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NEW CONTRA CONTINUITY AND SEPARATION AXIOMS

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ABSTRACT

In this paper, I_g-$\ast$-closed sets and I_g-$\ast$-open sets are used to define and investigate a new class of functions called contra I_g-$\ast$-continuous functions in ideal topological spaces. We discuss the relationships with some other related functions.

1. INTRODUCTION AND PRELIMINARIES

By a space, we always mean a topological space (X, τ) with no separation properties assumed. If $A \subset X$, $cl(A)$ and $int(A)$ will denote the closure and interior of A in (X, τ) respectively.

Definition 1.1: *A subset A of a space (X, τ) is said to be regular open* [16] *(resp. preclosed* [3]*) if $A = int(cl(A))$ (resp. $cl(int(A)) \subseteq A$).*

Definition 1.2: *A subset A of a space (X,τ) is said to be g-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

Definition 1.3: *A space (X, τ) is called locally indiscrete* [12] *if every open set is closed.*

The collection of all clopen subsets of X will be denoted by CO(X). We set $CO(X, x) = \{V \in CO(X) \mid x \in V\}$ for $x \in X$.

Definition 1.4 [15]: *A space (X, τ) is said to be*
1) *Ultra Hausdorff if for each pair of distinct points x and y in X there exist $U \in CO(X, x)$ and $V \in CO(X, y)$ such that $U \cap V = \phi$$.*
2) *Ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.*

Definition 1.5: *A function $f : (X,\tau) \to (Y, \sigma)$ is called preclosed* [3] *if the image of every closed subset of X is preclosed in Y.*

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies
1) $A \in I$ and $B \subseteq A$ imply $B \in I$ and
2) $A \in I$ and $B \in I$ imply $A \cap B \in I$ $.

Given a topological space (X, τ) with an ideal I on X and if **P**(X) is the set of all subsets of X, a set operator $(.)^* : \mathbf{P}(X) \to \mathbf{P}(X)$, called a local function [6] of A with respect to τ and I, is defined as follows: for $A \subset X$, $A^*(\tau, I) = \{x \in X \mid U \cap A \notin I$ $ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator cl*(.) for a topology $\tau^*(\tau, I)$, called the $\ast$-topology, finer than τ, is defined by $cl^*r(A) = A \cup A^*(\tau, I)$$, [17]. When there is no chance of confusion, we will simply write A* for $A^*(\tau, I)$$ and $\tau^\star$ for $\tau^*(\tau, I)$. If I is an ideal on X then (X, τ, I) is called an ideal topological space or an ideal space.

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In this paper, we introduce the notion of contra I_g-$*$-continuity in ideal topological spaces and discuss its properties and various characterizations.

Lemma 1.6 [5]: *Let (X, τ, I) be an ideal space and A, B subsets of X. Then the following properties hold:*
1) $A \subseteq B \to A^* \subseteq B^*$,
2) $A^* = cl(A^*) \subseteq cl(A)$,
3) $(A^*)^* \subseteq A^*$,
4) $(A \cup B)^* = A^* \cup B^*$,
5) $(A \cap B)^* \subseteq A^* \cap B^*$.

Definition 1.7: *A subset A of an ideal topological space (X,τ, I) is said to be $*$-closed (τ^*-closed)* [5] *if $A^* \subseteq A$. The complement of a $*$-closed set is called $*$-open.*

Definition 1.8 [8]: *Let (X,τ, I) be an ideal topological space and $A \subseteq X$. Then A is said to be $*$-g-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $*$-open in X. The complement of a $*$-g-closed set is called a $*$-g-open set.*

Definition 1.9: *A subset A of an ideal space (X, τ, I) is said to be I_g-$*$-closed* [13] *if $A^* \subseteq U$ whenever $A \subseteq U$ and U is $*$-open in X.*
The complement of an I_g-$*$-closed set is called I_g-$*$-open.
The family of all I_g-$*$-open sets of $(X,\tau, I_g)$$ is denoted by IG O(X).*

