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(r, 2, (r-n)(r-1)) - regular graphs

N. R. SANTHI MAHESWARI*

G. Venkataswamy Naidu College, Kovilpatti-628502, India.

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ABSTRACT

A graph G is (r, 2, (r-n)(r - 1)) - regular, for any $r \geq n$ if each vertex in the graph G is distance one from r vertices and each vertex in the graph G is distance two from exactly (r- n) (r -1) number of vertices. In this paper, we have suggests a method to construct (r, 2, (r-n)(r-1)) - regular graphs, for all $r \geq n \geq 2$.

Keywords: *Degree of a graph, Regular graph, Distance, Distance degree regular graphs, (2, k) -regular graphs, k-semiregular graphs.*

Mathematics subject code classification (2010): *05C12.*

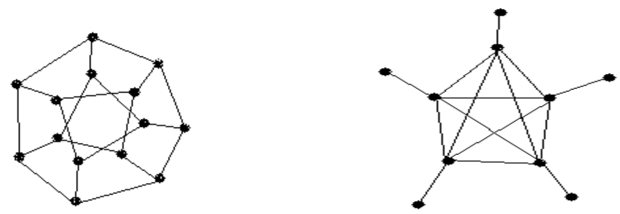
1. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. Notations and terminology that we do not define here can be found in Harary [6} and J.A. Bondy and U.S.R.Murty [4]. We denote the graph *G* by (V(*G*), E(*G*)). The **degree** of a vertex *v* is the number of edges incident at *v* and we denote it by d(v). A graph *G* is **regular** if all its vertices have the same degree. The set of all vertices at a distance one from r is denoted by N (v).

In a connected graph *G*, the **distance** between two vertices *u* and *v* is the length of a shortest (*u*, *v*) path in G and is denoted by d (u, v). Consequently, we define the degree of a vertex *v* is the number of vertices at a distance 1 from *v*. This observation suggests a generalization of degree. That is, $\mathbf{d_d(v)}$ is defined as the number of vertices at a distance d from *v*. Hence $d_1(v) = d(v)$ and $\mathbf{N_d(v)}$ denote the set of all vertices that are at a distance *d* away from *v* in a graph *G*. Hence $N_1(v) = N(v)$.

A graph is said to be distance d-regular [5] if every vertex of G has the same number of vertices at a distance d from it. A graph G is called (d, k)-regular if every vertex of G has k number of vertices at a distance d from it. The (1, k)-regular graphs are nothing but our usual k- regular graphs.

A graph G is (2, k)-regular if $d_2(v)=k$, for all v in G. The concept of the semiregular graph was introduced and studied by Alison Northup [2]. A graph G is said to be k-semiregular graph if each vertex of G is at a distance two away from exactly k vertices of G. We observe that (2, k)-regular graphs are k-semiregular graphs. Note that a (2, k)-regular graph may be regular or non-regular. Among the two (2, k)-regular graphs given in figure 1, (i) is regular whereas (ii) is non-regular.



(i) (ii)

Figure-1

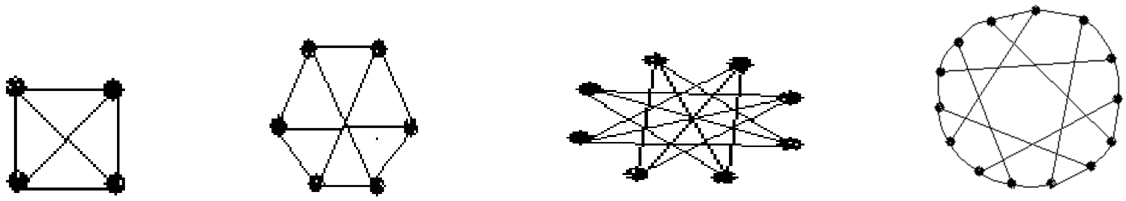
In this paper, we call r- regular graphs which are (2, k)-regular by (r, 2, k) - regular graph.

Corresponding Author: N. R. Santhi Maheswari*
G. Venkataswamy Naidu College, Kovilpatti-628502, India.

2. (r, 2, k) - regular graph

Definition 2.1: A graph G is called a (r, 2, k)-regular if each vertex in the graph G is at a distance one from exactly r vertices and at a distance two from exactly k vertices. That is, d(v) = r and $d_2(v) = k$, for all v in G.

Example 2.2: (r, 2, k) - regular graphs.



(3,2,(3-3)(3-1)) -regular (3,2,(3-2)(3-1)) -regular (3,2,(3-1)(3-1)) -regular (3,2,3(3-1) -regular

Figure-2

The following facts are known from literature.

Fact 1 [8] For any r > 1, a graph G is (r, 2, r(r-1)-regular if G is r-regular with girth at least five.

Fact 2 [9] For any odd r ≥ 3, there is no (r, 2, 1)-regular graph.

Fact 3 [9] Any (r, 2, k)-regular graph has at least k+r+1 vertices.

Fact 4 [9] If r and k are odd, then (r, 2, k)-regular graph has at least k+r+2 vertices.

Fact 5 [9] For any r ≥ 2 and k ≥ 1, G is a (r, 2, k)-regular graph of order r+k+1 if and only if diam (G) = 2.

Fact 6 [9] For any r > 1, if G is a (r, 2, (r-1)(r-1))-regular graph, then G has girth four.

