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SOME RESULTS ON DIFFERENTIABLE GRAPHS

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ABSTRACT

Operations have its own value and play a vital role in the development of results in the field of Mathematics. There are many operations by which new graphs are obtained from the old ones and are separated in to unary operations and binary operations. In this paper we try to get a new graph from the old one by introducing a new concept (operation), differentiability and study its various properties.

Keywords: *Operations, Derivative, Differentiable graphs.*

1. INTRODUCTION

The term "operation" plays a vital role in the scientific world especially in Mathematics. Many operations are defined in the field of Graph Theory to get new graphs from the old ones. They may be classified in to two classes namely, unary and binary operations.

Unary operations result a new graph from a given old one by a simple local change such as addition or deletion of a vertex or an edge, edge contraction and subdivision etc.

Binary operations result a new graph from the given two old ones such as union, intersection, conjunction, product, join and so on.

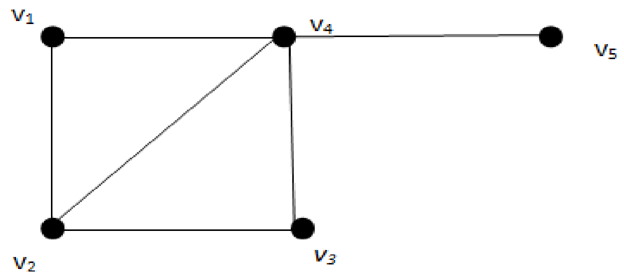
In this paper we try to get a new graph from the old one by introducing a new operation and study its various properties.

1.1 Construction

Let G be a graph of order p and $e = uv$ be an arbitrary edge in G. Now consider the graph $(G.e)$. Remove all the loops from $(G.e)$ if exists. The new graph is denoted as $G^{(1)}$. Next take $G^{(1)}$ and $e = uv$ be an arbitrary edge. Consider $(G^{(1)}.e)$. Remove all the loops from $(G^{(1)}.e)$ (if exists) and denote the new graph as $G^{(2)}$ and so on. Repeat the process till we get a positive integer 'r' such that $G^{(r)} \cong K_1$. $G^{(i)}$ is known as the derivative of $G^{(i-1)}$, $i = 1,2 \ldots, r$.Then this process is known as differentiation and symbolically it will be denoted as $(\frac{dG}{de}$ $where\ e \in E(G))$. Also 'r' is called as the order of differentiation.

Example 1.2:

G:



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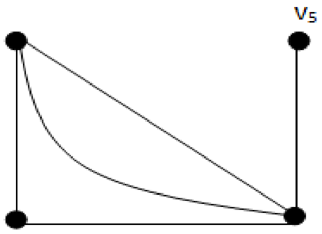
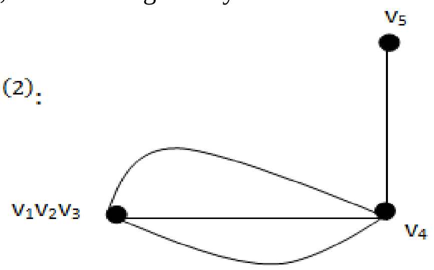
Let e = (v_1, v_2), then $G^{(1)}$ is given by,

$G^{(1)}$:



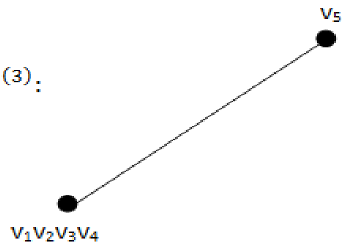
Now, let e=(v_1v_2, v_3), then $G^{(2)}$ is given by

$G^{(2)}$:



Now, let e=($v_1v_2v_3$, v_4), then $G^{(3)}$ is given by

$G^{(3)}$:



Let e=($v_1v_2v_3v_4$, v_5), then $G^{(4)}$ is given by

$G^{(4)}$: $\bullet$

$v_1v_2v_3v_4v_5$, which is isomorhic to K_1.

