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STOLARSKY3- MEAN LABELING OF ARBITRARY SUPER SUBDIVISIONS OF SOME STANDARD GRAPHS

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ABSTRACT

The Concept of Stolarsky-3 Mean labeling was already introduced. In this paper we introduced Arbitrary Super Subdivision in stolarsky-3 Mean graph. Further we investigate Arbitrary Super Subdivisions of Path, Cycle, H-graph (n$\leq$ 10 if n is even and n$\leq$ 11 if n is odd) and Middle graphs.

Keywords: *Graph Labeling, Stolarsky-3 mean labeling, Super subdivision of graphs, Arbitrary Super subdivision of graphs, H graph, Middle graph.*

1. INTRODUCTION

The graph G= (V, E) is considered here will be finite, simple and undirected. We follow Gallian[1] for all detailed survey of graph labeling and we refer Harary [2] for all other standard terminologies and notations. The concept of "Mean labeling" has been introduced by S.Somasundaram and R.Ponraj in 2004[7]. The concept of " Harmonic Mean Labeling" has been introduced by S. Somasundaram, R. Ponraj and S.S. Sandhya in [8].The concept of "Stolarsky-3 Mean Labeling" has introduced by S.S. Sandhya, E.Ebin Raja Merely and S.Kavitha in [6]. G. Sethuraman and P.Selvaraju have introduced a new method of construction called Super subdivision of graphs and prove that arbitrary super subdivisions of paths are graceful in [9]. Motivating these results we investigate Stolarsky-3 Mean labeling of Arbitrary super subdivisions of some standard graphs. Following definitions are useful for the present investigations.

Definition 1.1: A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1, 2,... q+1 and each edge e=uv is assigned the distinct labels f(e=uv) = $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ (or) $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G.

Definition 1.2: A walk in which all the vertices $u_1, u_2, \dots, u_n$ are distinct is called a path. It is denoted by P_n.

Definition 1.3: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.4: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, v_3, \dots, v_n$ and $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

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Definition 1.5: The Middle graph M (G) of a graph G is the graph whose vertex set is V(G) ∪E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 1.6: Let G be a graph. A graph H is called a super subdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph k_{2,m_i} for some m_i, $1\leq i \leq q$ in such a way that the ends of e_i are merged with the two vertices part of k_{2,m_i} after removing the edge e_i from graph G.

Definition 1.7: A Super subdivision H of G is said to be an Arbitrary super subdivision of G if every edge of G is replaced by an arbitrary $k_{2.m_i}$ where m_i may vary for each edge arbitrarily. It is denoted by ASS (G).

2. MAIN RESULTS

Theorem 2.1: Arbitrary Super subdivisions of path are Stolarsky- 3 mean graphs.

Proof: Let P_n be the path with successive vertices $u_1, u_2, u_3,\ldots,u_n$ and let e_i denote the edge u_iu_{i+1} of P_n for $1\leq i\leq n-1$

Let H be an arbitrary super subdivision of a path P_n, where each edge e_i of P_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer.

$$V(H) = \{u_1,u_2,u_3,..,u_n,\ v_1,v_2,v_3,\ldots,\ v_{m_1}, v_{m_1+1}, v_{m_1+2}, \ldots, v_{m_1+m_2}, \ldots,\ v_{m_1+m_2+m_3+\cdots+m_{n-1}}\}$$