Fact 7 [10] For any r ≥ 1, there exist a (r, 2, r-1)-regular graph of order 2r.

Fact 8 [10] For any r ≥ 1, there exist a (r, 2, 2 (r-1))- regular graph of order 4r-2.

Fact 9 [11] For any r > 2, there exist a (r, 2, r+n) - regular bipartite graph of order 2 (r + n + 1), for (0 ≤ n ≤ r).

Fact 11 [8] For any n≥ 5, (n ≠ 6,8) and any r > 1, tthere exists a (r, 2, r (r -1))-regular graph on n x 2^{r-2} vertices with girth five.

Fact 12 [9] For any r ≥ 2, there is a (r, 2, (r-1)(r-1))-regular graph on 4 x 2^{r-2} vertices.

F**act 13 [10] F**or any r ≥ 2 , there is a (r, 2, (r-2)(r-1)-regular graph on 3 x 2^{r-2} vertices.

Fact 14 [11] For any r ≥ 3, there is a (r, 2, (r-3)(r-1))-regular graph on 4x2^{r-3} vertices.

Fact 10 [7] If G is (r, 2, k)-regular graph, then 0 ≤ k ≤ r(r-1)

Is it possible to construct the (r, 2, k)-regular graphs for all values of k from 0 to r(r-1), for any r? . With this motivation, we have constructed the (r, 2, k) - regular graphs,for k = r(r-1)[8], k = (r-1)(r-1)[9], k = (r-2)(r-1)[10] and k = (r-3)(r-1)[11].

The constructions given in [10] and [11], motivate us to construct (r, 2, (r-n) (r-1)-regular graph, for any r ≥ n.

3. (r, 2, (r– n) (r– 1)) - regular graphs

In this section, we have given a method to construct a (r, 2, (r-n) (r-1))-regular graph with (n+1) x 2^{r-n} vertices, for any r ≥ n ≥ 2.

Definition 3.1: A graph G is called (r, 2, (r-n)(r-1))-regular graph, for r ≥ n if each vertex in the graph G is at a distance one from r vertices and each vertex in the graph G is at a distance two from (r-n)(r-1) vertices.

Theorem 3.2: Any r ≥ n ≥ 2, there exists a (r, 2, (r – n) (r - 1))- regular on (n+1) x 2^{r-n} vertices.

Proof: If r = n, Complete graph on (n+1) vertices is the required graph.

Let us prove this result by induction on r.

Let G be a graph with vertex set $V(G) = \{x_i^{(1)}, x_i^{(2)} / (0 \le i \le n)\}$ and edge set

$$E(G) = \{ x_i^{(1)} x_i^{(2)} / (0 \le i \le n)\} \bigcup_{i=0}^{n-1} \{ x_i^{(1)} x_{i+j}^{(1)} / (1 \le j \le n-i) \} \bigcup_{i=0}^{n-1} \{ x_i^{(2)} x_{i+j}^{(2)} / (1 \le j \le n-i)\}.$$

For $(0 \le i \le n)$, (Subscripts are taken modulo n).
$N_2(x_i^{(1)}) = \{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots\ldots x_{i+n}^{(2)}\}$ and $d_2(x_i^{(1)}) = n$.
$N_2(x_i^{(2)}) = \{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots\ldots x_{i+n}^{(2)}\}$ and $d_2(x_i^{(1)}) = n$.
G is ((n+1), 2, ((n+1)-(n)) (n+1-1)) - regular graph on $(n+1) \times 2^{n+1-n} = 2(n+1)$ vertices.

Step-1: Take another copy of G as G'. Let $V(G') = \{x_i^{(3)}, x_i^{(4)} / (0 \le i \le n)\}$ and $E(G') = \{x_i^{(3)} x_i^{(4)} / (0 \le i \le n)\} \bigcup_{i=0}^{n-1} \{x_i^{(4)} x_{i+j}^{(4)} / (1 \le j \le n-i)\} \bigcup_{i=0}^{n-1} \{x_i^{(3)} x_{i+j}^{(3)} / (1 \le i \le n-i)\}$

The desired graph G_1 has the vertex set $V(G_1) = V(G) \cup V(G')$. edge set $E(G_1) = E(G) \cup E(G') \cup \{ x_i^{(1)} x_{i+1}^{(4)}, x_i^{(2)} x_i^{(3)} / (0 \le i \le n)\}$ (Subscripts are taken modulo (n+1).Now the resulting graph G_1 is (n+2) regular graph having $(n+1) \times 2^{n+2-(n)} = 4(n+1)$ vertices.

Consider the edges $x_i^{(1)} x_{i+1}^{(4)}$ for $(0 \le i \le n)$.

