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# (k,1)-CONTRA HARMONIC MEAN LABELING OF GRAPHS

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***ABSTRACT***

*Let G =(V,E) be a graph with p vertices and q edges. Let f : V(G) $\rightarrow$ {0,1,2,…, k+(q-1)} be an injective function such that the induced edge labeling f(e = uv) is defined by f(e) =* $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ *or* $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ *is a bijection from E to {k, k+1, k+2,…,k+(q-1)}.Then f has a (k,1)- contra harmonic mean labeling. Any graph which admits a (k,1)- contra harmonic mean labeling is called a (k,1)-contra harmonic mean graph. In this paper we investigate the (k,1)- Contra Harmonic mean labeling for some path related graphs.*

**Keywords:** *Contra Harmonic mean labeling, (k, 1) - Contra Harmonic mean labeling.*

## 1. INTRODUCTION

Let G = (V, E) be a finite, simple, undirected graph with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harray [4].We will give a brief summary of definition and other information which are useful for the present investigation.

A graph G = (V,E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,...,q in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels . Here f is called a Contra Harmonic mean labeling of G.

S. Somasundaram and R. Ponraj introduced mean labeling of graphs and investigated mean labeling for some standard graph in [6]. These labeling patterns motivated us to introduced Contra Harmonic mean labeling [5]. In this paper we prove that some path related graphs admits (k,1)-Contra Harmonic mean labeling where k is any positive integer greater than or equal to 1.

**Definition 1.1:** Let G = (V, E) be a graph with p vertices and q edges . Let f : V(G) $\rightarrow$ {0,1, 2,…, k+(q-1)} be an injective function such that the induced edge labeling f(e = uv) is defined by f(e) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ is a bijection from E to {k, k+1, k+2,…,k+(q-1)}. Then f has a (k,1)- contra harmonic mean labeling. Any graph which admits a (k,1)- contra harmonic mean labeling is called a (k,1)-contra harmonic mean graph. Here k is a positive integer greater than or equal to 1.

**Definition 1.2:** A Triangular snake $T_n$ is obtained from a path $u_1 \ldots u_n$ by joining $u_i$ and to a vertex $v_i$ for $1 \le i \le n - 1$.



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**Definition 1.3:** A Quadrilateral snake $Q_n$ is obtained from a path $u_1 \ldots u_n$ by joining $u_i$ and $u_{i+1}$ to new vertices $v_i$, $w_i$, $1 \le i \le n-1$.

**Definition 1.4:** A middle graph M(G) of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

## 2. MAIN RESULTS

**Theorem 2.1:** A path has a (k, 1)-Contra Harmonic mean labeling for all k.

**Proof:** Let $u_1 \ldots u_n$ be the vertices of the path $P_n$.

We define an injective function $f: V(P_n) \rightarrow \{0, 1, 2, \ldots, k+(q-1)\}$ as follows

$f(u_1)=k-1$, $f(u_i)=k+i-2$, $2 \le i \le n$

The distinct edge labeling are as follows

$f(u_i u_{i+1}) = i-1+k$, $1 \le i \le n-1$

Hence, f is a (k, 1) -Contra Harmonic mean labeling of G.

Thus, the path admits a (k, 1)-Contra Harmonic mean graph for all k.

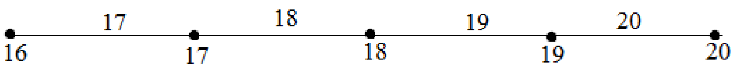


**Figure-1:** (17, 1) Contra Harmonic mean labeling of $P_5$

**Theorem 2.2:** A comb is a (k, 1) - Contra Harmonic mean graph for all k.

**Proof:** Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path and let $v_i$ be the pendant vertices attached to each $u_i$, $1 \le i \le n$.

Let G be a comb graph $P_n \odot K_1$.