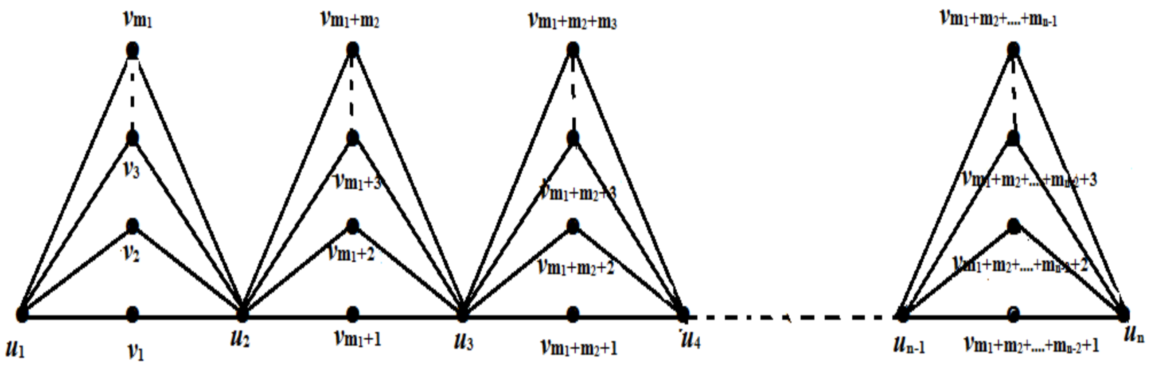

Figure-1: Arbitrary Super subdivision of P_n

Here we consider two different cases

Case-(i): $m_1=m_2=m_3= \ldots =m_{n-1} = 4$

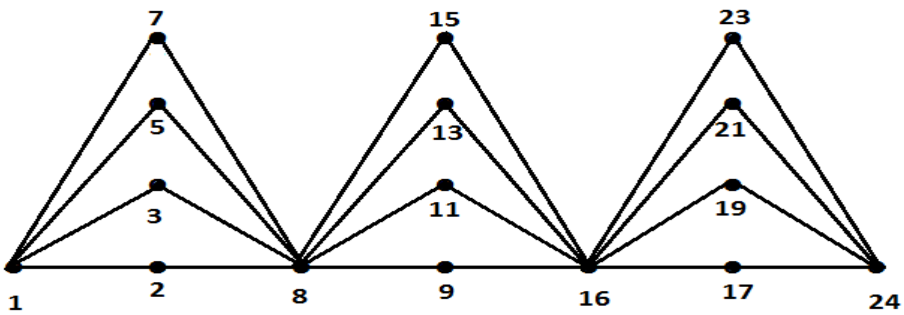

Figure-2: Arbitrary Super subdivision of P_4

Define φ: V(H)→{1,2, …, q+1} by

$\varphi(u_1) = 1.$

$\varphi(u_i) = 8(i-1),\ 2\leq i \leq n.$

$\varphi(v_1) = 2.$

$\varphi(v_i) = 2i-1,\ 2\leq i \leq m_1 + m_2 + \cdots + m_{n-1}.$

Then the edge labels are distinct.

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In this case H is Stolarsky-3 mean graph.

Case-(ii): $m_1=m_2=m_3= \dots =m_{n-1} =5$

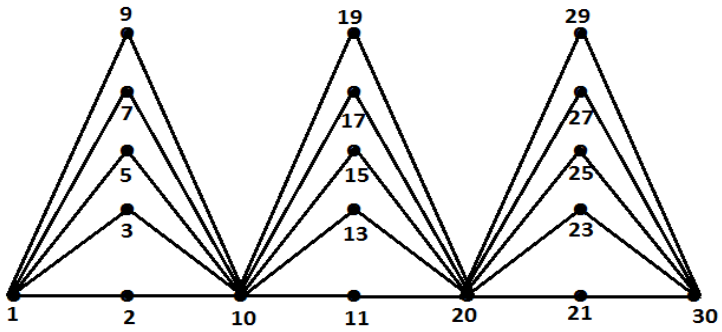

Figure-3: Arbitrary Super subdivision of P_4

Define φ: V(H)→{1, 2,…, q+1} by

$\varphi(u_1) = 1.$

$\varphi(u_i) = 10(i-1), 2 \le i \le n.$

$\varphi(v_1) = 2.$

$\varphi(v_i) = 2i-1, 2 \le i \le m_1 + m_2 + \cdots + m_{n-1}.$

Then the edge labels are distinct.

From case (i) and (ii) we can conclude that Arbitrary super subdivisions of path are Stolarsky-3 mean graphs.

Remark 2.2: The above result is true for all values of n's and m_i's with the condition $m_1 \le 8$ and m_i's, $2 \le i \le n-1$.

