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FUZZY PRE-CLOSURE (INTERIOR) OPERATORS AND FUZZY PRE-CONTINUITY IN SO-TOPOLOGICAL SPACES

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ABSTRACT

In the present paper, we introduce fuzzy pre open (closed) sets, fuzzy pre-closure (interior) operators and fuzzy pre-continuity in Sostak fuzzy topological space. Also we investigate their significant characteristic properties.

AMS Mathematics Subject Classification: *54A40.*

Keywords: *Fuzzy sets, fuzzy topological space, gradation of openness.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [9] and later Chang [1] defined fuzzy topological spaces. Sostak [8] introduced a new fuzzy topological space exploiting the idea of partial openness of fuzzy sets. This generalized fuzzy topological space was later rephrased by Chattopadhyay *et.al*. [2], Ramadan [6] etc.

The concepts of fuzzy strong preopen sets and strong pre continuity (see [4]), fuzzy preopen sets and fuzzy precontinuity (see [7]) etc. have been introduced in case of classical fuzzy topological spaces introduced by Chang [1]. In the present paper, we introduce fuzzy preopen (closed) sets, fuzzy ρ-pre closure and ρ-pre interior operators and fuzzy pre continuity in the Sostak fuzzy topological space redefined by Chattopdhyay [2]. We denote this generalized fuzzy topological space as So-fuzzy topological space for brevity of notation. We investigate significant characteristic properties of fuzzy pre open (closed) sets and also the ρ-pre closure (interior) operators. Further we establish interesting properties of fuzzy-ρ-pre continuous mappings.

2. PRELIMINARIES

Let X be a non-empty set and $I \equiv [0, 1]$ be the unit closed interval of real line. Let I^X denote the set of all fuzzy sets on X. A fuzzy set A on X is a mapping $A: X \to I$, where for any $x \in X$, $A(x)$ denotes the degree of membership of element x in fuzzy set A. The null fuzzy set 0 and whole fuzzy set 1 are the constant mappings from X to $\{0\}$ and $\{1\}$ respectively.

A family τ of fuzzy sets on X is called a fuzzy topology (see [1]) on X if (i) 0 and 1 belong to τ, (ii) Any union of members of τ is in τ, (iii) a finite intersection of members of τ is in τ. The system consisting of X equipped with fuzzy topology τ defined on it is called a fuzzy topological space and is denoted as (X, τ). Now we define the So-fuzzy topological space (see [2], [8]).

A So-fuzzy topology on a non-empty set X is a family τ of fuzzy sets on X satisfying the following axioms with respect to a mapping $\tau: I^X \to I$,

(i) $\tau(0) = \tau(1) = 1$;
(ii) $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$; for any $A, B \in I^X$;
(iii) $\tau\left(\cup_{i\in J} A\right) \geq \wedge_{i\in J} \tau(A_i)$, for any arbitrary family $\{A_i : i \in J\} \subseteq I^X$.

The system (X, τ) is called So-fuzzy topological space and the real number $\tau(A)$ is called the degree (or grade) of openness of fuzzy set A. We note that

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Proposition 2.1: *Let X be a non-empty set. Then the map $\tau : I^X \to I$ given by $\tau(0) = 1$ and $\tau(A) = inf\ \{A(x) : x \in supp\ A\}$ if $A \neq 0$, satisfies the axioms of gradation of openness.*

If (X, τ) is a So-fuzzy topological space, then we observe that (see [2]) for any $\rho \in [0, 1]$, the family $\tau_\rho \equiv \{ A \in I^X : \tau(A) \geq \rho\}$ is actually a fuzzy topology in sense of Chang [1] and it is called ρ-level fuzzy topology on X with respect to the gradation of openness τ. All fuzzy sets belonging to τ_ρ are called fuzzy-ρ-open sets and their complements are called fuzzy-ρ-closed sets.

For any fuzzy set A, the interior and closure of A with respect to τ_ρ are defined as follows:

$$Int_\rho(A) = \cup \{G \in I^X : G \subseteq A \text{ and } G \in \tau_\rho\}$$
$$Cl_\rho(A) = \cap \{K \in I^X : A \subseteq K \text{ and } K^c \in \tau_\rho\}$$

Proposition 2.2: *Clearly we observe that*
(i) $Int_\rho(A) = A$ *iff A is fuzzy-ρ-open set; and*
(ii) $Cl_\rho(A) = A$ *iff A is fuzzy-ρ-closed set.*

3. Fuzzy-ρ-Pre open (Closed) Sets

In this section, we define fuzzy-ρ-pre open sets and fuzzy-ρ-pre closed sets in So-fuzzy topological space and investigate their properties.