For $(0 \le i \le n)$,
$N(x_i^{(1)}) = \{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots\ldots x_{i+n}^{(1)}, x_i^{(2)}\}$ in G and $|N(x_i^{(1)})| = n+1$ in G.
$N(N(x_i^{(1)})) = \{x_{i+2}^{(4)}, x_{i+3}^{(4)}, x_i^{(4)}, \ldots.. x_{i+n}^{(4)}, x_i^{(3)}\}$ in G' and $|N(N(x_i^{1})| = n+1$ in G'.
$N(x_{i+1}^{(4)}) = \{x_i^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots\ldots x_{i+n}^{(4)}, x_{i+1}^{(3)}\}$ in G' and $|N(x_{i+1}^{(4)})| = n+1$ in G'.
$N(N(x_{i+1}^{(4)})) = \{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}, x_{i+1}^{(2)}\}$ in G and $|N(N(x_{i+1}^{4})| = n+1$ in G.

d_2 of each vertex in $C^{(1)}$, where $C^{(1)}$ is the cycle induced by the vertices $\{x_i^{(1)}/0 \le i \le n\}$
$N_2(x_i^{(1)})$ in $G_1 = N_2(x_i^{(1)})$ in G $\cup$ $N(x_{i+1}^{(4)})$ in G' $\cup$ $N(N(x_i^{(1)})$ in G'
$= N_2(x_i^{(1)})$ in G $\cup$ $\{x_i^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots x_{i+n}^{(4)}, x_{i+1}^{(3)}\}$ in G' $\cup \{x_{i+2}^{(4)}, x_{i+3}^{(4)}, x_i^{(4)}, \ldots.. x_{i+n}^{(4)}, x_i^{(3)}\}$ in G'.
$= N_2(x_i^{(1)})$ in G $\cup$ $\{x_{i+2}^{(4)}, x_{i+3}^{(4)}, x_i^{(4)}, \ldots. x_{i+n}^{(4)}, x_{i+1}^{(3)}, x_i^{(3)}\}$ in G'

Here $x_{i+2}^{(4)}, x_{i+3}^{(4)}, x_i^{(4)}, \ldots.., x_{i+n}^{(4)}$ are the common elements in $N(x_{i+1}^{(4)})$ in G' and $N(N(x_i^{(1)})$ in G'.
$d_2(x_i^{(1)})$ in $G_1 = d_2(x_i^{(1)})$ in G + $(d(x_{i+1}^{(4)})$ in G' + $|N(N(x_i^{1})|$ in G') - n.
$= n+(n+1+n+1)-(n) = 2(n+1) = [(n+2)-(n)[\ (n+2-1), (0 \le i \le n)$.

d_2- of each vertex in $C^{(4)}$, where $C^{(4)}$ is the cycle induced by the vertices $\{x_i^{(4)}/0 \le i \le n\}$
$N_2(x_{i+1}^{(4)})$ in $G_1 = N_2(x_{i+1}^{(4)})$ in G' $\cup$ $N(x_i^{(1)}))$ in G $\cup$ $N(N(x_{i+1}^{(4)}))$ in G
$= N_2(x_{i+1}^{(4)})$ in G' $\cup \{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}, x_i^{(2)}\}$ in G $\cup \{ x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots\ldots x_{i+n}^{(1)}, x_{i+1}^{(2)}\}$ in G.
$= N_2(x_{i+1}^{(4)})$ in G' $\cup \{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots\ldots\ldots x_{i+n}^{(1)}, x_i^{(2)}, x_{i+1}^{(2)}\}$ in G.

Here, $x_{i+2}^{(1)}, x_{i+1}^{(1)}, x_{i+3}^{(1)}, \ldots\ldots x_{i+n}^{(1)}$ are the common element in $N(x_i^{(1)}))$ in G and $N(N(x_{i+1}^{(4)}))$ in G.
$d_2(x_{i+1}^{(4)})$ in $G_1 = (d_2(x_{i+1}^{(4)})$ in G' $+(d(x_i^{(1)})$ in G + $|N(N(x_{i+1}^{4})|$ in G) - n
$= n + (n+1+n+1) - (n) = 2(n+1) = [(n+2) - (n)[\ (n + 2 - 1), (0 \le i \le n)$.

Next consider the edges $x_i^{(2)} x_i^{(3)}$, for $(0 \le i \le n)$.
For $(0 \le i \le n)$.
$N(x_i^{(2)}) = \{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots.. x_{i+n}^{(2)}, x_i^{(1)}\}$ in G and $|N(x_i^{(2)})| = n+1$ in G.
$N(N(x_i^{(2)})) = \{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}, x_{i+1}^{(4)}\}$ in G' and $|N(N(x_i^{2})| = n+1$ in G'.
$N(x_i^{(3)}) = \{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots\ldots x_{i+n}^{(3)}, x_i^{(4)}\}$ in G' and $|N(x_i^{(3)})| = n+1$ in G'.
$N(N(x_i^{(3)})) = \{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_{i+3}^{(1)}\}$ in G and $|N(N(x_i^{3}))| = n+1$ in G .

d_2- of each vertex in $C^{(2)}$, where$C^{(2)}$ is the cycle induced by the vertices$\{x_i^{(2)}/0\le i\le n\}$

$N_2(x_i^{(2)})$ in G_1 = $N_2(x_i^{(2)})$ in G U $N(x_i^{(3)})$ in G' U $N(N(x_i^{(2)})$in G'.

=$N_2(x_i^{(2)})$ in G U{ $x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}, x_i^{(4)}$ } in G' U {$x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}. x_{i+1}^{(4)}$} in G'.

=$N_2(x_i^{(2)})$ in G U { $x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}, x_i^{(4)}, x_{i+1}^{(4)}$} in G'.

Here, $x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)} \ldots x_{i+n}^{(3)}$ **are** the common elements in $N(x_i^{(3)})$ in G' and $N(N(x_i^{(2)})$in G'

$d_2(x_i^{(2)})$ in G_1= $d_2(x_i^{(2)})$ in G + ($d(x_i^{(3)})$ in G' + $\left|N(N(x_i^{2})\right|$ .in G') - n.