Theorem 2.3: Arbitrary Super subdivisions of cycle are Stolarsky- 3 mean graphs.

Proof: Let C_n be a cycle with consecutive vertices $u_1, u_2, u_3, \dots, u_n$ and let e_i denote the edge $u_{i-1}u_i$ of C_n for $1 \le i \le n-1$.

Let H be an arbitrary super subdivision of a cycle C_n, where each edge e_i of C_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer.

$V(H) = \{u_1, u_2, u_3, \dots, u_n, v_1, v_2, v_3, \dots, v_{m_1}, v_{m_1+1}, v_{m_1+2}, \dots, v_{m_1+m_2}, \dots, v_{m_1+m_2+m_3+\cdots+m_{n-1}}\}$

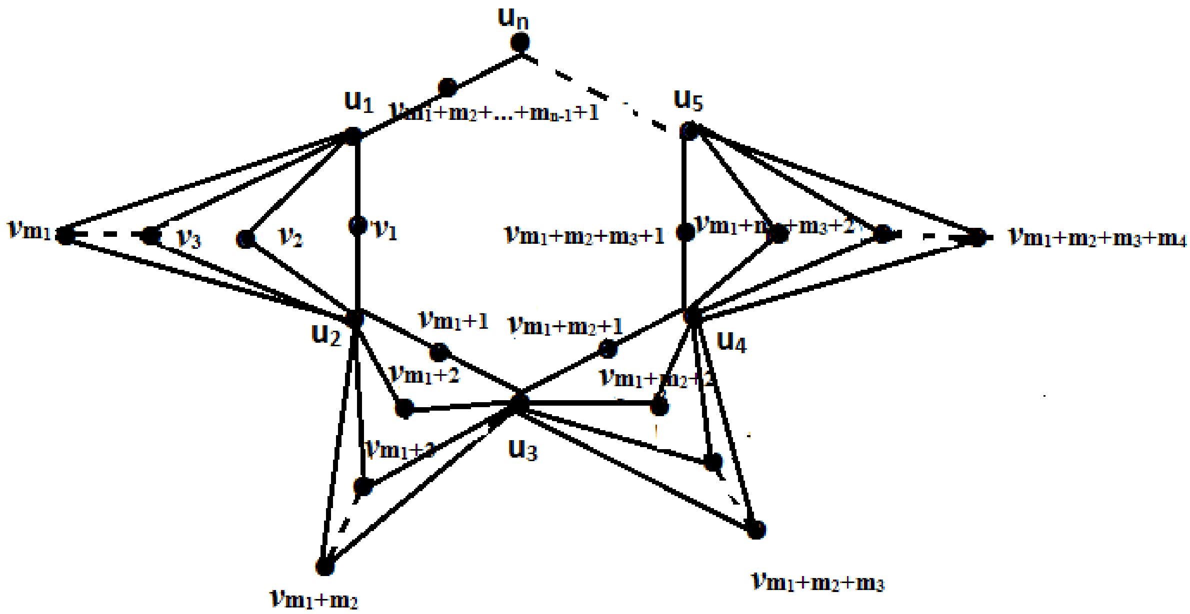

Figure-4: Arbitrary Super subdivision of C_n

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Case-(i): $m_1=m_2=m_3= ... =m_{n-1} = 3$