Definition 3.1: Let (X, τ) be a So-fuzzy topological space and $A \in I^X$ be a fuzzy set. Then for any $\rho \in I$, a fuzzy set A is said to be a
(i) Fuzzy-ρ-pre open set in X iff $A \subseteq Int_\rho(Cl_\rho\ (A))$
(ii) Fuzzy-ρ-pre closed set in X iff $A \supseteq Cl_\rho(Int_\rho\ (A))$

Clearly fuzzy sets 0 and 1 are both trivially fuzzy ρ-pre open as well as fuzzy ρ-pre closed sets in X.

Proposition 3.1: *In a So-fuzzy topological space, for any $\rho \in I$,*
(i) *Every fuzzy-ρ-open set is a fuzzy-ρ-pre open set;*
(ii) *Every fuzzy-ρ-closed set is a fuzzy-ρ-pre closed set.*
But converse of these may not be true in general.

Proof:
(i) Let (X, τ) be a So-fuzzy topological space and A be a fuzzy-ρ-open set on X, so that $\tau(A) \geq \rho$. Since $Int_\rho(A) = A$ (Proposition 2.1) and $A \subseteq Cl_\rho(A)$, we have $Int_\rho(A) \subseteq Int_\rho\left(Cl_\rho\ (A)\right)$, so that $Int_\rho(A) = A \subseteq Int_\rho\left(Cl_\rho\ (A)\right)$. Hence $A \subseteq Int_\rho\ \left(Cl_\rho\ (A)\right)$.
Thus A is a fuzzy-ρ-pre open set in X.
(ii) Let A be a fuzzy-ρ-closed set in So-fuzzy topological space (X, τ), so that $\tau(A^c) \geq \rho$ and $A = Cl_\rho(A)$. Also $Int_\rho\ (A) \subseteq A$, it follows $Cl_\rho\left(Int_\rho(A)\right) \subseteq Cl_\rho(A) = A$. Hence A is a fuzzy-ρ-pre closed set.

The fact that converse of (i) and (ii) may not be true in general can be shown through the following example.

Example 3.1: Let $X = \{a, b\}$ and $A, B, C, D, E, F \in I^X$ be fuzzy sets defined as follows:
$A = \{(a, 0.6), (b, 0.3)\}$ $B = \{(a, 0.4), (b, 0.5)\}$ $C = \{(a, 0.6), (b, 0.5)\}$
$D = \{(a, 0.4), (b, 0.3)\}$ $E = \{(a, 0.5), (b, 0.4)\}$ $F = \{(a, 0.5), (b, 0.6)\}$

Define a map $\tau : I^X \to I$ as follows:

$$\tau(F) = \begin{cases} 1, & if\ F = 0, 1 \\ 0.3, & if\ F = A, D \\ 0.4, & if\ F = B \\ 0.5 & if\ F = C \\ 0, & otherwise \end{cases}$$

Suppose ρ = 0.1. We see that fuzzy set E is a fuzzy-ρ-pre open set because $Cl_\rho(E) = B^c$ and $Int_\rho(B^c) = C$. Hence $Int_\rho\left(Cl_\rho(E)\right) = C \supseteq E$. Thus E is a fuzzy-ρ-pre open set, but it is not a fuzzy-ρ-open set (because $\tau(E) = 0 \ngeq 0.1$).

Similarly we observe that fuzzy set F is a fuzzy-ρ-pre closed set because $Cl_\rho\left(Int_\rho(F)\right) = Cl_\rho(B) = C^c \subseteq F$. Thus F is a fuzzy-ρ-pre closed set. But it is not a fuzzy-ρ-closed set.