= n+(n+1+n+1)-(n)=2(n+1)= [(n+2)-(n)[(n+2-1), $(0 \le i \le n)$.

d_2 of each vertex in $C^{(3)}$, where $C^{(3)}$ is the cycle induced by the vertices $\{x_i^{(3)}/0\le i\le n\}$

$N_2(x_i^{(3)})$ in G_1 = $N_2(x_i^{(3)})$ in G' U $N(x_i^{(2)})$ in GU $N(N(x_i^{(3)})$ in G

= $N_2(x_i^{(3)})$ in G' U {$x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_i^{(1)}$} in G U{$x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_{i+3}^{(1)}$} in G.

= $N_2(x_i^{(3)})$ in G' U{$x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_i^{(4)}, x_{i+1}^{(4)}$} in G'.

Here, $x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}$ are the common elements in $N(x_i^{(2)})$ in G and $N(N(x_i^{(3)})$ in G.

$d_2(x_i^{(3)})$ in G_1= $d_2(x_i^{(3)})$ in G' + ($d(x_i^{(2)})$ in G + $\left|N(N(x_i^{3}))\right|$ in G) - n.

= n+(n+1+n+1)-(n)=2(n+1)= [(n+2)-(n)[(n+2-1), $(0 \le i \le n)$.

In G_1,, for $(1 \le t \le 4)$, $d_2(x_i^{(t)})$ =[(n+2)-(n)] (n+2-1), $(0 \le i \le n)$.
G_1 is ((n+2), 2, ((n+2)-(n)) (n+2-1))-regular having (n+1) x $2^{n+2-(n)}$ = 4(n+1) vertices with the vertex set $V(G_1)$ ={ $x_i^{(t)}/(1\le t \le 2^{n+2}),(0 \le i \le n)$}and $E(G_1)$ =
E (G)U E (G')U{ $x_i^{(1)} x_{i+1}^{(4)}, x_i^{(2)} x_i^{(3)}$/ $(0 \le i \le n)$}.Therefore, the result is true for r = n+2.

Step-2: Take another copy of G_1 as G_1' with the vertex set V(G_1')={$x_i^{(t)}/(2^{5-3}+1\le t \le 2^{5-2})$, $(0 \le i \le n)$}and each $x_i^{(t)}$, ($2^{5-3}+1\le t\le 2^{5-3}$), corresponds to $x_i^{(t)}$, $(1 \le t \le 2^{5-3})$, for $(0 \le i \le n)$.

The desired graph G_2 has the vertex set $V(G_2)$ = $V(G_1)$U V(G_1') and edge set

$E(G_2)$ = $E(G_1)$U E(G_1')U{$x_i^{(1)} x_{i+1}^{(8)}, x_i^{(2)} x_i^{(7)}, x_i^{(3)} x_{i+1}^{(6)}, x_i^{(4)} x_i^{(5)}$/ $\mathbf{(0 \le i \le n)}$}(Subscripts are taken modulo (n+1).

Now the resulting graph G_2 is (n+3) regular graph having (n+1) x 2^{n+3-n} = 8 (n+1) vertices.

consider the edges $x_i^{(1)} x_{i+1}^{(8)}$, for $(0 \le i \le n)$.

For $(0 \le i \le n)$.
$N(x_i^{(1)})$ = {$x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}, x_i^{(2)}, x_{i+1}^{(4)}$ }in G_1 and $\left|N(x_i^{(1)})\right|$ = n+2 in G_1.

$N(N(x_i^{(1)}))$ = { $x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}, x_i^{(7)}, x_{i+1}^{(5)}$ } in G_1' and $\left|N(N(x_i^{1}))\right|$ = n+2, in G_1'.

$N(x_{i+1}^{(8)})$ = {$x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}, x_{i+1}^{(7)}, x_i^{(5)}$ } in G_1' and $\left|N(x_{i+1}^{(8)})\right|$ = n+2 in G_1'.

$N(N(x_{i+1}^{(8)}))$ = {$x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(8)}, x_{i+1}^{(2)}, x_i^{(4)}$} in G_1 and $\left|N(N(x_{i+1}^{8}))\right|$ = n+2 in G_1..

d_2- of each vertex in $C^{(1)}$, where $C^{(!)}$ is the cycle induced by the vertices $\{x_i^{(1)}/ 0 \le i \le n\}$

$N_2(x_i^{(1)})$ in G_2 = $N_2(x_i^{(1)})$ in G_1 U $N(x_{i+1}^{(8)})$ in G_1' U $N(N(x_i^{(1)})$ in G_1'

=$N_2(x_i^{(1)})$ in G_1U {$x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}, x_i^{(7)}, x_{i+1}^{(5)}$} in G_1' U { $x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}, x_i^{(7)}$
, $x_{i+1}^{(5)}$ } in G_1'

=$N_2(x_i^{(1)})$ in G_1 U {$x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}, x_i^{(7)}, x_{i+1}^{(5)}, x_i^{(5)}, x_{i+1}^{(7)}$} in G_1'.

Here $x_i^{(8)}, x_{i+2}^{(8)}, x_{i+3}^{(8)}, \ldots x_{i+n}^{(8)}$ are the common elements in $N(x_{i+1}^{(8)})$ in G_1' and $N(N(x_i^{(1)})$in G_1'.