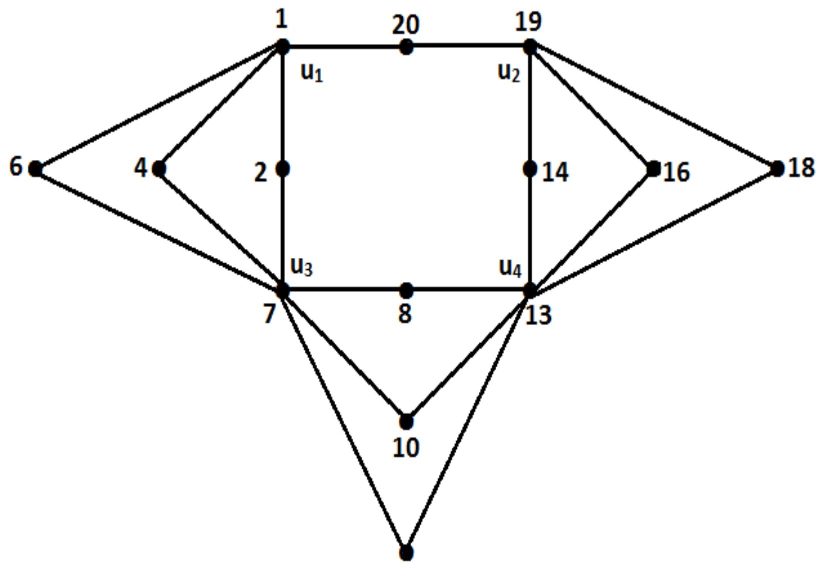

Figure 5: Arbitrary Super subdivision of C_4

Define φ: V(H)→{1, 2,…, q+1} by

$\varphi(u_1) = 1$.

$\varphi(u_i) = 6i-5,\ 2\le i \le n$.

$\varphi(v_i) = 2i,\quad 1\le i \le m_1 + m_2 + \cdots + m_{n-1}$.

$\varphi\left(v_{m_1+m_2+\cdots+m_{n-1}+1}\right) = \varphi(u_n) + 1$.

Then the edge labels are distinct.

In this case H is Stolarsky-3 mean graph.

Case-(ii): $m_1=m_2=m_3=\ldots\ldots=m_{n-1} = 4$

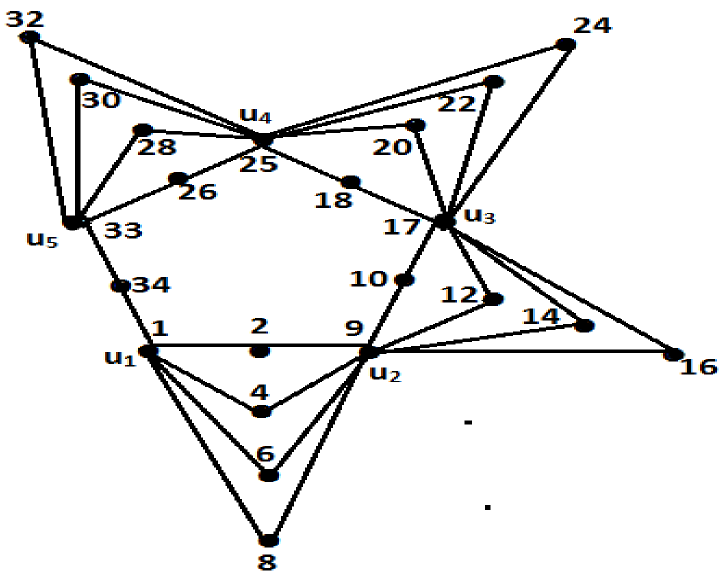

Figure-6: Arbitrary Super subdivision of C_5

Define φ: V(H)→{1, 2, …, q+1} by

$\varphi(u_1) = 1$.

$\varphi(u_i) = 8i-7,\ 2\le i \le n$.

$\varphi(v_i)$ = 2i, $1\leq i \leq m_1 + m_2 + \cdots + m_{n-1}$.
$\varphi(v_{m_1+m_2+\cdots+m_{n-1}+1)}$= $\varphi(u_n)$ +1.
Then the edge labels are distinct.

From case (i) and (ii) we can conclude that Arbitrary Super subdivisions of cycle are Stolarsky-3 mean graphs.

Remark 2.4: The above result is true for all values of n's and m_i's with the condition $m_1 \leq 4$ and m_i's, $2 \leq i \leq n-1$.

Theorem 2.5: Arbitrary Super subdivision of H-graph is Stolarsky-3 mean graph (n≤ 11 if n is odd, n≤ 10 if n is even).