Theorem 3.1: *Let (X,τ) be a So-fuzzy topological space. Then for any $\rho \in I$,*
a) *Any union of fuzzy-ρ-pre open sets is a fuzzy-ρ-pre open set;*
b) *Any intersection of fuzzy-ρ-pre closed sets is a fuzzy-ρ-pre closed set.*

Proof:
(a) Let $\{A_i : i \in J\}$ be an arbitrary collection of fuzzy-ρ-pre open sets in So-fuzzy topological space (X,τ). Then for each $i \in J$, we have $A_i \subseteq Int_\rho(Cl_\rho(A_i))$. Hence
$\cup_{i\in J} A_i \subseteq \cup_{i\in J} Int_\rho\left(Cl_\rho(A_i)\right) \subseteq Int_\rho(\cup_{i\in J} Cl_\rho(A_i)) \subseteq Int_\rho(Cl_\rho(\cup_{i\in J} A_i))$
Thus $\cup_{i\in J} A_i$ is a fuzzy-ρ-pre open set.
(b) Let $\{A_i : i \in J\}$ be an arbitrary collection of fuzzy-ρ-pre closed sets in So-fuzzy topological space (X,τ). Then for each $i \in J$, we have $A_i \supseteq Cl_\rho(Int_\rho(A_i))$.
Hence $\cap_{i\in J} A_i \supseteq \cap_{i\in J} Cl_\rho\left(Int_\rho(A_i)\right) \supseteq Cl_\rho(Int_\rho(\cap_{i\in J} A_i))$
Thus $\cap_{i\in J} A_{i\in J}$ is a fuzzy-ρ-pre closed set.

Definition 3.2: Let (X,τ) be a So-fuzzy topological space and $A \in I^X$ be a fuzzy set. Then for any $\rho \in I$, fuzzy-ρ-pre interior and fuzzy-ρ-pre closure of fuzzy set A denoted as $P\text{-}int_\rho(A)$ and $P\text{-}cl_\rho(A)$ are defined as follows:
$$P\text{-}int_\rho(A) = \cup\{G \in I^X : G \subseteq A \text{ and } G \text{ is a fuzzy-}\rho\text{-pre open set in } X\}$$
$$P\text{-}cl_\rho(A) = \cap\{K \in I^X : K \supseteq A \text{ and } K \text{ is a fuzzy-}\rho\text{-pre closed set in } X\}$$

Theorem 3.2: *Let (X,τ) be a So-fuzzy topological space and $A \in I^X$ be a fuzzy set. Then for any $\rho \in I$,*
(i) $P\text{-}cl_\rho(1-A) = 1 - P\text{-}int_\rho(A)$
(ii) $P\text{-}int_\rho(1-A) = 1 - P\text{-}cl_\rho(A)$

Proof:
(i) Suppose $\{G_i\}_{i\in J}$ is the family of all fuzzy-ρ-preopen sets in X contained in A. Then
$P\text{-}int_\rho(A) = \cup_{i\in J} G_i = 1 - \cap_{i\in J} G_i^c$

Since $G_i \subseteq A$, we have $G_i^c \supseteq A^c, \forall\, i \in J$. Thus $\{G_i^c\}_{i\in J}$ is the collection of all fuzzy-ρ-preclosed sets containing A^c.

Hence $\cap_{i\in J} G_i^c = P\text{-}cl_\rho(A^c) = P\text{-}cl_\rho(1-A)$.

Thus $P\text{-}int_\rho(A) = 1 - P\text{-}cl_\rho(1-A)$

Hence $P\text{-}cl_\rho(1-A) = 1 - P\text{-}int_\rho(A)$. This proves (i).
(ii) can be proved in a similar manner.

Theorem 3.3: *Let (X,τ) be a So-fuzzy topological space. Then for any $\rho \in I$, a fuzzy set $A \in I^X$ is a*
a) *Fuzzy-ρ-pre open set iff $P\text{-}int_\rho(A) = A$;*
b) *Fuzzy-ρ-pre closed set iff $P\text{-}cl_\rho(A) = A$.*

Proof:
(a) Let A be fuzzy-ρ-pre open set in X. Let $\{G_i\}_{i\in J}$ be the family of all fuzzy ρ-pre open sets which are contained in A. Since each $G_i \subseteq A,\ i \in J$, we have $\cup_{i\in J} G_i \subseteq A$. Therefore
$P\text{-}int_\rho = \cup_{i\in J}\{G_i \in I^X : G_i \subseteq A \text{ and } G_i \text{ is a fuzzy-}\rho\text{-preopen set}\} \subseteq A$ (3.3.1)

Since $A \subseteq A$ and A is a fuzzy-ρ-preopen set in X, hence $A \in \{G_i\}_{i\in J}$. Therefore
$A \subseteq \cup_{i\in J} G_i \equiv P\text{-}int_\rho(A)$ (3.3.2)

From equations (3.3.1) and (3.3.2), $A = P\text{-}int_\rho(A)$.