$d_2(x_i^{(1)})$ in G_1= $d_2(x_i^{(1)})$ in G_1 + ($d(x_{i+1}^{(8)})$ in G_1' + $\left|N(N(x_i^{1}))\right|$ in G_1') - n.

= 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

d_2- of each vertex in $C^{(8)}$,where $C^{(8)}$ is the cycle induced by the vertices$\{x_i^{(8)}/0\le i\le n\}$

$N_2(x_{i+1}^{(8)})$ in $G_2 = N_2(x_{i+1}^{(8)})$ in G_1' U $N(x_i^{(1)})$ in G_1 U $N(N(x_{i+1}^{(8)})$in G_1.

$=N_2(x_{i+1}^{(8)})$ in G_1' U $\{\mathbf{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}}, x_i^{(2)}, x_{i+1}^{(4)}\}$in G_1 U$\{\mathbf{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}}, x_{i+1}^{(2)}, x_i^{(4)}\}$ in G_1 .

$= N_2(x_{i+1}^{(8)})$ in G_1' U $\{\mathbf{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}}, x_i^{(2)}, x_{i+1}^{(4)} x_{i+1}^{(2)}, x_i^{(4)}\}$ in G_1

Here $\mathbf{x_{i+1}^{(1)}, x_{i+2}^{(1)}, x_{i+3}^{(1)}, \ldots x_{i+n}^{(1)}}$ are the common elements in $N(x_i^{(1)})$ in $G_1$and $N(N(x_{i+1}^{(8)})$in G_1.

$d_2(x_{i+1}^{(8)})$ in $G_2 = (d_2(x_{i+1}^{(8)})$ in $G_1' + (d(x_i^{(1)})$ in G_1 + $\left|N(N(x_{i+1}^{8}))\right|$ in G_1)-n

$d_2(x_{i+1}^{(8)})$ in G_2 = 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

Next consider the edge $x_i^{(2)} x_i^{(7)}$, for $(0 \le i \le n)$.
For $(0 \le i \le n)$.
$N(x_i^{(2)}) = \{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2}, \ldots x_{i+n}^{(2)}, x_i^{(1)}, x_i^{(3)}\}$in $G_1$and $|N(x_i^{(2)})|$ =n+2 in G_1.

$N(N(x_i^{(2)})) = \{x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots x_{i+n}^{(7)}, x_{i+1}^{(8)}, x_{i+1}^{(6)}\}$in G_1' and $\left|N(N(x_i^{2}))\right|$ = n+2 in G_1'.

$N(x_i^{(7)}) = \{x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots x_{i+n}^{(7)}, x_i^{(8)}, x_i^{(6)}, \}$ in G_1' and $|N(x_i^{(7)})|$ = n+2 in G_1'.

$N(N(x_i^{(7)})) = \{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(7)}, x_{i+3}^{(1)}, x_{i+3}^{(3)}\}$ in G_1 and $\left|N(N(x_i^{7}))\right|$ = n+2 in G_1.

d_2- of each vertex in $C^{(2)}$,where $C^{(2)}$ is the cycle induced by the vertices$\{x_i^{(2)}/ 0\le i\le n\}$

$N_2(x_i^{(2)})$ in $G_2 = N_2(x_i^{(2)})$ in G_1 U $N(x_i^{(7)})$ in G_1' U $N(N(x_i^{(2)})$in G_1'

$= N_2(x_i^{(2)})$ in G_1 U $\{\mathbf{x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots x_{i+n}^{(7)}}, x_i^{(8)}, x_i^{(6)}\}$ in G_1' U$\{\mathbf{x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots x_{i+n}^{(7)}}, x_{i+1}^{(8)}, x_{i+1}^{(6)}\}$in G_1'.

$= N_2(x_i^{(2)})$ in G_1 U $\{x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots x_{i+n}^{(7)}, x_i^{(6)}, x_i^{(8)}, x_{i+1}^{(8)}, x_{i+1}^{(6)}\}$ in G_1'.

Here , $x_{i+1}^{(7)}, x_{i+2}^{(7)}, x_{i+3}^{(7)}, \ldots, x_{i+n}^{(7)}$ are the common elements in $N(x_i^{(7)})$ in G_1' and $N(N(x_i^{(2)})$ in G_1'

$d_2(x_i^{(2)})$ in $G_2 = d_2(x_i^{(2)})$ in G_1 + $(d(x_i^{(7)})$ in G_1' + $\left|N(N(x_i^{2}))\right|$in G_1') - n.

= 2(n+1)+(n+2+n+2)-(n)=3(n+2) = [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

d_2- of each vertex in $C^{(7)}$, where $C^{(7)}$ is the cycle induced by the vertices$\{x_i^{(7)}/ 0\le i\le n\}$

$N_2(x_i^{(7)})$ in $G_2 = N_2(x_i^{(7)})$ in G_1' U $N(x_i^{(2)})$ in G_1 U $N(N(x_i^{(7)})$ in G_1.

$= N_2(x_i^{(7)})$ in G_1' U $\{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_i^{(1)}, x_i^{(3)}\}$in G_1U $x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_{i+3}^{(1)}, x_{i+3}^{(3)}\}$ in G_1.

$= N_2(x_i^{(7)})$ in G_1' U $\{x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}, x_i^{(3)}, x_i^{(1)}, x_{i+3}^{(3)}, x_{i+3}^{(1)}\}$ in G_1.
Here $x_{i+1}^{(2)}, x_{i+2}^{(2)}, x_{i+3}^{(2)}, \ldots x_{i+n}^{(2)}$ are the common elements in $N(x_i^{(2)})$ in G_1 and $N(N(x_i^{(7)})$ in G_1.