Definition 1.10: *A subset A of an ideal space $(X, \tau, I)$$ is said to be*
1) I_g-$*$-closed [10, 11] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
2) I_g-$*$-closed [2, 9] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Remark 1.11: *In ideal space, the following properties hold.*
1) *every closed set is $*$-closed but not conversely* [8].
2) *every $*$-g-closed set is g-closed but not conversely* [8].
3) *every g-closed set is I_g-closed but not conversely* [2, 9].
4) every I_g-closed set is I_{rg}--closed but not conversely [10,11].

Definition 1.12: *A function $f: (X, \tau, I) \to (Y,\sigma)$ is called* Ig-continuous [4] *if the inverse image of every closed set in Y is I_g-closed in X.*

Definition 1.13: *A function $f : (X, \tau, I) \to (Y, \sigma)$$ is said to be*
1) *contra continuous* [1] *if for each open set V in Y, $f^{1}(V)$ is closed in X,*
2) *contra $*$-g-continuous* [13] *if for each open set V in Y, $f^{1}(V)$ is $*$-g-closed in X.*
3) *I_g-$*$-continuous* [13] *if $f^{1}(V)$ is I_g-$*$-closed in (X,τ, I) for each closed set V in (Y, σ).*
4) *contra I_g-continuous* [14] *if $f^{1}(V)$ is I_g-closed in (X,τ, I) for each open set V in (Y, σ).*

2. CONTRA I_g-$*$-CONTINUITY

Definition 2.1: *An ideal topological space (X, τ, I) is said to be*
1) *$**$-g-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $*$-g-open sets.*
2) *I_g-$*$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint I_g-$*$-open sets.*

Example 2.2:
1) *Let X = {a, b, c}, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $I = \{\phi\}$. Then $*$-g-open sets are ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Therefore (X, τ, I) is $**$-g-normal.*
2) *Let X = {a, b, c}, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi, \{a\}\}$ $. Then $*$-g-open sets are ϕ, X, {a}, {b}, {a, b}. Therefore (X, τ, I) is not $*$-g-normal.*

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Example 2.3:
1) *In Example* 2.2, I_g-$*$-*open sets are* $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. *Therefore* (X, τ, I) *is* I_g-$*$-*normal.*
2) *In Example* 2.2, I_g-$*$-*open sets are* $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. *Therefore* (X, τ, I) *is not* I_g-$*$-*normal.*

Definition 2.4: *A function* $f : (X, \tau, I) \to (Y, \sigma)$ *is said to be contra* I_g-$*$-*continuous if* $f^{-1}(V)$ *is* I_g-$*$-*closed in* (X, τ, I) *for each open set V in* (Y, σ).

Proposition 2.5: *Every* I_g-$*$-*closed set is* I_g-*closed.*

Proof: The proof follows from the fact that every open set is $*$-open.

However, converse need not be true as seen from the following Example.

Example 2.6: *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ *and* $I = \{\phi, \{c\}\}$. *Then* $\{b\}$ *is an* I_g-*closed set but not* I_g-$*$-*closed.*

Proposition 2.7: *Every* $*$-*g-closed set is* I_g-$*$-*closed.*

Proof: The proof follows from the fact that $A^* = cl(A)$.

However, converse need not be true as seen from the following Example.

Example 2.8: *Let* $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{b\}, \{b, c, d\}\}$ *and* $I = \{\phi, \{c\}\}$. *Then* $\{c\}$ *is an* I_g-$*$-*closed set but not* $*$-*g-closed.*

Proposition 2.9: *Every contra* $*$-*g-continuous function is contra* I_g-$*$-*continuous.*

Proof: Let $f: (X, \tau, I) \to (Y, \sigma)$ be a contra $*$-g-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is $*$-g-closed in X. Since every $*$-g-closed set is I_g-$*$-closed, $f^{-1}(V)$ is I_g-$*$-closed in X. Therefore f is contra I_g-$*$-continuous.