Conversely; suppose A is a fuzzy set in So-fuzzy topological space (X,τ) such that $A = P\text{-}int_\rho(A)$. Then
$A = P\text{-}int_\rho(A) = \cup\ \{G_i \in I^X : G_i \subseteq A \text{ and } G_i \text{ is a fuzzy-}\rho\text{-pre open set}\}$ (3.3.3)

Since any union of fuzzy-ρ-preopen sets is a fuzzy-ρ-preopen set, in view of (3.3.3), set A is a fuzzy-ρ-pre open set in X.
(b) This can be proved in a similar manner.

Theorem 3.4: *Let (X, τ) be a So-fuzzy topological space. Then for any $\rho \in I$, the following properties hold for fuzzy-ρ-pre closure:*
(i) $P\text{-}cl_\rho(0) = 0$
(ii) $P\text{-}cl_\rho(A)$ *is a fuzzy-ρ-pre closed set in X*
(iii) $P\text{-}cl_\rho(A) \subseteq P\text{-}cl_\rho(B),\ if\ A \subseteq B$
(iv) $P\text{-}cl_\rho(P\text{-}cl_\rho(A)) = P\text{-}cl_\rho(A)$
(v) $P\text{-}cl_\rho(A \cup B) \supseteq P\text{-}cl_\rho(A) \cup P\text{-}cl_\rho(B)$
(vi) $P\text{-}cl_\rho(A \cap B) \leq P\text{-}cl_\rho(A) \cap P\text{-}cl_\rho(B)$

Proof: Let (X, τ) be a So-fuzzy topological space and A, B be fuzzy sets on X. Let $\rho \in I$.

(i) In view of Definition 3.2,
$P\text{-}cl_\rho(0) = \cap\ \{K_i \in I^X : K_i \supseteq 0\ and\ K_i\ is\ a\ fuzzy\text{-}\rho\text{-}pre\ closed\ set\ in\ X\}$
We know 0 is a fuzzy-ρ-pre closed set. Thus 0 is a fuzzy-ρ-pre closed set containing 0. Hence $0 \in \{K_i\}_{i\in J}$.
Therefore $\cap_{i\in J} K_i = 0$. Thus $P\text{-}cl_\rho(0) = 0$.

(ii) By definition, $P\text{-}cl_\rho(A)$ is the intersection of all fuzzy-ρ-pre closed sets containing A and in view of Theorem 3.1, any intersection of fuzzy-ρ-pre closed sets is a fuzzy-ρ-pre closed set. Thus $P\text{-}cl_\rho(A)$ is a fuzzy-ρ-pre closed set in X.

(iii) Let A and B be two fuzzy sets in X such that $A \subseteq B$. Consider the collection $\{K_i\}_{i\in J}$ such that K_i is a fuzzy ρ-pre closed set and contains A for each $i \in J$; so that $P\text{-}cl_\rho(A) = \cap_{i\in J} \{K_i\}$. Now consider $P\text{-}cl_\rho(B)$. We know that
$P\text{-}cl_\rho(B) = \cap_{l\in L} \{F_l : F_l \supseteq B\ and\ F_l\ is\ a\ fuzzy\text{-}\rho\text{-}pre\ closed\ set\ in\ X\}$
Since $B \supseteq A, F_l \supseteq A, \forall\ l \in L$. Therefore $\cap_{l\in L} F_l \supseteq B \supseteq A$. Thus $P\text{-}cl_\rho(B)$ is a fuzzy ρ-pre closed set in X, which contains A. Therefore $P\text{-}cl_\rho(B) \in \{K_i\}_{i\in J}$.
Hence $\cap_{i\in J} K_i \subseteq P\text{-}cl_\rho(B)$.
Thus $P\text{-}cl_\rho(A) \subseteq P\text{-}cl_\rho(B)$. This proves (iii).

(iv) We know by (ii) that for every A in I^X, $P\text{-}cl_\rho(A)$ is a fuzzy ρ-preclosed set in X. Therefore in view of Theorem 3.3 (b), we conclude that $P\text{-}cl_\rho(P\text{-}cl_\rho(A)) = P\text{-}cl_\rho(A)$.

