) = \rho_e$ (say) and $\rho_e : V \to [0, 1]$, each element x_i is mapped to $\rho_e(x_i)$ and we call (A, ρ), a fuzzy soft vertex.

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(ii) $\mu : A \to F(V \times V)$, collection of all fuzzy subsets in $V \times V$, which mapped each element e to $\mu(e) = \mu_e$ (say) and $\mu_e : V \times V \to [0,1]$, which mapped each element (x_i, x_j) to $\mu_e(x_i, x_j)$, and we call (A, μ) as a fuzzy soft edge.

Then $((A,\rho),(A,\mu))$, is called a ***fuzzy soft graph*** if and only if $\mu_e(x_i, x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)\ \forall e \in A$ and $\forall i, j = 1,2,3,\cdots n$, this fuzzy soft graph is denoted by $G_{A,V}$.

Definition 2.4[6]: A fuzzy soft graph $H_{A,V}(\tau_e, \sigma_e)$ is called a ***fuzzy soft sub graph*** of $G_{A,V}(\rho_e, \mu_e)$ if $\forall e \in A$ $\tau_e(x_i) \le \rho_e(x_i)\ \forall x_i \in V$ and $\sigma_e(x_i, x_j) \le \mu_e(x_i, x_j)\ \forall x_i, x_j \in V$.

Definition 2.5[7]: The underlying ***crisp graph of a fuzzy soft graph*** $G_{A,V} = ((A,\rho),(A,\mu))$ is denoted by $G^* = (\rho^*, \mu^*)$, where $\rho^* = \{x_i \in V;\ \rho_e(x_i) > 0\ \ for\ some\ e \in E\}$ and

$\mu^* = \{(x_i, x_j) \in V \times V;\ \mu_e(x_i, x_j) > 0\quad for\ some\ e \in E\}$.

Definition 2.5 [6]: A fuzzy soft graph $G_{A,V}$ is called a ***strong fuzzy soft graph*** if

$\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \forall (x_i, x_j) \in \mu^*, \forall e \in A.$ and is called a ***complete fuzzy soft graph*** if

$\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \forall x_i, x_j \in \rho^*, \forall e \in A.$

Definition 2.6 [6]: Let $G_{A,V}$ be a fuzzy soft graph. Then the ***order*** of $G_{A,V}$ is defined as $O(G_{A,V}) = \sum_{e \in A}\left(\sum_{x_i \in V} \rho_e(x_i)\right)$ and ***size*** of $G_{A,V}$ is defined as $S(G_{A,V}) = \sum_{e \in A}\left(\sum_{x_i, x_j \in V} \mu_e(x_i, x_j)\right)$ and the ***degree*** of a vertex x_i is defined as $d_{(G_{A,V})}(x_i) = \sum_{e \in A}\left(\sum_{x_j \in V\ x_i \ne x_j} \mu_e(x_i, x_j)\right)$.

Definition 2.7: ***Degree*** of a fuzzy soft graph $G_{A,V}$ is defined as $D_{G_{A,V}} = \max\{d_{G_{A,V}}(x_i);\ x_i \in V\}$

Definition 2.8: A fuzzy soft graph $G_{A,V}$ is said to be ***regular fuzzy soft graph*** if the fuzzy graph corresponding to each parameter $e \in A$ is a regular fuzzy graph.

Definition 2.9: A fuzzy soft sub graph $H_{A,V}(\tau_e, \sigma_e)$ is said to be a ***spanning fuzzy soft sub graph*** if $\forall e \in A$ $\tau_e(x_i) = \rho_e(x_i)\ \forall x_i \in V$ and $\sigma_e(x_i, x_j) \le \mu_e(x_i, x_j)\ \forall x_i, x_j \in V$.

Definition 2.10: A fuzzy soft sub graph $H_{A,V}(\tau_e, \sigma_e)$ is said to be an ***induced fuzzy soft graph*** if $\forall e \in A$ $\sigma_e(x_i, x_j) = \tau_e(x_i) \wedge \tau_e(x_j) \wedge \mu_e(x_i, x_j)\ \forall x_i, x_j \in V$ and is denoted by $\langle \tau_e \rangle$. In other words a fuzzy soft sub graph induced by τ_e is the maximal fuzzy soft sub graph that has a fuzzy soft vertex set τ_e.

Definition 2.11: A ***path*** of length 'n' in a fuzzy soft graph is a sequence of distinct vertices $x_1, x_2, x_3, \ldots. x_n$ such that $\forall e \in A\ \mu_e(x_{i-1}, x_i) > 0 and\ \forall i = 1,2,3,\ldots,n.$

Definition 2.12: If any two vertices of a fuzzy soft graph can be connected by a path for each parameter $e \in A$, then it is said to be a ***fuzzy soft connected graph***.

Definition 2.13[9]: Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be two vertices of $G_{A,V}$. If $\mu_e(x_i, x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$, then we say that x_i dominates x_j in $G_{A,V}$. A subset S of V is called a ***fuzzy soft dominating set*** if for every $x_j \in V - S,$ there exist a vertex $x_i \in S$ such that x_i dominates x_j.

Definition 2.14[9]: A fuzzy soft dominating set S of a fuzzy soft graph $G_{A,V}$ is said to be a ***minimal fuzzy soft dominating set*** if for each parameter $e \in A$, deletion of an element from S is not a fuzzy soft dominating set.

Definition 2.15[9]: The minimum cardinality of all minimal fuzzy soft dominating set is called the ***fuzzy soft domination number*** and is denoted by $\gamma_{fs}(G_{A,V})$.