Proof: Let G be a H-graph with vertices $v_1, v_2, \dots, v_n$and $u_1, u_2, \dots, u_n$ and let $u_i u_{i+1}$, $v_i v_{i+1}$,$1\leq i \leq n-1$, $v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}$ if n is odd and $v_{\frac{n}{2}+1} u_{\frac{n}{2}}$ if n is even be the edges of a H-graph G.

Let H be an Arbitrary super subdivision of a H-graph G, where each edge ei of G is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer.

V(H) = {v_1,v_2,v_3,…., v_n,u_1,u_2,u_3,….u_n, $w_1, w_2, \dots, w_{m_1}$
$w_{m_1+1}, w_{m_1+2}, \dots, w_{m_1+m_2}, w_{m_1+m_2+m_3+\cdots+m_{n-1}}$, x_1,x_2,x_3,…, x_{m_1},
$x_{m_1+1}, x_{m_1+2}, \dots, x_{m_1+m_2}, x_{m_1+m_2+m_3+\cdots+m_{n-1}}, x_{m_k}$}

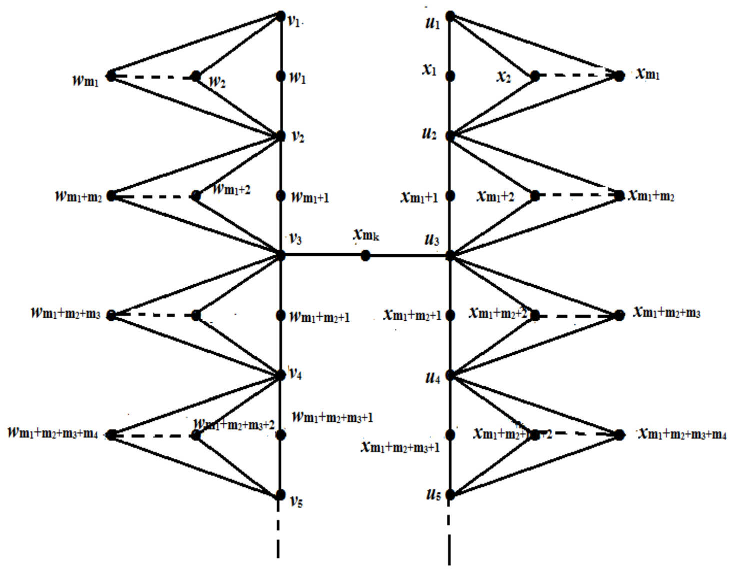

Figure-7: Arbitrary Super Subdivision of H- graph (n is Odd)

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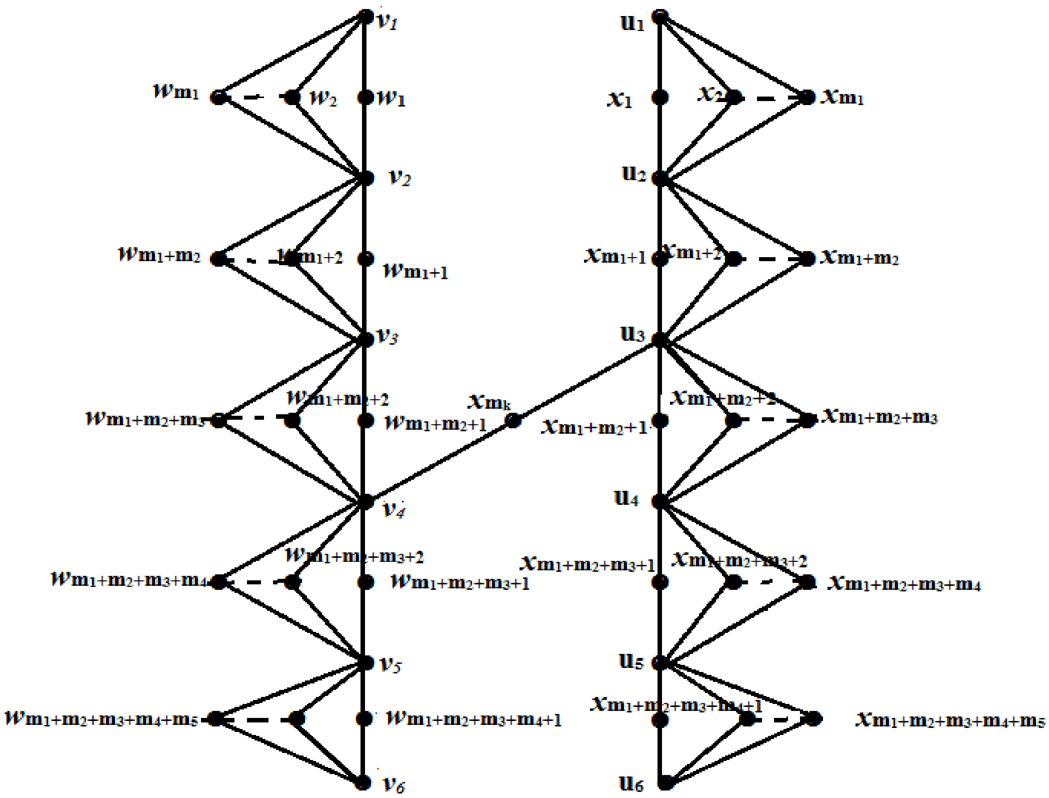