We define $f: V(G) \rightarrow \{0, 1, 2, \ldots, k+(q-1)\}$ as follows

$f(u_i)=2i-3+k$, $1 \le i \le n$

$f(v_i)=2i-2+k$, $1 \le i \le n$

The distinct edge labeling are as follows

$f(u_i u_{i+1})=2i-1+k$, $1 \le i \le n-1$

$f(u_i v_i)=2i-2+k$, $1 \le i \le n$

Hence, f is a (k, 1)-Contra Harmonic mean labeling of G.

Then, the comb is a (k, 1)-Contra Harmonic mean graph for all k.

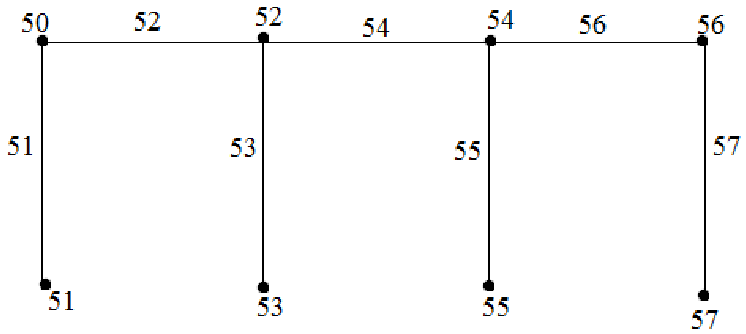


**Figure-2:** (51, 1)-Contra Harmonic mean labeling of $P_4 \odot K_1$




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**Theorem 2.3:** $P_n \odot K_{1,2}$ admits (k,1)-Contra Harmonic mean labeling for all k.

**Proof:** Let $P_n$ be the path $u_{1,}$ $u_{2,}$ $u_{3,}\ldots,u_n$ and $v_i$ , $w_i$ be the vertices of $K_{1,2}$ which are attached to the vertex $u_i$ of $P_n$, $1\le i\le n$

Let G =$P_n \odot K_{1,2}$

We define f: V(G)$\rightarrow${0, 1, 2…,k+(q-1)} as

$f(u_i)=3i-3+k$, $1\le i\le n$
$f(v_i)=3i-4+k$, $1\le i\le n$
$f(x_i)=3i-2+k$, $1\le i\le n$

The distinct edge labeling are as follows

$f(u_iu_{i+1})=3i-1+k$, $1\le i\le n-1$
$f(u_iv_i)=3i-3+k$, $1\le i\le n$
$f(u_ix_i)=3i-2+k$, $1\le i\le n$

Hence, the function f is a (k,1)-Contra Harmonic mean labeling of G.

Thus, $P_n \odot K_{1,2}$ is a (k-1)-Contra Harmonic mean graph for all k.

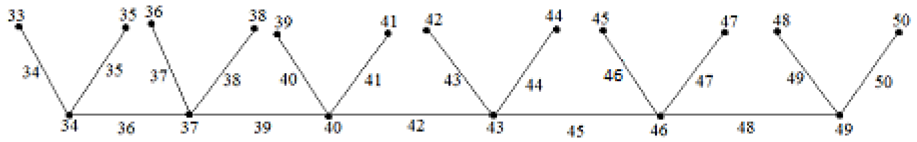


**Figure-3:** (34, 1)-Contra Harmonic mean labeling of $P_6 \odot K_{1,2}$

**Theorem 2.4:** $P_n \odot K_{1,3}$ admits (k,1)-Contra Harmonic mean labeling for all k.

**Proof:** Let $P_n$ be the path with vertices $u_{1,}$ $u_{2,}$ $u_{3,}\ldots,u_n$ and $v_i$, $w_i$, $z_i$ be the vertices of $K_{1,3}$ which are joined to the vertices $u_i$ of the path $P_n$,$1 \le i \le n$.

Let G = $P_n \odot K_{1,3}$.