3. FSC, FSS AND FSPS DOMINATION

Definition 3.1: Let $G_{A,V}$ be a fuzzy soft graph and $S \subseteq V$ be a fuzzy soft dominating set. If $\langle S \rangle$ is connected then S is called ***fuzzy soft connected dominating set***. The minimum fuzzy soft cardinality of a fuzzy soft connected dominating set is called the ***fuzzy soft connected domination number*** and is denoted by $\gamma_{fsc}(G_{A,V})$.

Definition 3.2: A fuzzy soft dominating set $S \subseteq V$ of a fuzzy soft graph $G_{A,V}$ is called ***fuzzy soft set dominating set*** if for every $V_1 \subseteq V - S$ there exist a set $S_1 \subseteq S$ such that $\langle V_1 \cup S_1 \rangle$ is a connected fuzzy soft graph. The minimum fuzzy soft cardinality of a fuzzy soft set dominating set is called the ***fuzzy soft set domination number*** and is denoted by $\gamma_{fss}(G_{A,V})$.

Definition 3.3: A fuzzy soft dominating set $S \subseteq V$ of a fuzzy soft graph $G_{A,V}$ is called ***fuzzy soft point set dominating set*** if for every $V_1 \subseteq V - S$ there exist a vertex $x_i \in S$ such that $\langle V_1 \cup \{x_i\} \rangle$ is a connected fuzzy soft graph. The minimum fuzzy soft cardinality of a fuzzy soft point set dominating set is called the ***fuzzy soft point set domination number*** and is denoted by $\gamma_{fsp}(G_{A,V})$.

Theorem 3.4: In a fuzzy soft graph any fuzzy soft point set dominating set is a fuzzy soft set dominating set.

Proof: Let $S \subseteq V$ be a fuzzy soft point set dominating set. That is for any $V_1 \subseteq V - S$ there exist a vertex $x_i \in S$ such that $\langle V_1 \cup \{x_i\} \rangle$ is a connected fuzzy soft graph .Since singleton sets are subsets of S, S is a fuzzy soft set dominating set.

Remark 3.5: The converse of the above theorem is true only when the fuzzy soft graph is complete.

Theorem 3.6: In a complete fuzzy soft graph, every fuzzy soft point set dominating set is a fuzzy soft set dominating set and vice versa.

Proof: Suppose S is a complete fuzzy soft graph. Proof of first part of the theorem is same as in theorem3.4.

Conversely suppose that S^1 is an FSS dominating set. Then for every $V_1^1 \subseteq V - S^1$ there exist a set $S_1^1 \subseteq S^1$ such that $\langle V_1^1 \cup S_1^1 \rangle$ is a connected fuzzy soft graph and in a complete fuzzy soft graph every single vertex set is a dominating set. So S^1 is a FSPS dominating set.

Theorem 3.7: In a fuzzy soft graph
(i) $\gamma_{fs}(G_{A,V}) \le \gamma_{fss}(G_{A,V}) \le \gamma_{fsc}(G_{A,V})$.
(ii) $\gamma_{fs}(G_{A,V}) \le \gamma_{fsp}(G_{A,V}) \le \gamma_{fsc}(G_{A,V})$.

Proof:

(i) Since every fuzzy soft dominating set is not a fuzzy soft set dominating set and from the definition of a fuzzy soft set domination number together will imply $\gamma_{fs}(G_{A,V}) \le \gamma_{fss}(G_{A,V})$. Again since every fuzzy soft set dominating set is not a fuzzy soft connected dominating set and the definition of both together will imply $\gamma_{fss}(G_{A,V}) \le \gamma_{fsc}(G_{A,V})$.

(ii) Proof is similar to (i).

Theorem 3.8: Every fuzzy soft connected dominating set is a fuzzy soft set dominating set.

Proof: Let S be a fuzzy soft connected dominating set, then $\langle S \rangle$ is connected. Since $\langle S \rangle$ itself is connected and any addition of the fuzzy soft sub graph induced by any subset of $V - S$ will produce a connected fuzzy soft graph. Hence S is a fuzzy soft set dominating set.

Remark 3.9: Converse of the above theorem is true only when the fuzzy soft graph is complete.

Theorem 3.10: In a fuzzy soft complete graph $\gamma_{fs}(G_{A,V}) = \gamma_{fss}(G_{A,V}) = \gamma_{fsc}(G_{A,V}) = \gamma_{fsp}(G_{A,V}) =$

$$\wedge \left\{ \sum_{e \in A} \rho_e(x_i) \quad \forall x_i \in V \right\}.$$

Proof: Let be a fuzzy soft complete graph. Then every vertex is adjacent to the remaining all vertices. So every single vertex set is a fuzzy soft dominating set. There for fuzzy soft domination number is the minimum fuzzy soft cardinality of singleton vertex set. So $\gamma_{fs}(G_{A,V}) = \wedge \left\{ \sum_{e \in A} \rho_e(x_i) \quad \forall x_i \in V \right\}$.

In a fuzzy soft complete graph each singleton vertex set is a FSC set. So $\gamma_{fsc}(G_{A,V})$ is the minimum fuzzy soft cardinality of singleton vertex set and there for $\gamma_{fsc}(G_{A,V}) = \wedge \left\{ \sum_{e \in A} \rho_e(x_i) \quad \forall x_i \in V \right\}$. Similar proof will hold for $\gamma_{fss}(G_{A,V}) = \gamma_{fsp}(G_{A,V})$. Hence the proof.

5. CONCLUSION

In this paper we defined new concepts such as Fuzzy soft connected domination, Fuzzy soft connected domination number, fuzzy soft set domination, fuzzy soft set domination number, fuzzy soft point set domination, fuzzy soft point set domination number and proved some theorems related to this. Fuzzy soft connected domination, fuzzy soft set domination, fuzzy soft point set domination are very useful for solving wide range of problems.

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