Figure-8: Arbitrary Super Subdivision of H- graph (n is Even)

Case-(i): When n is odd ($n \leq 11$) and $m_1=m_2=m_3= \ldots =m_{n-1} = 3$

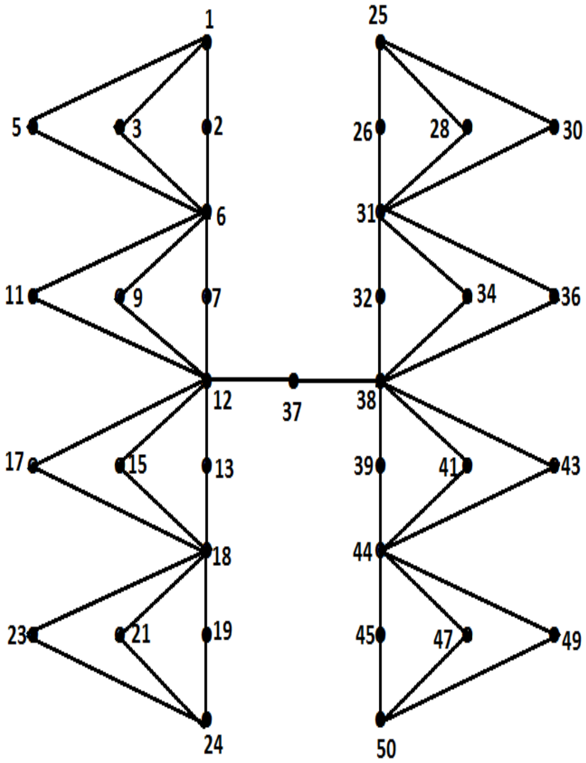

Figure-9: Arbitrary Super subdivision of H-graph with n=5

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Define a function φ: V(H)→{ 1,2,…, q+1} by

$\varphi(v_1) = 1$.
$\varphi(v_i) = 6(i-1)$, $2\leq i \leq n$.
$\varphi(u_1) = \varphi(v_n)+1$.
$\varphi(u_i) = \varphi(u_{i-1}) +6$, $1\leq i < \frac{n+1}{2}$ & $\left(\frac{n+1}{2}\right)+1\leq i \leq n$.
$\varphi(u_i) = \varphi(u_{i-1}) +7$, $i = \frac{n+1}{2}$.
$\varphi(w_1) = 2$.
$\varphi(w_i) = 2i-1$, $2\leq i \leq m_1+m_2+m_3+\ldots+m_{n-1}$.
$\varphi(x_1) = \varphi(u_1)+1$.
$\varphi(x_i) = \varphi(x_{i-1}) + 2$, $2\leq i \leq m_1+m_2$ & $m_1+m_2+2\leq i \leq m_1+m_2+m_3+\ldots+m_{n-1}$
$\varphi(x_{(m_1+m_2)+1}) = \varphi\left(u_{\frac{n+1}{2}}\right) + 1$.
$\varphi(x_{(m_k)}) = \varphi(x_{m_1+m_2}) + 1$.