However, converse need not be true as seen from the following Example.

Example 2.10: *Let* $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\phi, X, \{a\}\}$, $I = J = \{\phi, \{a\}\}$. *Then the identity function* $f: (X,\tau, I) \to (Y, \sigma, J)$ *is contra* I_g-$*$-*continuous but not contra* $*$-*g-continuous.*

Remark 2.11: *The following Example shows that* I_g-$*$-*continuity and contra* I_g-$*$-*continuity are independent.*

Example 2.12:
1) *Let R be the set of all real numbers and* τ_u *the usual topology on R. The identity function* $f : (R, \tau_u, I = \{\phi\}) \to (R, \tau_u)$ *is continuous and hence* I_g-$*$-*continuous but not contra* I_g-$*$-*continuous, since the preimage of each singleton fails to be* I_g-$*$-*open.*
2) *Let* $X = \{a, b\}$ *be the Sierpinski space with the topology* $\tau = \{\phi, \{a\}, X\}$ *and* $I = \{\phi\}$. *Let* $f : (X, \tau, I) \to (X, \tau)$ *be defined by* $f(a) = b$ *and* $f(b) = a$. *Since the inverse image of every open set is* I_g-$*$-*closed, then f is contra* I_g-$*$-*continuous, but* $f^{-1}(\{b\})$ *is not* I_g-$*$-*closed in* $(X, \tau, I)$$. *Therefore f is not* I_g-$*$-*continuous.*

Proposition 2.13: *Every contra* I_g-$*$-*continuous function is contra* I_g-*continuous.*

Proof: The proof follows from the fact that every I_g-$*$-closed set is I_g-closed.

However, converse need not be true as seen from the following Example.

Example 2.14: *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\},\{c\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{b\}, \{a, c\}, X\}$ *and* $I = \{\phi, \{c\}\}$. *Then the identity function* $f : (X, \tau, I) \to (X, \sigma)$ *is contra* I_g-*continuous but not contra* I_g-$*$-*continuous.*

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Proposition 2.15: *Every contra continuous function is contra I_g-$\ast$-continuous.*

Proof: Let $f : (X, \tau, I) \to (Y, \sigma)$ be a contra continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is closed in X. Since every closed set is I_g-$\ast$-closed by Remark 1.11 (1) and Proposition 2.7, $f^{-1}(V)$ is I_g-$\ast$-closed in X.

However, converse need not be true as seen from the following Example.

Example 2.16: *In Example 2.10, the identity function $f : (X, \tau, I) \to (Y, \sigma, J)$ is contra I_g-$\star$-continuous but not contra continuous.*

Theorem 2.17: *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function. Then the following are equivalent:*
1) *f is contra I_g-$\ast$-continuous.*
2) *The inverse image of each closed set in Y is I_g-$\ast$-open in X.*
3) *For each point x in X and each closed set V in Y with $f(x) \in V$, there is an*
4) *I_g-$\ast$-open set U in X containing x such that $f(U) \subset V$.*

Proof:
(1) ⇒ (2): Let F be closed in Y. Then Y-F is open in Y. By definition of contra I_g-$\ast$-continuity, $f^{-1}(Y-F)$ is I_g-$\ast$-closed in X. But $f^{-1}(Y-F) = X-f^{-1}(F)$. This implies $f^{-1}(F)$ is I_g-$\ast$-open in X.

(2) ⇒ (3): Let $x \in X$ and V be any closed set in Y with $f(x) \in V$. By (2), $f^{-1}(V)$ is I_g-$\ast$-open in X. Set $U = f^{-1}(V)$. Then there is an I_g-$\ast$-open set U in X containing x such that $f(U) \subset V$.