$d_2(x_i^{(7)})$ in $G_2 = (d_2(x_i^{(7)})$ in G_1' + $(d(x_i^{(2)})$ in G_1 + $\left|N(N(x_i^{7}))\right|$in G_1)-n

$d_2(x_i^{(7)})$ in G_2 = 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

Next consider the edge $x_i^{(3)} x_{i+1}^{(6)}$, for $(0 \le i \le n)$.
$N(x_i^{(3)}) = \{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}, x_i^{(4)}, x_i^{(2)}\}$in G_1 and $|N(x_i^{(3)})|$ = n+2 in G_1.

$N(N(x_i^{(3)})) = \{x_i^{(6)}, x_{i+2}^{(6)} x_{i+3}^{(6)}, \ldots x_{i+n}^{(6)}., x_i^{(5)}, x_i^{(7)}\}$ in $G_1'$and $\left|N(N(x_i^{3}))\right|$ = n+2 in G_1'.

$N(x_{i+1}^{(6)}) = \{x_i^{(6)}, x_{i+2}^{(6)}, x_{i+3}^{(6)}, \ldots x_{i+n}^{(6)}, x_{i+1}^{(5)}, x_{i+1}^{(7)}\}$ in G_1' and $|N(x_{i+1}^{(6)})|$ = n+2 in G_1'.

$N(N(x_{i+1}^{(6)}))= \{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(30}, x_{i+1}^{(4)}, x_{i+1}^{(2)}\}$ in G_1 and $\left|N(N(x_{i+1}^{6}))\right|$ = n+2 in G_1.

d_2- of each vertex in $C^{(3)}$, where $C^{(3)}$ is cycle induced by the vertices $\{x_i^{(3)} / 0\le i\le n\}$

$N_2(x_i^{(3)})$ in $G_2 = N_2(x_i^{(3)})$ in G_1 U $N(x_{i+1}^{(6)})$ in G_1' U $N(N(x_i^{(3)})$ in G_1'

$=N_2(x_i^{(3)})$ in G_1 U$\{\mathbf{x_i^{(6)}, x_{i+2}^{(6)}, x_{i+3}^{(6)}, \ldots x_{i+n}^{(6)}}, x_{i+1}^{(5)}, x_{i+1}^{(7)}\}$ in G_1' U $\{\mathbf{x_{i+2}^{(6)} x_{i+3}^{(6)}, x_i^{(6)}, \ldots x_{i+n}^{(6)}}, x_i^{(5)}, x_i^{(7)}\}$ in G_1'

$=N_2(x_i^{(3)})$ in G_1 U$\{\mathbf{x_i^{(6)}, x_{i+2}^{(6)}, x_{i+3}^{(6)}, \ldots x_{i+n}^{(6)}}, x_{i+1}^{(5)}, x_{i+1}^{(7)}, x_i^{(5)}, x_i^{(7)}\}$ in G_1'.

Here $x_i^{(6)}, x_{i+2}^{(6)}, x_{i+3}^{(6)}, \ldots x_{i+n}^{(6)}$ are the common elements in $N(x_{i+1}^{(8)})$ in G_1' and $N(N(x_i^{(1)})$ in G_1'.

$d_2(x_i^{(3)})$ in $G_1 = d_2(x_i^{(3)})$ in G_1 + ($d(x_{i+1}^{(6)})$ in G_1' + $|N(N(x_i^{3}))|$ in G_1') - n.

= 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

d_2- of each vertex in $C^{(6)}$, where $C^{(6)}$ is the cycle induced by the vertices $\{x_i^{(6)}/ 0 \le i \le n\}$

$N_2(x_{i+1}^{(6)})$ in $G_2 = N_2(x_{i+1}^{(6)})$ in G_1' U $N(x_i^{(3)})$ in G_1 U $N(N(x_{i+1}^{(6)})$ in G_1.

$=N_2(x_{i+1}^{(6)})$ in G_1' U $\{\mathbf{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}}, x_i^{(4)}, x_i^{(2)}\}$in G_1 U $\{\mathbf{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}}, x_{i+1}^{(4)}, x_{i+1}^{(2)}\}$ in G_1 .

$= N_2(x_{i+1}^{(6)})$ in G_1' U$\{\mathbf{x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}}, x_i^{(4)}, x_i^{(2))}, x_{i+1}^{(4)}, x_{i+1}^{(2)}\}$ in G_1

Here, $x_{i+1}^{(3)}, x_{i+2}^{(3)}, x_{i+3}^{(3)}, \ldots x_{i+n}^{(3)}$ are the common elements in $N(x_i^{(1)})$ in G_1 and $N(N(x_{i+1}^{(8)})$in G_1.

$d_2(x_{i+1}^{(6)})$ in G_2 = ($d_2(x_{i+1}^{(6)})$ in G_1' + ($d(x_i^{(3)})$ in G_1 + $|N(N(x_{i+1}^{6}))|$ in G_1)-n.

$d_2(x_{i+1}^{(6)})$ in G_2 = 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

Next consider the edge $x_i^{(4)}$ $x_i^{(5)}$ for $(0 \le i \le n)$.

For $(0 \le i \le n)$.