Then the edge labels are distinct.

Case-(ii): When n is even (n≤10) and $m_1=m_2=m_3= \ldots =m_{n-1} = 3$

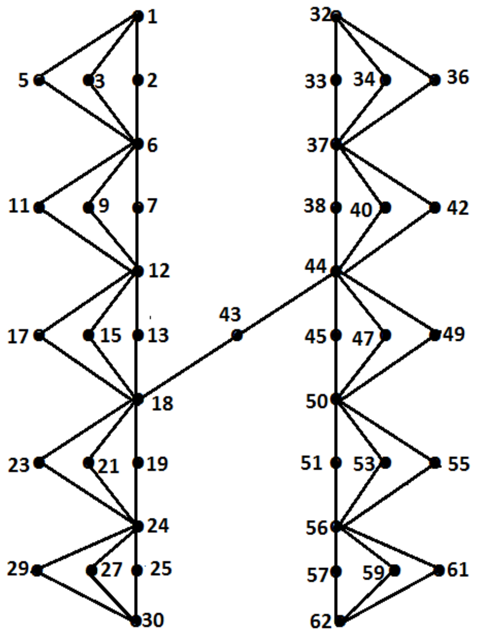

Figure-10: Arbitrary Super subdivision of H-graph with n=6

Define a function φ : V (H)→{ 1, 2, …, q+1} by

$\varphi(v_1)= 1$.
$\varphi(v_i) = 6(i-1)$, $2\leq i\leq n$.
$\varphi(u_1) =\varphi(v_n) + 1$.
$\varphi(u_i) = \varphi(u_{i-1}) + 6$, $2\leq i < \frac{n}{2}$ & $\left(\frac{n}{2}+1\right) +1\leq i \leq n$.
$\varphi(u_{\frac{n}{2}}) = \varphi(u_{\frac{n}{2}-1}) +7$.
$\varphi(w_1) = 2$.
$\varphi(w_i) = 2i-1$, $2\leq i \leq m_1+m_2+m_3+\ldots+m_{n-1}$.
$\varphi(x_1) = \varphi(w_{m_1+m_2+\cdots+m_{n-1}}) +3$.
$\varphi(x_i) = \varphi(x_{i-1}) + 2$, $2\leq i \leq m_1+m_2$ & $m_1+m_2+2\leq i \leq m_1+m_2+m_3+\ldots+m_{n-1}$.
$\varphi(x_{m_1+m_2} + 1) = \varphi(u_{\frac{n}{2}}) + 1$.
$\varphi(x_{(m_k)}) = \varphi(x_{m_1+m_2}) + 1$.

Then the edge labels are distinct.

From case (i) and (ii), Arbitrary Super subdivision of H-graph is stolarsky-3 Mean graph.

Remark 2.6: The above result is true for all values of n's if n is odd ($n \leq 11$) and n is even ($n \leq 10$) and m_i's with the condition $m_1 \leq 8$ and m_i's, $2\leq i \leq n-1$.

Theorem 2.7: Arbitrary Super Subdivision of Middle graph $M(P_n)$ is Stolarsky-3 Mean graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n and $G=M(P_n)$ be the middle graph of path P_n. $V(G) = V(P_n) \cup E(P_n)$ and let u_ix_i $(1 \le i \le n-1)$, u_ix_{i-1}, $2 \le i \le n$, x_ix_{i+1}, $1 \le i \le n-2$ be the edges of $M(P_n)$.

Let H be an arbitrary super subdivision of $M(P_n)$, where each edge of $M(P_n)$ is replaced by a complete bipartite graph $k_{2.m_i}$ where m_i is any positive integer.

$V(H) = \{u_1, u_2, \ldots, u_n, x_1, x_2, \ldots, x_{n-1}, v_1, v_2, \ldots, v_{m_1},$
$v_{m_1+1}, v_{m_1+2}, \ldots, v_{m_1+m_2}, v_{m_1+m_2+m_3+\cdots+m_{2n-2}}, w_1^{(k)}, w_2^{(k)}, \ldots, w_{n-2}^{(k)}\}$.

