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MORE RESULTS ON SUPER HERONIAN MEAN LABELING

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ABSTRACT

A function f is called a ***SuperHeronian Mean Labeling*** *of a graph G=(V,E) with p vertices and q edges if there is a injection f:V(G)→{1,2,3,...,p+q} such that the induced edge labeling f*(e=uv) is defined by,*
f(e)=* $\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ *or* $\left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil$*.Any graph which satisfies the Super Heronian Mean Labeling is called the* ***Super Heronian Mean graph****. In this paper we present Flag graph, Dumbellgraph, Polygonalsnake, Balloon of the Triangular snake graphs are Super Heronian Mean graphs.*

Key words: *Graph, Heronian mean graph, Flag graph, Dumbell graph, Balloon of the triangular snake.*

1. INTRODUCTION

The graphs which are used here are finite, undirectedgraphs. HereV(G) indicates vertices and E(G) indicates edges. For all described view of Graph Labeling we refer to J.A. Gallian [1] and all other standard terminology and notations we follow Harary [2] .The notion of "Heronian mean Labeling" has introduced by S.S Sandhya., E.Ebin Raja Merly.and S.D.Deepa. The concept of Super Heronian Mean Labeling has introduced by S.SSandhya., E Ebin Raja Merly.and G.D.Jemi.

Definition1.1: Let f: V(G)→{1,2, . . . ,p+q} be an injective function. For a vertex Labeling "f" the induced edge Labeling f*(e=uv) is defined by,f*(e)=$\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$or$\left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil$.Then "f" is called a **Super Heronian Mean Labeling** if {f(V(G)} U {f(e): e ϵ E(G)={1,2, ...,p+q}}. A graph which admits Super Heronian Mean Labeling is called

Super Heronian Mean Graph

Definition 1.2: The **Flag graph Fl_m** is obtained by joining one vertex of C_m to an extra vertex is called the root.

Definition 1.3: The **Dumbell graph $D_{n,m}$** is obtained by joining two disjoint cycles with a chord.

Definition 1.4: The **Balloon of the triangular snake $T_n(C_m)$** is the graph obtained from C_m by identifying an end vertex of the basic path in T_n at a vertex of C_m.

Theorem 1.5: Any Path P_n is a Super Heronian Mean Graph.

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Theorem 1.6: Any Cycle C_n is a Super Heronian Mean Graph.

2. MAIN RESULTS

Theorem 2.1: The **Flag** $\mathbf{Fl_m}$ graph is a Super Heronian Mean graph.

Proof: Let $\{v_o$ and $v_i : 1 \le i \le m\}$ be the vertices and $\{e_i: 1 \le i \le m+1\}$ be the edges of Fl_m Here v_o is a root vertex.

Define $f(e_{m+1}) = \begin{cases} f\left(v_0 v_{m+1/_2}\right); if\ m\ is\ odd \\ f\left(v_0 v_{m+2/_2}\right); if\ m\ is\ even \end{cases}$

Define f: $V(Fl_m) \rightarrow \{1,2, \ldots, p+q\}$ by

$$f(v_i) = \begin{cases} 2(m+1) & ;\ i = 0 \\ 1 & ;\ i = 1 \\ 4i - 2 & 2 \le i \le \frac{m+1}{2};\ if\ m\ is\ odd \\ & and\ \ 2 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ 4(m - i + 1) & \frac{m+3}{2} \le i \le m;\ if\ m\ is\ odd \\ & and\ \ \frac{m+2}{2} \le i \le m; if\ m\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_i) = \begin{cases} 4i - 1 & 1 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ & and\ \ 1 \le i \le \frac{m-1}{2};\ if\ m\ is\ odd \\ 4m - 4i + 1 & \frac{m+2}{2} \le i \le m - 1;\ if\ m\ is\ even \\ & and\ \ \frac{m+1}{2} \le i \le m - 1; if\ m\ is\ odd \\ 2 & ;\ i = m \\ 2m + 1 & ; i = m + 1 \end{cases}$$

Hence the Flag Fl_m graph is a Super Heronian mean graph.

Example: 2.2A Super Heronian Mean labeling of Flag graph Fl_8 is given below.

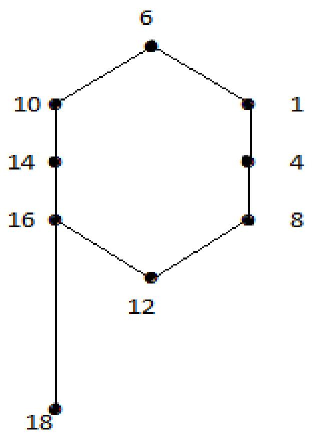

Figure-1

Theorem 2.3: The graph **Dumbell graph** $\mathbf{D_{n,m}}$ is a Super Heronian Mean graph for any n, m≥3.