Let f : V(G) $\rightarrow${0, 1, 2,…,k+(q-1)} be defined by

$f(u_i)=4i-4+k$, $1\le i\le n$
$f(v_i)=4i-4+k-1$, $1\le i\le n$
$f(w_i)=4i-3+k$, $1\le i\le n$
$f(x_i)=4i-2+k$, $1\le i\le n$

The distinct edge labeling are as follows

$f(u_iu_{i+1})=4i-1+k$, $1\le i\le n-1$
$f(u_iv_i)=4i-4+k$, $1\le i\le n$
$f(u_iw_i)=4i-3+k$, $1\le i\le n$
$f(u_ix_i) = 4i-2+k$, $1\le i\le n$

Hence, the function f is a (k, 1)-Contra Harmonic mean labeling of G.

Then, $P_n \odot K_{1,3}$ is a (k-1)-Contra Harmonic mean graph.

 

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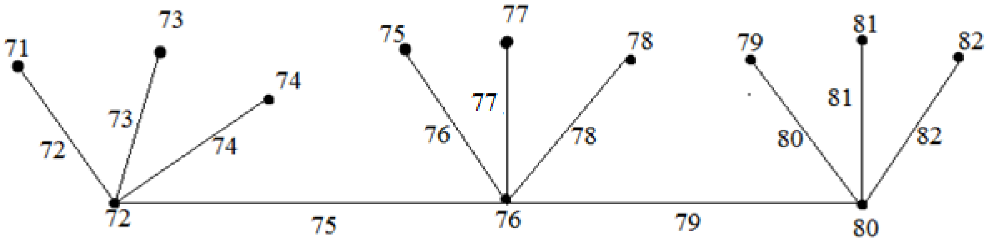


**Figure-4:** (72, 1)- Contra Harmonic mean labeling of $P_3 \odot K_{1,3}$

**Theorem 2.5:** A Ladder is a (k, 1) - Contra Harmonic mean graph for all k.

**Proof:** Let G = $P_2 x P_n$ be a ladder graph. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of G.

Let us define f: V(G) $\rightarrow$ {0, 1, 2…,k+(q-1)} as follows

$f(u_i)=3i-3+k,\ 1\le i\le n$

$f(v_i) = k+3i-4$, if i is odd

$f(v_i) = k+3i-5$ if *i* is even

The distinct edge labeling are as follows

$f(u_iu_{i+1})=3i-1+k,\ 1\le i\le n-1$

$f(v_iv_{i+1})=3i-2+k,\ 1\le i\le n-1$

$f(u_iv_i)=3i-3+k,\ 1\le i\le n$

Hence, the function f is a (k, 1)- Contra Harmonic mean labeling of G.

Then, $P_2 x P_n$ is a (k-1)-Contra Harmonic mean graph for all k.

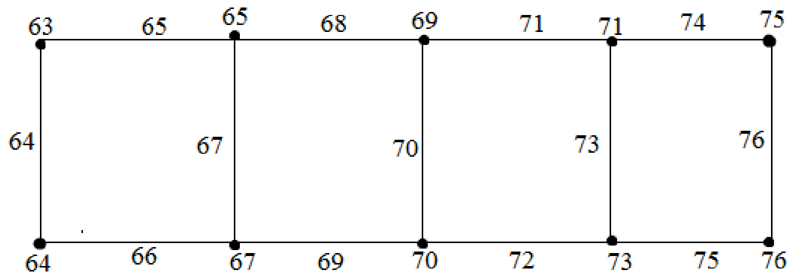


**Figure-5:** (64,1) – Contra Harmonic mean labeling of $P_2 x P_5$

**Theorem 2.6:** $(P_n \odot K_1) \odot K_{1,2}$ admits (k,1)-Contra Harmonic mean graph for all k.

**Proof:** Let G = $(P_n \odot K_1) \odot K_{1,2}$, where $P_n$ is a path with vertices $u_1, u_2, u_3, \ldots, u_n$. Let $v_i$ be a vertex adjacent to $u_i$, $1\le i\le n$.