(v) In view of (iii), we know that if $P \subseteq Q$ in X, then $P\text{-}cl_\rho(P) \subseteq P\text{-}cl_\rho(Q)$. Now $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
Therefore $P\text{-}cl_\rho(A \cup B) \supseteq P\text{-}cl_\rho(A)$ and $P\text{-}cl_\rho(A \cup B) \supseteq P\text{-}cl_\rho(B)$.
Hence $P\text{-}cl_\rho(A \cup B) \supseteq P\text{-}cl_\rho(A) \cup P\text{-}cl_\rho(B)$

(vi) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$. We have
$P\text{-}cl_\rho(A \cap B) \subseteq P\text{-}cl_\rho(A)$ and $P\text{-}cl_\rho(A \cap B) \subseteq P\text{-}cl_\rho(B)$
Therefore, $P\text{-}cl_\rho(A \cap B) \subseteq P\text{-}cl_\rho(A) \cap P\text{-}cl_\rho(B)$.

Similarly we have the following:

Theorem 3.5: *Let (X, τ) be a So-fuzzy topological space and $A, B \in I^X$ be fuzzy sets. Then for any $\rho \in I$,*
(i) $P\text{-}int_\rho(1) = 1$
(ii) $P\text{-}int_\rho(A)$ *is a fuzzy-ρ-pre open set in X*
(iii) $P\text{-}int_\rho(A) \subseteq P\text{-}int_\rho(B),\ if\ A \subseteq B$
(iv) $P\text{-}int_\rho(P\text{-}int_\rho\ (A)) = P\text{-}int_\rho(A)$
(v) $P\text{-}int_\rho(A \cup B) \supseteq P\text{-}int_\rho(A) \cup P\text{-}int_\rho(B)$
(vi) $P\text{-}int_\rho(A \cap B) \subseteq P\text{-}int_\rho(A) \cap P\text{-}int_\rho(B)$

4. Fuzzy-ρ-Pre Continuous Map

In this section, we define a fuzzy-ρ-pre continuous map from one So-fuzzy topological space to another and investigate its characteristic properties. We know fuzzy-ρ-continuous map is defined (see [2]) as follows:

Definition 4.1: Let (X, τ) and (Y, σ) be two So-fuzzy topological spaces. A map $f: X \to Y$ is said to fuzzy-ρ-continuous map if $\tau\big(f^{-1}(B)\big) \geq \sigma(B)$, for each fuzzy set $B \in I^Y$ such that $\sigma(B) \geq \rho$.

Now we define fuzzy-ρ-pre continuous map as follow:

Definition 4.2: Let (X, τ) and (Y, σ) be two So-fuzzy topological spaces. A map f from X to Y is called a fuzzy-ρ-pre continuous map iff $f^{-1}(B)$ is a fuzzy-ρ-pre open set for any fuzzy set $B \in I^Y$ such that $\sigma(B) \geq \rho$.

Proposition 4.1: *Every fuzzy-ρ-continuous map is a fuzzy-ρ-pre continuous map, but converse may not be true.*

Proof: Let (X,τ) and (Y,σ) be two So-fuzzy topological spaces and $f:X\to Y$ be a fuzzy-ρ-continuous map. Suppose $B\in I^Y$ is a fuzzy-ρ-open set in Y, so that $\sigma(B)\ge\rho$. Then $\tau\big(f^{-1}(B)\big)\ge\sigma(B)\ge\rho$. Hence $f^{-1}(B)$ is a fuzzy-ρ-open set in X. Since every fuzzy-ρ-open set is a fuzzy-ρ-pre open set, $f^{-1}(B)$ is a fuzzy-ρ-pre open set in X. Thus f is a fuzzy-ρ-pre continuous map.
But converse of this may not be true in general. This can be exemplified as follows:

Example 4.1: Let $X=\{a,b\}$, $Y=\{u,v\}$ and $A,B,C,D\in I^X,\in I^Y$ be fuzzy sets defined as follows:

$A=\{(a,0.7),(b,0.2)\}$ $\quad B=\{(a,0.5),(b,0.6)\}$ $\quad C=\{(a,0.7),(b,0.6)\}$