Observation 1.3:

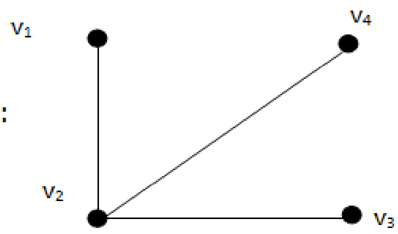
1) If G is an empty graph then $r = 0$
2) If G is a disconnected graph with 'm' components ,then there exists an 'r' such that $G^{(r)} \cong mK_1$.

Remark 1.4: If at each $(G^{(i)}.e)$, $i = 1, 2, .., r-1$, the resultant graph is simple then the given graph G is said to be simply differentiable and in all other cases it may be called partially differentiable.

Definition 1.5: A graph G is said to be differentiable if it is either simply differentiable or partially differentiable.

Example 1.6:

G:



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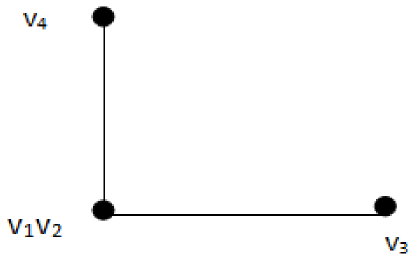
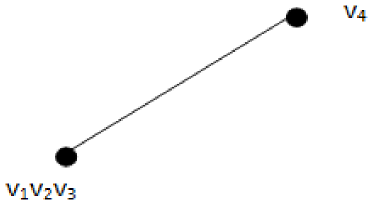
$G^{(1)}$:



$G^{(2)}$:



$G^{(3)}$:

Observation 1.7: Every graph is differentiable.

2. SOME RESULTS

In this section we give some results related to this concept.

Result 2.1: Let G be a connected graph of order p. If there exists an 'r' such that $G^{(r)} \cong K_1$, then r must be equal to $p-1$.

Proof: Let G be a graph of order p. If we contract an edge in G and delete all loops from it, we get a new graph $G^{(1)}$ of order $p-1$. Again we contract an edge in $G^{(1)}$and delete all loops from it we get a new graph say $G^{(2)}$ of order $p-2$. Continue this process until we get an'r' such that $G^{(r)} \cong K_1$

Now $G^{(r)}$ is a graph of order $p-r$. Since $G^{(r)} \cong K_1$, $p-r$ must be equal to1.

That is $p-r=1$

Therefore $r = p-1$.

Result 2.2: Let G be a disconnected graph of order p with 'm' components, each of whose order is $p_1, p_2, \dots, p_m$ respectively. If there exists an 'r' such that $G^{(r)} \cong mK_1$, then 'r' must be equal to $p-m$.

Proof: Let $G=(p,q)$ be a disconnected graph with m components $G_1, G_2, \dots G_m$ of order $p_1, p_2, \dots p_m$ respectively. Since each G_i is connected, then by the above result, if there exists an 'r_1' such that ${G_1}^{(r_1)} \cong K_1$, then $r_1 = p_1 - 1$. Again since G_2 is connected if there exists an 'r_2' such that ${G_2}^{(r_2)} \cong K_1$ then $r_2 = p_2 - 1$. Continuing like this there exists an'r_m' such that ${G_m}^{(r_m)} \cong K_1$ then $r_m = p_m - 1$

Let $r = r_1 + r_2 + \cdots . + r_m$. Thus we get $G^{(r)} \cong mK_1$.

Since $r_i = p_i - 1$, it follows that
$$\begin{aligned} r &= p_1 - 1 + p_2 - 1 + \cdots + p_m - 1 \\ &= p_1 + p_2 + \cdots + p_m - m \\ &= p - m. \end{aligned}$$

Theorem 2.3: Let G_1 and G_2be two graphs with m and n components respectively. Then
$${G_1}^{(r_1)} \vee {G_2}^{(r_2)} \cong Km,n\,.$$

Proof: Let G_1 be a graph with 'm' components and G_2 be a graph with 'n' components respectively.