The resultant graph is $(P_n \odot K_1)$. Let $x_i$, $w_i$, $z_i$ be the vertices of $i^{th}$ copy of $K_{1,2}$ with $z_i$ the central vertex. Identify the vertex $z_i$ with $v_i$ we get the resultant graph G.

That is, G is a graph obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb.

Let us define f: V (G) $\rightarrow$ {0, 1, 2….k+(q-1)} as

$f(u_i)=4i-3+k,\ 1\le i\le n$

$f(v_i)=4i-4+k,\ 1\le i\le n$

$f(w_i)=4i-5+k,\ 1\le i\le n$

$f(x_i)=4i-2+k,\ 1\le i\le n$

Then the distinct edge labeling are

$f(u_iu_{i+1})= 4i-1+k,\ 1\le i\le n-1$

$f(u_iv_i)=4i-3+k,\ 1\le i\le n$

$f(v_iw_i)=4i-4+k,\ 1\le i\le n$

$f(v_ix_i)=4i-2+k,\ 1\le i\le n$

 

Then f provides a (k, 1) – Contra Harmonic mean labeling of G.

Hence $(P_n \odot K_1) \odot K_{1,2}$ is a (k-1) Contra Harmonic mean graph.

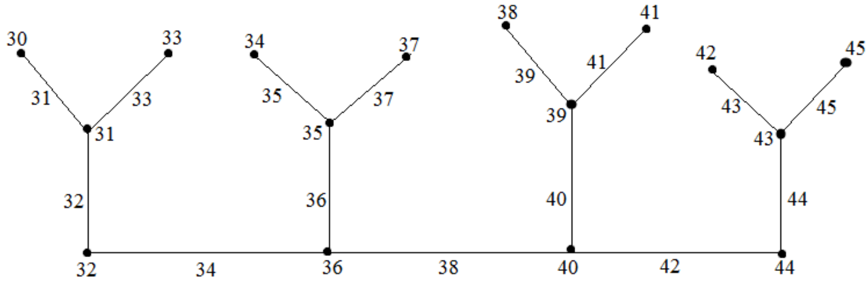


**Figure-6:** (31,1)- Contra Harmonic mean labeling of $(P_4 \odot K_1) \odot K_{1,2}$

**Theorem: 2.7:** Any Triangular snake is a (k, 1)-Contra Harmonic mean graph for all k.

**Proof:** Let G = $T_n$, where $T_n$ is a Triangular snake obtained from a path $u_1, u_2, u_3 \ldots, u_n$ by joining $u_i$ to $v_{i+1}$ to a new vertex $v_i$ for $1 \le i \le n-1$.

Let us define f: $V(G) \rightarrow \{0, 1, 2..., k+(q-1)\}$ as follows

$f(u_i) = 3i-4+k,\ 1 \le i \le n$

$f(v_i)) = 3i-2+k,\ 1 \le i \le n-1$

The distinct edge labeling are as follows

$f(u_iu_{i+1}) = 3i-2+k,\ 1 \le i \le n-1$

$f(u_iv_i) = 3i-3+k,\ 1 \le i \le n-1$

$f(v_iu_{i+1}) = 3i-1+k,\ 1 \le i \le n-1$

Then f is a (k, 1)- Contra Harmonic mean labeling of G.

Hence, any Triangular snake is a (k, 1)-Contra Harmonic mean Graph.

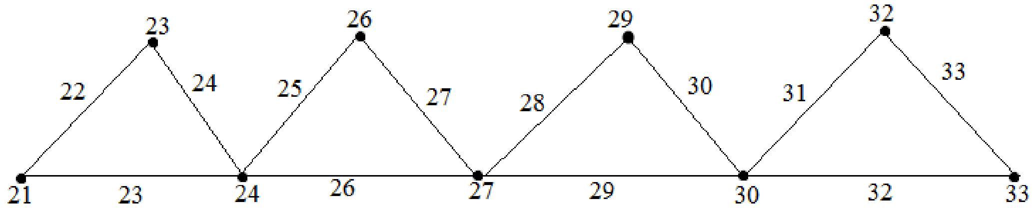


**Figure-7:** (22, 1) – Contra Harmonic mean labeling of $T_4$

**Theorem 2.1:8:** Any Quadrilateral Snake is a (k, 1) – Contra Harmonic mean graph for all k.

**Proof:** Let G be a Quadrilateral snake obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by joining $u_i$ and $u_{i+1}$ to new vertices $v_i$ and $w_i$ respectively and joining the vertices $v_i$ and $w_i$ $1 \le i \le n-1$