Proof: Let $\{v_i: 1 \le i \le n$ & $u_i: 1 \le i \le m\}$ be the vertices of $D_{n,m}$ and let $\{e_i: 1 \le i \le n$ &$e'_i: 1 \le i \le m\}$ be the edges of $D_{n,m.}$

Define $f(e) = \begin{cases} f\left(u_1 v_{\frac{n+1}{2}}\right); if\ n\ is\ odd \\ f\left(u_1 v_{n/_2}\right); if\ n\ is\ even \end{cases}$

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Define f: $V(D_{n,m}) \rightarrow \{1,2,\ldots,p+q\}$ by

$$f(v_i) = \begin{cases} 1 & ; i = 1 \\ 4i-2 & 2 \le i \le \frac{n+1}{2};\ if\ n\ is\ odd \\ & and\ \ 2 \le i \le \frac{n}{2};\ if\ n\ is\ even \\ 4(n-i+1) & \frac{n+3}{2} \le i \le n;\ if\ n\ is\ odd \\ & and\ \ \frac{n+2}{2} \le i \le n; if\ n\ is\ even \end{cases}$$

$$f(u_i) = \begin{cases} 2\,(m+1) & ;\ i = 1 \\ 2m+4i-1 & 2 \le i \le \frac{m+1}{2};\ if\ m\ is\ odd \\ & and\ \ 2 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ 6m-4i+5 & \frac{m+3}{2} \le i \le m;\ if\ m\ is\ odd \\ & and\ \ \frac{m+2}{2} \le i \le m\,; if\ m\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_i) = \begin{cases} 4i-1 & 1 \le i \le \frac{n}{2};\ if\ n\ is\ even \\ & and\ \ 1 \le i \le \frac{n-1}{2};\ if\ n\ is\ odd \\ 4n-4i+1 & \frac{n+2}{2} \le i \le n-1;\ if\ n\ is\ even \\ & and\ \ \frac{n+1}{2} \le i \le n-1; if\ n\ is\ odd \\ 2 & ;\ i = n \end{cases}$$

f(e) = 2n+1

$$f(e'_i) = \begin{cases} 2(m+2i) & 1 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ & and\ \ 1 \le i \le \frac{m-1}{2};\ if\ m\ is\ odd \\ 2m+4i-6 & \frac{m+2}{2} \le i \le m-1;\ if\ m\ is\ even \\ & and\ \ \frac{m+1}{2} \le i \le m-1; if\ m\ is\ odd \\ 2i+3 & ;\ i = m \end{cases}$$

Hence $D_{n,m}$ is a Super Heronian mean graph for n, m≥3

Example 2.4: A Super Heronian Mean labeling of dumbell graph $D_{9,10}$ is given below

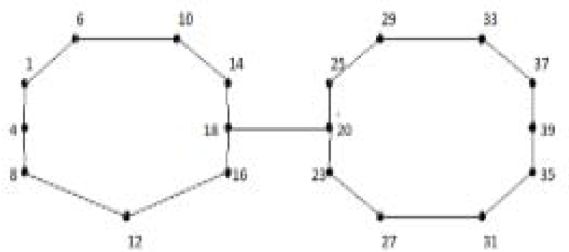

Figure-2

Theorem 2.5: The balloon of the triangular snake T_n (C_m) is a Super Heronian Mean graph.

Proof: Let $u_1,u_2,\ldots,u_m$; $v_1,v_2,\ldots,v_{n+1}$ and $w_1,w_2,\ldots,w_n$ be the vertices of T_n (C_m).
Define f: $V(T_n (C_m)) \rightarrow \{1,2,\ldots,p+q\}$ by

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$$f(u_i) = \begin{cases} 1 & ; i = 1 \\ 4i-2 & 2 \le i \le \frac{m+1}{2};\ if\ m\ is\ odd \\ & and\ \ 2 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ 4(m-i+1) & \frac{m+3}{2} \le i \le m;\ if\ m\ is\ odd \\ & and\ \ \frac{m+2}{2} \le i \le m; if\ m\ is\ even \end{cases}$$

$f(v_i) = 2m+5(i-1); 1 \le i \le n+1$
$f(w_i) = 2(m+1) +5(i-1);\ 1 \le i \le n$
Then the induced edge labels are

$$f(u_i\ u_{i+1}) = \begin{cases} 4i-1 & 1 \le i \le \frac{m}{2};\ if\ m\ is\ even \\ & and\ \ 1 \le i \le \frac{m-1}{2};\ if\ m\ is\ odd \\ 4m-4i+1 & \frac{m+2}{2} \le i \le m-1;\ if\ m\ is\ even \\ & and\ \ \frac{m+1}{2} \le i \le m-1; if\ m\ is\ odd \\ 2 & ;\ i = m \end{cases}$$

$f(v_i\ v_{i+1})\ \ 2m +5i-2\ ;\ 1 \le i \le n$
$f(v_i\ v_{i+1})\ \ 2m +5i-4\ ;\ 1 \le i \le n$
$f(v_i\ v_{i+1})\ \ 2m +5i-1\ ;\ 1 \le i \le n$
Hence balloon of the triangular snake T_n (C_m) is a Super Heronian Mean graph.

Example 2.6: Super Heronian Mean labeling of T_4 (C_6) is given below

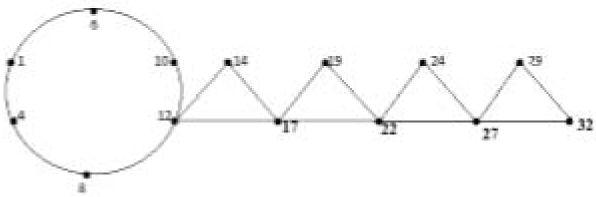

Figure-3

CONCLUSION

In this publication we discussed more results on Super Heronian Mean Labeling of graphs and we investigated on Flag graph, Dumbellgraph, Balloon of the triangular snake graphs. Extending the study to other systematic formations of graph families is an open area of research.

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