Then ${G_1}^{(r_1)} \cong mK_1$ and ${G_2}^{(r_2)} \cong nK_1$

Clearly ${G_1}^{(r_1)} \vee {G_2}^{(r_2)}$ has $(m+n)$ verticesand each of m vertices in ${G_1}^{(r_1)}$ is adjacent to all 'n' vertices in ${G_2}^{(r_2)}$.

Therefore, ${G_1}^{(r_1)} \vee {G_2}^{(r_2)} \cong Km, n.$

.

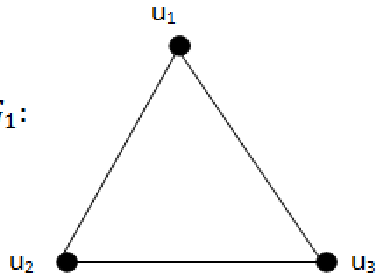
Remark 2.4:

1) If G_1 and G_2 are connected graphs then, ${G_1}^{(r_1)} \vee {G_2}^{(r_2)} \cong K_2$ that is $K_{1,1}$
2) Let G_1 and G_2be two graphs such that one is connected and the other has n components then ${G_1}^{(r_1)} \vee {G_2}^{(r_2)} \cong K_{1,n}$.

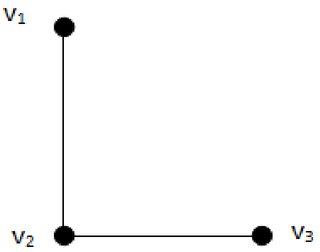
Result 2.5: Let G_1 and G_2 be arbitrary graphs. Then $(G_1 \vee G_2)^{(r)} \ncong {G_1}^{(r_1)} \vee {G_2}^{(r_2)}$.

Proof: We shall prove this by exhibiting the following example

G_1:



G_2:



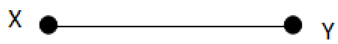
Then ${G_1}^{(r_1)} \cong K_1$ and ${G_2}^{(r_2)} \cong K_1$.

Let

${G_1}^{(r_1)}$:

${G_2}^{(r_2)}$:

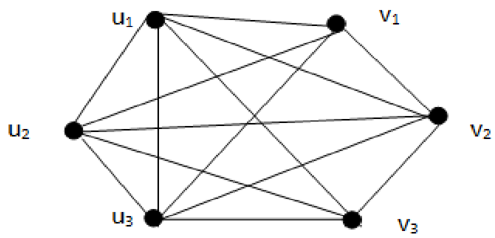
Then,${G_1}^{(r_1)} \vee {G_2}^{(r_2)}$ is



That is ${G_1}^{(r_1)} \vee {G_2}^{(r_2)}$is K_2 (or $K_{1,1}$).

Now,

$G_1 \vee G_2$:



Therefore $(G_1 \vee G_2)^{(r)} \cong K_1$

Hence $(G_1 \vee G_2)^{(r)} \ncong {G_1}^{(r_1)} \vee {G_2}^{(r_2)}$.

Theorem 2.6: If G_1 and G_2 be two arbitrary graphs, then $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$.

Proof: To prove this theorem, we have to consider two cases.

Case-(i):G_1 and G_2 are connected

Then $G_1 \cup G_2$ has two components

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Therefore $(G_1 \cup G_2)^{(r)} \cong 2K_1$.

Also, ${G_1}^{(r_1)} \cong K_1$ and ${G_2}^{(r_2)} \cong K_1$

Therefore ${G_1}^{(r_1)} \cup {G_2}^{(r_2)} \cong 2K_1$

In this case $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$.

Case-(ii): G_1 and G_2 are disconnected graphs

Let G_1 has m components and G_2 has n components.

Then ${G_1}^{(r_1)} \cong mK_1$ and ${G_2}^{(r_2)} \cong nK_1$.

Since $G_1 \cap G_2 = \phi$, $G_1 \cup G_2$ has $(m+n)$ components.

Therefore $(G_1 \cup G_2)^{(r)} \cong (m+n)K_1$.