$D=\{(a,0.5),(b,0.2)\}$ $\quad E=\{(u,0.8),(v,0.7)\}$

We define fuzzy topologies $\tau: I^X\to I$ and $\sigma: I^Y\to I$ as follows:

$$\tau(F)=\begin{cases}1, & if\ F=0,1\\ 0.2, & if\ F=A,D\\ 0.5, & if\ F=B\\ 0.6 & if\ F=C\\ 0, & otherwise\end{cases}$$

$$\sigma(F)=\begin{cases}1, & if\ F=0,1\\ 0.7, & if\ F=E\\ 0, & otherwise\end{cases}$$

Consider a map $f:(X,\tau)\to(Y,\sigma)$ defined as $f(a)=u, f(b)=v$. Suppose $\rho=0.1$. We see that $Cl_\rho\big(f^{-1}(E)\big)=1$ and $Int_\rho\Big(Cl_\rho\big(f^{-1}(E)\big)\Big)=1$. Thus $f^{-1}(E)\subseteq Int_\rho(Cl_\rho(f^{-1}(E)))$. Hence $f^{-1}(E)$ is a fuzzy-ρ-pre open set. Similarly $f^{-1}(0)\equiv 0$ and $f^{-1}(1)\equiv 1$ are also fuzzy-ρ-pre open sets. Thus f is a fuzzy-ρ-pre continuous map. But f is not a fuzzy-ρ-continuous map because we observe that $f^{-1}(E)=\{(a,0.8),(b,0.7)\}$ and $\tau\big(f^{-1}(E)\big)=0\ngeq\sigma(E)\equiv 0.7$.

Theorem 4.2: *Let (X,τ) and (Y,σ) be two So-fuzzy topological spaces and $\rho\in I$ be a real number. If $f:X\to Y$ be a mapping such that $\tau^*\big(f^{-1}(B)\big)\ge\rho$, for each $B\in I^Y$ with $\sigma^*(B)\ge\rho$, then f is a fuzzy-ρ-pre continuous map.*

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a map such that $\tau^*\big(f^{-1}(B)\big)\ge\rho$, for each $B\in I^Y$ for which $\sigma^*(B)\ge\rho$. Since $f^{-1}(B)\in I^X$ and $\tau^*\big(f^{-1}(B)\big)=\tau\big((f^{-1}(B))^c\big)=\tau\big(f^{-1}(B^c)\big)\ge\rho$, we conclude that $f^{-1}(B^c)$ is a fuzzy-ρ-open set in X. Since every fuzzy ρ-open set is a fuzzy ρ-pre open set, $f^{-1}(B^c)$ is a fuzzy-ρ-pre open set in X. Further $\sigma(B^c)=\sigma^*(B)\ge\rho$. Thus $f^{-1}(B^c)$ is a fuzzy-ρ-pre open set in X for each $B^c\in I^Y$ such that $\sigma(B^c)\ge\rho$. Therefore f is a fuzzy-ρ-pre continuous map.

Theorem 4.3: *Let $f:(X.\tau)\to(Y,\sigma)$ be a map from one So-fuzzy topological space to another. Then for any $\rho\in I$, following statements are equivalent:*

a) *f is a fuzzy-ρ-pre continuous map;*
b) *$f^{-1}(B)$ is a fuzzy-ρ-pre closed set for each fuzzy-ρ-closed set B in Y;*
c) *$Cl_\rho\Big(Int_\rho\big(f^{-1}(B)\big)\Big)\subseteq f^{-1}(Cl_\rho(B))$, for each fuzzy set B in Y;*
d) *$f\Big(Cl_\rho\Big(Int_\rho(A)\Big)\Big)\subseteq Cl_\rho(f(A))$, for each fuzzy set A in X.*

Proof: Let (X,τ) and (Y,σ) be two So-fuzzy topological spaces. We will prove this theorem in following steps:

(i) $(a)\Rightarrow(b)$: Let $f:X\to Y$ be a fuzzy-ρ-pre continuous map for any $\rho\in I$. Let B be a fuzzy-ρ-closed set in Y. Then B^c is a fuzzy-ρ-open set in Y so that $\sigma(B^c)\ge\rho$. Since f is a fuzzy ρ-continuous map, we find that $f^{-1}(B^c)$ is a fuzzy-ρ-pre open set in X. Therefore $\big(f^{-1}(B^c)\big)^c=f^{-1}(B)$ is a fuzzy-ρ-pre closed set in X for each $B\in I^Y$ which is a fuzzy ρ-closed set.
Thus (a)⇒(b).