$N(x_i^{(4)}) = \{x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots x_{i+n}^{(4)}, x_i^{(3)}, x_{i+3}^{(1)}\}$ in G_1 and $|N(x_i^{(4)})|$ = n+2 in G_1.

$N(N(x_i^{(4)})) = \{x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)} \ldots x_{i+n}^{(5)}, x_{i+1}^{(6)}, x_i^{(8)}\}$ in G_1' and $|N(N(x_i^{4}))|$ = n+2 in G_1'.

$N(x_i^{(5)}) = \{x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)}, \ldots x_{i+n}^{(5)}, x_i^{(6)}, x_{i+1}^{(8)}\}$ in G_1' and $|N(x_i^{(5)})|$ = n+2 in G_1'.

$N(N(x_i^{(5)})) = \{x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots x_{i+n}^{(4)}, x_{i+2}^{(3)}, x_i^{(1)}\}$ in G_1 and $|N(N(x_i^{5}))|$ = n+2 in G_1.

d_2- of each vertex in $C^{(4)}$, where $C^{(4)}$ is the cycle induced by the vertices $\{x_i^{(4)}/ 0 \le i \le n\}$

$N_2(x_i^{(4)})$ in $G_2 = N_2(x_i^{(4)})$ in G_1 U $N(x_i^{(5)})$ in G_1' U $N(N(x_i^{(4)})$in G_1'

$=N_2(x_i^{(4)})$ in G_1U$\{\mathbf{x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)}, \ldots x_{i+n}^{(5)}}, x_i^{(6)}, x_{i+1}^{(8)}\}$ in G_1' .U$\{x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)} \ldots\ldots x_{i+n}^{(5)}, x_{i+1}^{(6)}, x_i^{(8)}\}$ in G_1'

$=N_2(x_i^{(2)})$ in G_1 U$\{x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)}, \ldots x_{i+n}^{(5)}, x_i^{(6)}, x_{i+1}^{(8)}, x_{i+1}^{(6)}, x_i^{(8)}\}$ in G_1'.

Here $x_{i+1}^{(5)}, x_{i+2}^{(5)}, x_{i+3}^{(5)}, \ldots x_{i+n}^{(5)}$ are the common elements in $N(x_i^{(5)})$ in G_1' and $N(N(x_i^{(4)})$ in G_1'

$d_2(x_i^{(4)})$ in $G_2 = d_2(x_i^{(4)})$ in G_1 + ($d(x_i^{(5)})$ in G_1' + $|N(N(x_i^{4}))|$ in G_1') - n.

= 2(n+1)+(n+2+n+2)-(n)=3(n+2)= [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

d_2- of each vertex in $C^{(5)}$, where $C^{(5)}$ is the cycle induced by the vertices $\{x_i^{(5)}/ 0 \le i \le n\}$

$N_2(x_i^{(5)})$ in $G_2 = N_2(x_i^{(5)})$ in G_1' U $N(x_i^{(4)})$ in G_1 U $N(N(x_i^{(5)})$ in G_1.

$= N_2(x_i^{(5)})$ in G_1' U$\{\mathbf{x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots x_{i+n}^{4}}, x_i^{(3)}, x_{i+3}^{(1)}\}$in G_1U $\{\mathbf{x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{4}, \ldots x_{i+n}^{4}}, x_{i+2}^{(3)}, x_i^{(1)}\}$ in G_1.

$= N_2(x_i^{(5)})$ in G_1' U $\{x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{(4)}, \ldots x_{i+n}^{4}, x_i^{(3)}, x_{i+3}^{(1)}, x_{i+2}^{(3)}, x_i^{(1)}\}$ in G_1.

Here $x_{i+1}^{(4)}, x_{i+2}^{(4)}, x_{i+3}^{4}, \ldots\ldots x_{i+n}^{4}$ are the common elements in $N(x_i^{(4)})$ in G_1 and $N(N(x_i^{(5)})$ in G_1.

$d_2(x_i^{(5)})$ in G_2 = ($d_2(x_i^{(5)})$ in G_1' + ($d(x_i^{(4)})$ in G_1 + $|N(N(x_i^{5}))|$ in G_1) - n

$d_2(x_i^{(5)})$ in G_2 =2(n+1)+(n+2+n+2)-(n) = 3(n+2) = [(n+3)-(n)[(n+3-1), $(0 \le i \le n)$.

In G_2, for $(1 \le t \le 8)$, $d_2(x_i^{(t)})$ = [(n+3)-(n)[(n+3-1), for $(0 \le i \le n)$.

G_2 is (n+3, 2, ((n+3)-(n)) (n+3-1))-regular on (n+1) x 2^{n+3-n} = 8(n+1) vertices with the vertex set $V(G_2) = \{x_i^{(t)}/(1 \le t \le 2^{n+3-n}), (0 \le i \le n)\}$and $E(G_2) = E(G_1)$U $E(G_1')$U$\{x_i^{(1)} x_{i+1}^{(8)}, x_i^{(2)} x_i^{(7)}, x_i^{(3)} x_{i+1}^{(6)}, x_i^{(4)} x_i^{(5)} /(0 \le i \le n\}$.

Therefore, the result is true for r = n+3.