Also, ${G_1}^{(r_1)} \cup {G_2}^{(r_2)} \cong (m+n)K_1$.

In this case also $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$.

Theorem 2.7: If $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$, then $r = r_1 + r_2$.

Proof:

Case-(i): G_1 and G_2 are connected

Let G_1 and G_2 be of order p_1 and p_2 respectively.

Then ${G_1}^{(p_1-1)} \cong K_1$ and ${G_2}^{(p_2-1)} \cong K_1$

Also, $G_1 \cup G_2$ is of order p_1+p_2 and has two components.

Therefore $(G_1 \cup G_2)^{(p_1+p_2-2)} \cong 2K_1$.

${G_1}^{(p_1-1)} \cup {G_2}^{(p_2-1)} \cong 2K_1$.

Since $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$, $(G_1 \cup G_2)^{(p_1+p_2-2)} \cong {G_1}^{(p_1-1)} \cup {G_2}^{(p_2-1)}$

Now, $p_1 + p_2 - 2 = p_1 - 1 + p_2 - 1$

That is $r = r_1 + r_2$.

Case-(ii): G_1 and G_2 are disconnected graphs.

Let order of G_1 be p_1 and order of G_2 be p_2.

Let G_1 has m components each of whose order is $m_1, m_2, \dots, m_m$ and G_2 has n components, each of whose order is $n_1, n_2, \dots, n_n$.

Then ${G_1}^{(p_1-1)} \cong mK_1$ and ${G_2}^{(p_2-1)} \cong nK_1$.

Therefore, ${G_1}^{(p_1-\mathrm{m})} \cup {G_2}^{(p_2-\mathrm{n})} \cong (m+n)K_1$

Now $G_1 \cup G_2$ has $(m+n)$ components and order of $G_1 \cup G_2$ is p_1+p_2.

Therefore, $(G_1 \cup G_2)^{(p_1+p_2-(m+n))} \cong (m+n)K_1$

Since $(G_1 \cup G_2)^{(r)} \cong {G_1}^{(r_1)} \cup {G_2}^{(r_2)}$, $(G_1 \cup G_2)^{(p_1+p_2-(m+n))} \cong {G_1}^{(p_1-m)} \cup {G_2}^{(p_2-n)}$

Now, $p_1 + p_2 - (m+n) = p_1 - m + p_2 - n$.

That is $r = r_1 + r_2$.

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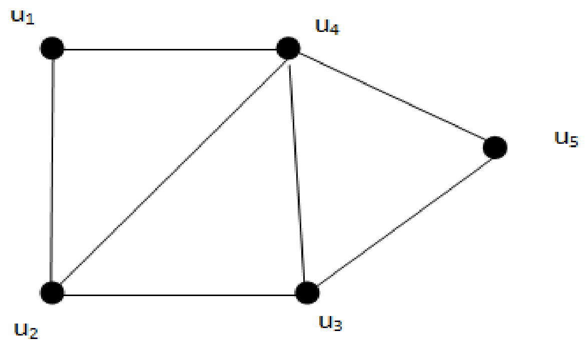
Result 2.8: Let G_1 and G_2 be two subgraphs of G such that they have some vertices in common. Then $(G_1 \cup G_2)^{(r)}) \ncong G_1^{(r_1)} \cup G_2^{(r_2)}$.

Proof: We shall prove this by exhibiting an example.

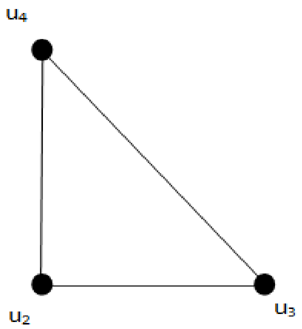
Let G be the graph given by

G:

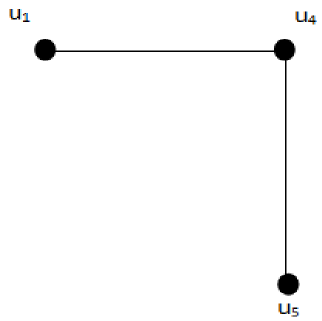


Let G_1 and G_2 be two subgraphs of G,

G_1:

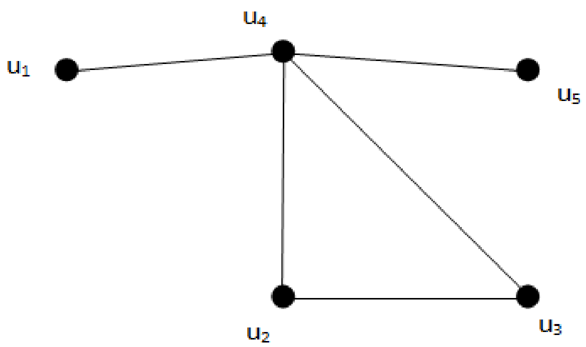


G_2:



Now $G_1 \cup G_2$ is,

$G_1 \cup G_2$:



Here $(G_1 \cup G_2)^{(r)} \cong K_1$, $G_1^{(r_1)} \cong K_1$, $G_2^{(r_2)} \cong K_1$, $G_1^{(r_1)} \cup G_2^{(r_2)} \cong 2K_1$.

Therefore,$(G_1 \cup G_2)^{(r)}) \ncong G_1^{(r_1)} \cup G_2^{(r_2)}$.

Result 2.9: Let $G_1, G_2, \ldots, G_n$ be n disjoint graphs of order $p_1, p_2, \ldots, p_n$ respectively. Then

$$(G_1 \cup G_2 \cup \ldots \cup G_n)^{(r)} \cong G_1^{(r_1)} \cup G_2^{(r_2)} \cup \ldots \cup G_n^{(r_n)},$$

where $r = r_1 + r_2 + r_3 + \cdots + r_n$.

Proof: We will prove the result by using mathematical induction.

When n=1, $G_1^{(r)} \cong G_1^{(r)}$.

Therefore the result is true for n=1.

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When n=2, $(G_1 \cup G_2)^{(r)} \cong G_1^{(r_1)} \cup G_2^{(r_2)}$ and $r = r_1 + r_2$ (From previous theorem)

Therefore, the result is true for n=2.

Now assume that the result is true for fewer than n graphs

That is,
$(G_1 \cup G_2 \cup \ldots \cup G_{n-1})^{(r)} \cong G_1^{(r_1)} \cup G_2^{(r_2)} \cup \ldots \cup G_{n-1}^{(r_{n-1})}$ where,

$$r = r_1 + r_2 + r_3 + \cdots + r_{n-1}$$
$$= p_1 + p_2 + p_3 + \cdots + p_{n-1} - (n-1).$$

Next consider n graphs. Then we have,

$$(G_1 \cup G_2 \cup \ldots \cup G_n)^{(r)} = [(G_1 \cup G_2 \cup \ldots \cup G_{n-1}) \cup G_n]^r$$
$$\cong (G_1 \cup G_2 \cup \ldots \cup G_{n-1})^{(r_1)} \cup G_n^{(r_2)}\ ;\ r = r_1 + r_2$$
$$\cong G_1^{(r_1{}')} \cup G_2^{(r_2{}')} \cup \ldots \cup G_{n-1}^{(r_{n-1}{}')} \cup G_n^{(r_2)} \text{ and}$$

$r_1 = r_1^{(\prime)} + r_2^{(\prime)} + \cdots + r_{n-1}^{(\prime)}$ by assumption.

Now, $r = r_1 + r_2$

$$= r_1^{(\prime)} + r_2^{(\prime)} + \cdots + r_{n-1}^{(\prime)} + r_2$$
$$= p_1 + p_2 + p_3 + \cdots + p_{n-1} - (n-1) + p_n - 1$$
$$= p_1 + p_2 + p_3 + \cdots + p_{n-1} + p_n - n$$
$$= r_1 + r_2 + r_3 + \cdots + r_n.$$

Therefore the result is true for n graphs. Hence the result is true for all n≥1.