(3) ⇒ (1): Let $x \in X$ and V be any closed set in Y with $f(x) \in V$. Then Y-V is open in Y with $f(x) \in V$. By (3), there is an I_g-$\ast$-open set U in X containing x such that $f(U) \subset V$. This implies $U = f^{-1}(V)$\$.
Therefore, $X - U = X - f^{-1}(V) = f^{-1}(Y - V)$ which is I_g-$\ast$-closed in X.

Theorem 2.18: *Let $f : (X, \tau, I) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \mu)$\$. Then the following properties hold:*
1) *If f is contra I_g-$\ast$-continuous and g is continuous then g o f is contra I_g-$\ast$-continuous.*
2) *If f is contra I_g-$\ast$-continuous and g is contra continuous then g o f is I_g-$\ast$-continuous.*
3) *If f is I_g-$\ast$-continuous and g is contra continuous then g o \$ is contra I_g-$\ast$-continuous.*

Proof:
(1) Let V be a closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is contra I_g-$\ast$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is I_g-$\ast$-open in X. Therefore g o f is contra I_g-$\ast$-continuous.
(2) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is contra I_g-$\ast$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is I_g-$\ast$-closed in X. Therefore g o f is I_g-$\ast$-continuous.
(3) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is I_g-$\ast$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is I_g-$\ast$-open in X. Therefore g o f is contra I_g-$\ast$-continuous.

Theorem 2.19: *If a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra I_g-$\ast$-continuous and Y is regular, then f is I_g-$\ast$-continuous.*

Proof: Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $cl(W) \subset V$. Since f is contra I_g-$\ast$-continuous, by Theorem 2.17, there exists an I_g-$\ast$-open set U containing x such that $f(U) \subset cl(W)$. Thus $f(U) \subset cl(W) \subset V$. Hence f is I_g-$\ast$-continuous.

Definition 2.20: *A space (X, τ, I) is said to be an I_g-$\ast$-space if every I_g-$\ast$-open set of X is open in X.*

Example 2.21:
1) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi, \{a, c\}\}$. Then I_g-$\ast$-open sets are $\{a\}, \{b\}, \{a, b\}, \phi, X$. Then (X, τ, I) is I_g-$\ast$-space.*
2) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{a, c\}\}$ and $I = \{\phi, \{c\}\}$. Then (X, τ, I) is not I_g-$\ast$-space.*

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Theorem 2.22: *If a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra Ig-$*$-continuous and X is an I_g-$*$-space then f is contra continuous.*

Proof: Let *V* be a closed set in *Y*. Since *f* is contra I_g-$*$-continuous, $f^{-1}(V)$ is I_g-$*$-open in *X*. Since *X* is an I_g-$*$-space, $f^{-1}(V)$ is open in *X*. Therefore *f* is contra continuous.

Definition 2.23: *An ideal topological space (X,τ, I) is said to be I_g-$*$-T_2 space if for each pair of distinct points x and y in (X, τ, I), there exist an I_g-$*$-open set U containing x and an I_g-$*$-open set V containing y such that $U \cap V = \phi$.*

Example 2.24:

1) *Let X = {a, b, c}, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $I = \{\phi\}$. Then (X, τ, I) is $\mathcal{I} g$-$*$-T_2 space.*
2) *Let X = {a, b, c}, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi\}$. Then (X, τ, I) is not I_g-$*$-T_2 space.*

Theorem 2.25: *If (X, τ, I) is an ideal topological space and for each pair of distinct points x_1, x_2 in X, there exists a function f from (X, τ, I) into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is contra I_g-$*$-continuous at x_1 and x_2, then X is I_g-$*$-T_2.*