Let us define f: $V(G) \rightarrow \{0, 1, 2\ldots, k+(q-1)\}$ as follows

$f(u_i) = 4i-5+k,\ 1 \le i \le n$

$f(v_i) = 4i-4+k,\ 1 \le i \le n-1$

$f(w_i) = 4i-3+k,\ 1 \le i \le n-1$

Then the distinct edge labeling are as follows

$f(u_iu_{i+1}) = 4i-2+k,\ 1 \le i \le n-1$

$f(u_iv_i) = 4i-4+k,\ 1 \le i \le n-1$

$f(v_iw_i) = 4i-3+k,\ 1 \le i \le n-1$

$f(w_iu_{i+1}) = 4i-1+k,\ 1 \le i \le n-1$

Hence, f is a (k, 1)-Contra Harmonic mean labeling for G.

Thus any Quadrilateral snake is a (k, 1) – Contra Harmonic mean graph.




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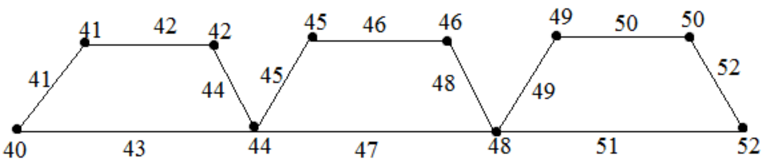


**Figure-8:** (41, 1) – Contra Harmonic mean labeling of $Q_3$

**Theorem: 2.9:** The middle graph of path $P_n$ ($n\geq3$) is a (k, 1) – Contra Harmonic mean graph for all k.

**Proof:** Let $V(P_n) =\{v_1, v_2, \ldots, v_n\}$ and $E(P_n) =\{e_i = v_i v_{i+1}, 1\leq i\leq n-1\}$ be the vertex set and edge set of the path $P_n$. Then $V(G) =\{v_1, v_2, \ldots, v_n, e_1, e_2, e_3, \ldots, e_n\}$ and $E(G) =\{v_i e_i, e_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{e_i e_{i+1}, 1\leq i \leq n-2\}$.

Let us define $f: V(G) \rightarrow \{0, 1, 2 \ldots, k+(q-1)\}$ by

$f(e_i) =3i-3+k, 1\leq i\leq n$

$f(v_i) =3i-5+k, 1\leq i\leq n+1$

Then the distinct edge labels are

$f(e_i e_{i+1}) =3i-1+k, 1\leq i\leq n-1$

$f(e_i v_i) =3i-3+k, 1\leq i\leq n$, $f(e_i v_{i+1}) =3i-2+k, 1\leq i\leq n$

Clearly, f provides a (k,1)- contra Harmonic mean labeling for G.

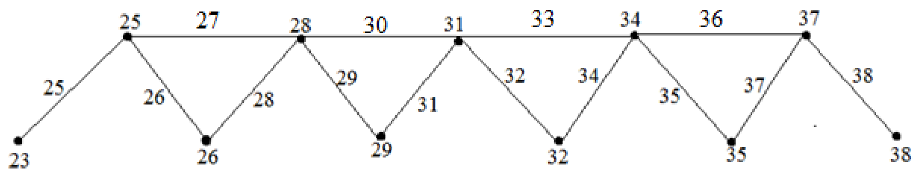


**Figure-9:** (25, 1) – Contra Harmonic mean labeling of $M(P_5)$

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