Theorem 2.10: Let G_1 and G_2 be two connected graphs. Then, $(G_1[G_2])^{(r)} \cong \left(G_1^{(r_1)}[G_2^{(r_2)}]\right)$.

Proof:
Since G_1 and G_2 are connected graphs, $G_1[G_2]$ is also connected.

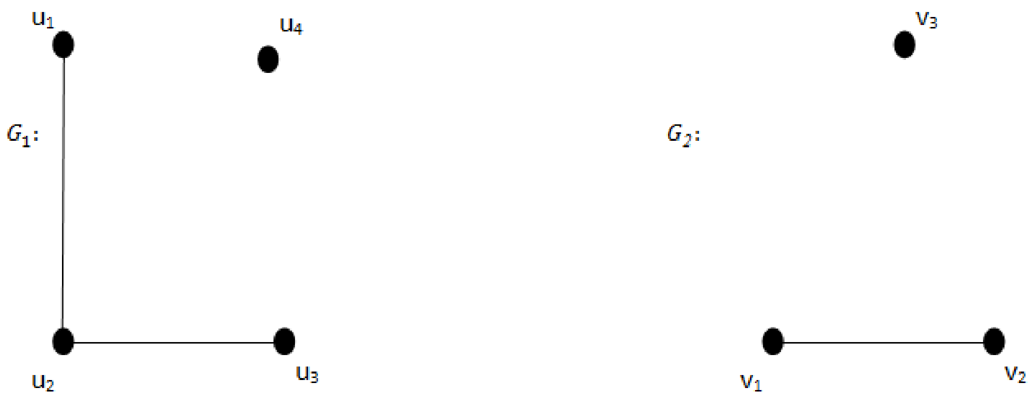
Therefore,$(G_1[G_2])^{(r)} \cong K_1$.

Since G_1 is connected, $G_1^{(r_1)} \cong K_1$and since G_2 is connected, $G_2^{(r_2)} \cong K_1$.

Therefore, $G_1^{(r_1)}[G_2^{(r_2)}] \cong K_1$.Hence the theorem.

Remark 2.11: If any one of G_1 and $G_2$is disconnected, then $(G_1[G_2])^{(r)}$ need not be isomorphic to $\left(G_1^{(r_1)}[G_2^{(r_2)}]\right)$.

Example 2.12:



Here, $G_1^{(r_1)} \cong 2K_1$ and $G_2^{(r_2)} \cong 2K_1$.

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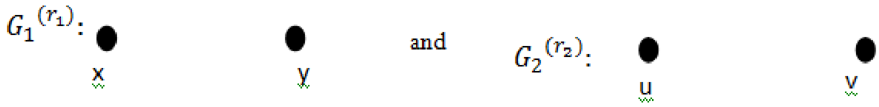
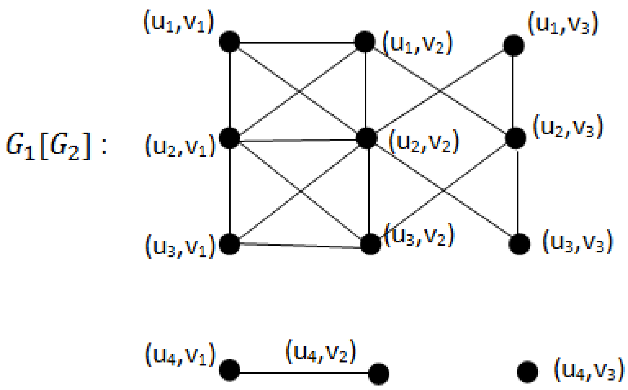
Let



Therefore, $G_1^{(r_1)}[G_2^{(r_2)}]$ is

That is $G_1^{(r_1)}[G_2^{(r_2)}] \cong 4K_1$.

Now,



Therefore, $(G_1[G_2])^{(r)} \cong 3K_1$

Hence $(G_1[G_2])^{(r)} \ncong (G_1^{(r_1)}[G_2^{(r_2)}])$

Theorem 2.13: Let G_1 and G_2 be any two graphs. Then $(G_1 \times G_2)^{(r)} \cong G_1^{(r_1)} \times G_2^{(r_2)}$.