Proof: Let x_1 and x_2 be any two distinct points in *X*. Then by hypothesis, there is a function $f : (X, \tau, I) \to (Y, \sigma)$, such that $f(x_1) \neq f(x_2)$. Let $y_i = f(x_i)$ for $i = 1, 2$. Then $y_1 \neq y_2$. Since *Y* is Urysohn, there exist open neighbourhoods V_{y_1} and V_{y_2} of y_1 and y_2 respectively in *Y* such that $cl(V_{y_1}) \cap cl(V_{y_2}) = \phi$. Since *f* is contra I_g-$*$-continuous, there exists an I_g-$*$-open set U_{x_i} of x_i in *X* such that $f(U_{x_i}) \subset cl(V_{y_i})$ for $i = 1, 2$. Hence we get $U_{x_1} \cap U_{x_2} = \phi$ because $cl(V_{y_1}) \cap cl(V_{y_2}) = \phi$. Thus *X* is I_g-$*$-T_2.

Corollary 2.26: *If f is a contra I_g-$*$-continuous injection of an ideal topological space (X, τ, I) into a Urysohn space (Y, σ), then (X,τ, I) is I_g-$*$-T_2.*

Proof: Let x_1 and x_2 be any pair of distinct points in *X*. Since *f* is contra I_g-$*$-continuous and injective, we have $f(x_1) \neq f(x_2)$. Therefore by Theorem 2.25, *X* is I_g-$*$-T_2.

Corollary 2.27: *If f is a contra I_g-$*$-continuous injection of an ideal topological space (X,τ, I) into a Ultra Hausdorff space (Y, σ), then (X, τ, I) is I_g-$*$-T_2.*

Proof: Let x_1 and x_2 be any two distinct points in *X*. Then since *f* is injective and *Y* is Ultra Hausdorff, $f(x_1) \neq f(x_2)$ and there are two clopen sets V_1 and V_2 in *Y* such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. Then $x_i \in f^{-1}(V_i) \in IG^* O(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus *X* is I_g-$*$-T_2.

Theorem 2.28: *If $f : (X, \tau, I) \to (Y, \sigma)$ is a contra I_g-$*$-continuous, closed injection and Y is Ultra normal, then (X,τ, I) is I_g-$*$-normal.*

Proof: Let F_1 and F_2 be disjoint closed subsets of *X*. Since *f* is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of *Y*. Since *Y* is Ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2 respectively. Hence $F_i \subset f^{-1}(V_i)$, $f^{-1}(V_i) \in IG^* O(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus *X* is I_g-$*$-normal.

Definition 2.29: *A graph G(f) of a function $f : (X, \tau, I) \to (Y,\sigma)$ is said to be contra I_g-$*$-closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist an $U \in IG^* O(X)$ containing x and a closed set V of (Y, σ) containing y such that $f(U) \cap V = \phi$.*

Theorem 2.30: *If $f : (X, \tau, I) \to (Y, \sigma)$ is contra I_g-\continuous and (Y, σ) is Urysohn, then G(f) is contra I_g-\closed in $X \times Y$.*

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Proof: Let $(x, y) \in (X \times Y) \setminus G(f)$, then $f(x) \neq y$ and there exist open sets V, W such that $f(x) \in V$, $y \in W$ and $cl(V) \cap cl(W) = \phi$. Since f is contra I_g-$*$-continuous there exists $U \in IG^* O(X)$ containing x such that $f(U) \subset cl(V)$.

Since $cl(V) \cap cl(W) = \phi$, we have $f(U) \cap cl(W) = \phi$. This shows that $G(f)$ is contra I_g-$*$-closed in $X \times Y$.

Remark 2.31: *The following Example shows that the condition Urysohn on the space (Y, σ) in Theorem 2.30 cannot be dropped.*

Example 2.32: *Let $X = Y = \{a, b, c\}$, \tau $= \{\phi, \{a\}, X\} = \sigma$ and $I = \{\phi, \{a\}\}$. Then X is not a Urysohn space. Also the identity function $f : (X, \tau, I) \to (Y,\sigma)$ is contra I_g-$*$-continuous but not contra I_g-$*$-closed.*

Corollary 2.33: *If $f : (X, \tau) \to (Y, \sigma)$ is contra $*$-g-continuous function and (Y, σ) is a Urysohn space, then $G(f)$ is contra-$*$-g-closed in $X \times Y$.*

Proof: The proof follows from the Theorem 2.30 if $I = \{\phi\}$.