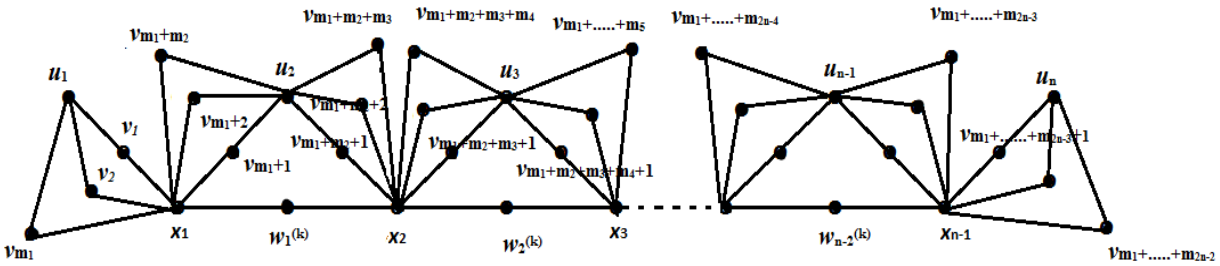

Figure-11: Arbitrary Super subdivision of $M(P_n)$

When $m_1=m_2=m_3= \ldots = m_{2n-2} =3$.

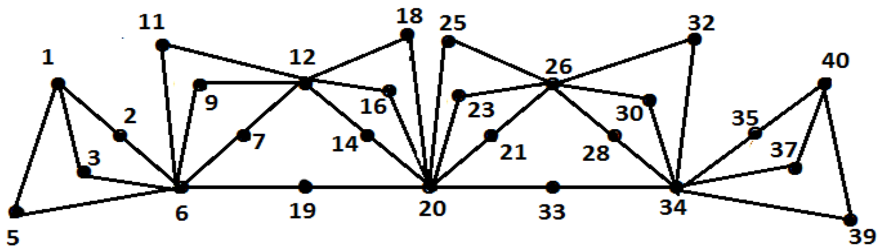

Figure-12: Arbitrary Super subdivision of $M(P_3)$

Define a function $\varphi: V(H) \to \{1, 2, \ldots, q+1\}$ by

$\varphi(u_1) = 1.$
$\varphi(u_i) = 14(i-1) - 2,\ 1 \le i \le n.$
$\varphi(x_1) = 6.$
$\varphi(x_i) = 14(i-1) + 6,\ 1 \le i \le n-1.$
$\varphi(v_1) = \varphi(u_1) + 1.$
$\varphi(v_2) = \varphi(v_1) + 1.$
$\varphi(v_i) = \varphi(v_{i-1}) + 2,\ 3 \le i \le m_1 + m_2 + m_3 + \cdots + m_{2n-2}$
But $i \ne (m_1 + m_2)+1, (m_1 + m_2 + m_3)+1, \ldots$
$(m_1 + m_2 + m_3 + \cdots + m_{2n-3}) + 1$
$\varphi(w_i^{(k)}) = 14i+5,\ 1 \le i \le n-2.$
$\varphi(v_{(\sum_{i=1}^{k} m_i)+1}) = \varphi(v_{\sum_{i=1}^{k} m_i}) + 3, \quad k = 2,3,4,\ldots\ldots,2n-3.$

Then the edge labels are distinct.

Hence Arbitrary Super subdivision of Middle graph $M(P_n)$ is stolarsky-3 Mean graph.

Remark 2.8: The above result is true for all values of n's and m_i's with the condition $m_1 \le 8$ and m_i's, $2 \le i \le n-1$.

CONCLUSION

In this paper we studied the stolarsky-3 Mean labeling behavior of Arbitrary Super subdivision of some standard graphs. The authors are of the opinion that the study of Stolarsky-3 Mean labeling of Arbitrary Super subdivision of some standard graphs shall be quite interesting and also will lead to new results.

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