Proof: We shall consider two cases.

Case-(i): Let G_1 and G_2be connected graphs.

Then, $G_1 \times G_2$ is also connected and $(G_1 \times G_2)^{(r)} \cong K_1$

Now, $G_1^{(r_1)} \cong K_1$and $G_2^{(r_2)} \cong K_1$.

Therefore, $G_1^{(r_1)} \times G_2^{(r_2)} \cong K_1$

Hence, $(G_1 \times G_2)^{(r)} \cong G_1^{(r_1)} \times G_2^{(r_2)}$

Case-(ii): Let G_1 has m components and G_2has n components.

Then, $G_1 \times G_2$ has mn components.

Therefore,$(G_1 \times G_2)^{(r)} \cong mnK_1$.

Also, $G_1^{(r_1)} \cong mK_1$and $G_2^{(r_2)} \cong nK_1$.

Therefore, $G_1^{(r_1)} \times G_2^{(r_2)} \cong mnK_1$

Hence, $(G_1 \times G_2)^{(r)} \cong G_1^{(r_1)} \times G_2^{(r_2)}$.

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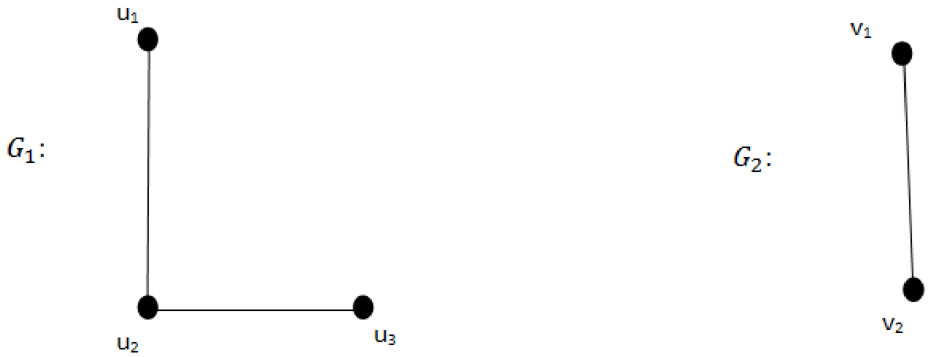
Conjunction [2]: The conjunction of two graphs G_1 and G_2, denoted by $G_1 \wedge G_2$ is a graph with vertex set $V_1 \times V_2$ and $u = (u_1, u_2)$ is adjacent to $v = (v_1, v_2)$ if u_1 is adjacent to v_1 in G_1 and u_2 is adjacent to v_2 in G_2.

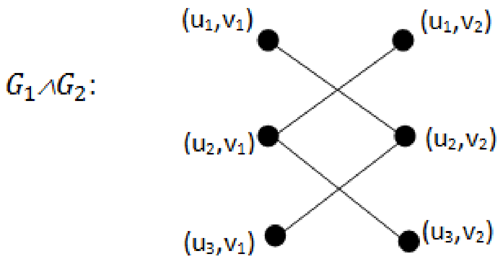
Result 2.14: Let G_1 and G_2 be two arbitrary graphs. Then, $(G_1 \wedge G_2)^{(r)} \ncong {G_1}^{(r_1)} \wedge {G_2}^{(r_2)}$.

Proof: We shall prove this by exhibiting an example.

Let G_1 and G_2 be,



Then,



Therefore, $(G_1 \wedge G_2)^{(r)} \cong 2K_1$

Now, ${G_1}^{(r_1)} \cong K_1$ and ${G_2}^{(r_2)} \cong K_1$

Therefore, ${G_1}^{(r_1)} \wedge {G_2}^{(r_2)} \cong K_1$

Hence, $(G_1 \wedge G_2)^{(r)} \ncong {G_1}^{(r_1)} \wedge {G_2}^{(r_2)}$.

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