Definition 2.34: *An ideal topological space (X, τ, I) is said to be I_g-$*$-connected if (X, τ, I) cannot be expressed as the union of two disjoint nonempty I_g-$*$-open subsets of (X, τ, I).*

Example 2.35:

1) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi\}$. Then I_g-$*$-open sets are $\{a\}$,$\{b\}$,$\{c\}$, $\{a, b\}$, $\{a, c\}$, ϕ, X is I_g-$*$-connected.*
2) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$ and $I = \{\phi, \{c\}\}$. Then (X, τ, I) is not I_g-$*$-connected.*

Theorem 2.36: *A contra I_g-$*$-continuous image of a I_g-$*$-connected space is connected.*

Proof: Let $f : (X, \tau, I) \to (Y, \sigma)$ be a contra I_g-$*$-continuous function of an I_g-$*$-connected space (X, τ, I) onto a topological space (Y, σ). If possible, let Y be disconnected. Let A and B form a disconnection of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \phi$. Since f is contra I_g-$*$-continuous, $X = f^1(Y) = f^1(A \cup B) = f^1\}(A) \cup f^1(B)$, where $f^1(A)$ and $f^1(B)$ are nonempty I_g-$*$-open sets in X. Also $f^1(A) \cap f^1(B) = \phi$. Hence X is not I_g-$*$-connected. This is a contradiction. Therefore Y is connected.

Definition 2.37: *An ideal topological space (X, τ, I) is said to be $*$-g-connected if (X, τ, I) cannot be exp*ressed as the union of two disjoint non-empty $*$-g-open subsets of (X,τ, I).*

Example 2.38:

1) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$ and $I = \{\phi, \{c\}\}$. Then (X,τ, I) is $*$-g-connected.*
2) *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $I = \{\phi\}$. Then (X, τ, I) is not $*$-g-connected.*

Corollary 2.39: *A contra $*$-g-continuous image of a $*$-g-connected space is connected.*

Proof: The proof follows from the Theorem 2.36 if $I = \{\phi\}$.

Lemma 2.40: *For an ideal topological space (X, τ, I)\$, the following are equivalent.*

1) *X is I_g-$*$-connected.*
2) *The only subset of X which are both I_g-$*$-open and I_g-$*$-closed are the empty set ϕ and X.*

Proof:

(1) ⇒ (2): Let F be an I_g-$*$-open and I_g-$*$-closed subset of X. Then $X - F$ is both I_g-$*$-open and I_g-$*$-closed. Since X is I_g-$*$-connected, X can be expressed as union of two disjoint nonempty I_g-$*$-open sets X and $X - F$, which implies $X - F$ is empty.

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(2) ⇒ (1): Suppose $X = U \cup V$ where U and V are disjoint nonempty Ig-∗-open subsets of X. Then U is both Ig-∗-open and I_g-∗-closed. By assumption either $U = \phi$ or X which contradicts the assumption that U and V are disjoint nonempty I_g-∗-open subsets of X. Therefore X is I_g-∗-connected.

Theorem 2.41: *Let $f : (X, \tau, I) \rightarrow (Y,\sigma)$ be a surjective preclosed contra I_g-∗-continuous function. If X is an I_g-∗-space, then Y is locally indiscrete.*

Proof: Suppose that V is open in Y. Since f is contra Ig-∗-continuous, $f^1(V) = U$ is I_g-∗-closed in X. Since X is an Ig-∗-space, U is closed in X. Since f is preclosed, then V is preclosed in Y. Now we have $cl(V) = cl(int(V)) \subset V$. This means that V is closed and hence Y is locally indiscrete.

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