Let us assume this result is true for r = m+n+1

That is , there exist (m+n+1, 2, (m+1)(m+n))- regular on (n+1) x 2^{m+1} vertices with the vertex set $V(G_m)=\{x_i^{(t)}/(1 \le t \le 2^{m+1}), (0 \le i \le n)\}$ and $E(G_m)= E(G_{m-1}) \cup E(G'_{m-1}) \bigcup_{t=1}^{2^m}\{x_i^{(t)} x_{i+t(\mathrm{mod}\,2)}^{2^{m+1}-t+1} / (0 \le i \le n)\}$.

That is, for$(1\le t \le 2^{m+1})$, $d_2(x_i^{(t)}) = (m+1)(m+n)$, for $(0 \le i \le n)$ and $d(x_i^{(t)}) = m+n+1$.

Take another copy of G_m as G'_m with the vertex set.
$V(G'_m)=\{x_i^{(t)}/(2^{m+1}+1 \le t \le 2^{m+2}), (0 \le i \le n)\}$ and each $x_i^{(t)}$, $(2^{m+1}+1 \le t \le 2^{m+2})$, corresponds to $x_i^{(t)}$, $(1 \le t \le 2^{m+1})$, for $(0 \le i \le n)$.

The desired graph G_{m+1} has the vertex set $V(G_{m+1}) = V(G_m) \cup V(G'_m)$ and

edge set $E(G_{m+1}) = E(G_m) \cup E(G'_m) \bigcup_{t=1}^{2^{m+1}}\{x_i^{(t)} x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1} /(0 \le i \le n)\}$.

Now the resulting graph G_{m+1} is (m+n+2) regular graph having (n+1) x 2^{m+2} vertices.

Consider the edges $\bigcup_{t=1}^{2^{m+1}}\{x_i^{(t)} x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1} /(0 \le i \le n)\}$.

For $(1 \le t \le 2^{m+1})$, d_2- of each vertex in $C^{(t)}$, where $C^{(t)}$ is the cycle induced by the vertices$\{x_i^{(t)}/ 0\le i\le n\}$.
$N_2(x_i^{(t)})$ in G_{m+1} = $N_2(x_i^{(t)})$ in G_m U $N(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1})$ in G'_m U $N(N(x_i^{(t)})$ in G'_m .

$d_2(x_i^{(t)})$ in G_{m+1} = $d_2(x_i^{(t)})$ in G_m + $d(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1})$ in G'_m + $|N(N(x_i^t))|$ in G'_m .
= (m+1) (m+n) + ((m+n+1)+(m+n+1))-n, for (0 ≤ i ≤ n).
= (m+2) (m+n+1), for (0 ≤ i ≤ n).
d_2 of each vertex in $C^{(2^{m+2}-t+1)}$, where $C^{(2^{m+2}-t+1)}$ is the cycle induced by the vertices$\{x_i^{(2^{m+2}-t+1)}/0\le i\le n\}$.
$N_2(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1})$ in G_{m+1}=$N_2(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1})$ in G'_m +$N(x_i^{(t)})$ in G_m + $|N(N(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1}))|$ in G_m.
$d_2(x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1})$ in G_{m+1} = (m+1)(m+n) +((m+n+1)+(m+n+1))-n, for(0 ≤ i ≤ n).
= (m+2) (m+n+1), for (0 ≤ i ≤ n).

In $G_{m+1,}$ for$(1 \le t \le 2^{m+2})$, $deg_2(x_i^{(t)})$=(m+2)(m+n+1), for (0 ≤ i ≤ n).

That is ,there exist (m+n+2, 2, (m+2)(m+n+1)) regular on (n+1) x 2^{m+2} vertices with the vertex set $V(G_{m+1}) = \{x_i^{(t)}/(1 \le t \le 2^{m+2}), (0 \le i \le n)\}$ and $E(G_{m+1}) = E(G_m) \cup E(G'_m) \bigcup_{t=1}^{2^{m+1}}\{x_i^{(t)} x_{i+t(\mathrm{mod}\,2)}^{2^{m+2}-t+1} / (0 \le i \le n)\}$.
That is, for$(1\le t \le 2^{m+2})$, $d_2(x_i^{(t)})$ =(m+2)(m+n+1), for (0 ≤ i ≤ n) and $d(x_i^{(t)})$=m+n+2.

If the result is true for r=m+n+1, then it is true for r=m+n+2.

Therefore, the result is true for all r ≥ n.

That is, for any r ≥ n ≥ 2, there is a (r, 2, (r – n) (r - 1)) - regular on (n+1) x 2^{r-n} vertices.

Corollary 3.4: For any r ≥ 2, there is a (r, 2, (r-2) (r-1)) - regular graph on 3 x 2^{r-2} vertices [10].

Corollary.3.5: For any r ≥ 3, there is a (r, 2, (r-3) (r-1)) - regular graph on 4 x 2^{r-3} vertices. [11].

Corollary 3.6: For any r ≥ 4, there is a (r, 2, (r-4) (r-1)-regular graph on 5 x 2^{r-4} vertices

Summary 3.7: In theorem 3.3, if we put n = 2, 3, 4, …. r, then we get (r, 2, (r-2)(r-1))-regular graph, (r, 2, (r-3)(r-1)) – regular graph, (r, 2,(r-4)(r-1) – regular graph , (r, 2, (r-5)(r-1)) – regcular graph …..(r, 2, 4(r-1))-regular graph, (r,2,3(r-1))-regular graph, (r,2,2(r-1))-regular graph, (r, 2, (r-1))-regular graph, (r, 2, 0)-regular graph. …..

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