(ii) $(b)\Rightarrow(a)$: Let $f:X\to Y$ be a map such that $f^{-1}(B)$ is a fuzzy ρ-pre closed set in X if B is a fuzzy ρ-closed set in Y. Let C be any fuzzy ρ-open in Y. Then C^c is a fuzzy ρ-closed set in Y. Therefore by (b), $f^{-1}(C^c)$ is a fuzzy ρ-pre closed set in X. Therefore $(f^{-1}(C^c))^c\equiv f^{-1}(C)$ is fuzzy ρ-pre open set in X. Thus if C is fuzzy ρ-open set in Y, then $f^{-1}(C)$ is fuzzy ρ-pre open set in X. Hence map $f:X\to Y$ is fuzzy ρ-pre continuous map.

(iii) $(b)\Rightarrow(c)$: Let B be a fuzzy set in Y, then $Cl_\rho(B)$ is a fuzzy-ρ-closed set in Y and hence by (b), $f^{-1}(Cl_\rho(B))$ is a fuzzy-ρ-pre closed set in X. Therefore by definition, $f^{-1}\Big(Cl_\rho(B)\Big)\supseteq Cl_\rho\bigg(Int_\rho\Big(f^{-1}\Big(Cl_\rho(B)\Big)\Big)\bigg)\supseteq Cl_\rho(Int_\rho(f^{-1}(B)))$, because $B\subseteq Cl_\rho(B)$. Thus $Cl_\rho\Big(Int_\rho\big(f^{-1}(B)\big)\Big)\subseteq f^{-1}(Cl_\rho(B))$.

(iv) $(c)\Rightarrow(d)$: Let $A\in I^X$ be any fuzzy set, then $f(A)\in I^Y$. Now by (c),

$$Cl_\rho\Big(Int_\rho\big(f^{-1}(f(A))\big)\Big)\subseteq f^{-1}(Cl_\rho(f(A)))$$

It implies $Cl_\rho\left(Int_\rho(A)\right) \subseteq f^{-1}(Cl_\rho(f(A)))$.

Now $Cl_\rho(f(A)) \supseteq f\left(f^{-1}\left(Cl_\rho(f(A))\right)\right)$

Therefore $f\left(Cl_\rho\left(Int_\rho(A)\right)\right) \subseteq f\left(f^{-1}\left(Cl_\rho(f(A))\right)\right) \subseteq Cl_\rho(f(A))$

Hence $f\left(Cl_\rho\left(Int_\rho(A)\right)\right) \subseteq Cl_\rho(f(A))$ for each A in X.

(v) $(d) \Rightarrow (b)$: Let $B \in I^Y$ be a fuzzy-ρ-closed set, then $f^{-1}(B) \in I^X$. Now from (d) we have

$$f\left(Cl_\rho\left(Int_\rho(f^{-1}(B))\right)\right) \subseteq Cl_\rho\left(f(f^{-1}(B))\right) \subseteq Cl_\rho(B) = B.$$

Thus $f\left(Cl_\rho\left(Int_\rho(f^{-1}(B))\right)\right) \subseteq B$

Therefore $f^{-1}(f\left(Cl_\rho\left(Int_\rho(f^{-1}(B))\right)\right) \subseteq f^{-1}(B)$

It implies $Cl_\rho\left(Int_\rho(f^{-1}(B))\right) \subseteq f^{-1}(B)$.

Thus $f^{-1}(B)$ is a fuzzy-ρ-pre closed set in X for each fuzzy-ρ-closed set B in Y.

This completes the proof of the theorem.

Theorem 4.4: *Let (X,τ), (Y,σ) and (Z,δ) be three So-fuzzy topological spaces and let $\rho \in I$ be any real number. If $f: X \to Y$ is a fuzzy-ρ-pre continuous map and $g: Y \to Z$ is a fuzzy-ρ-continuous map, then $g \circ f: X \to Z$ is a fuzzy-ρ-pre continuous map.*

Proof: Suppose $(X,\tau), (Y,\sigma)$ and (Z,δ) are So-fuzzy topological spaces and supoose $f: X \to Y$ is a fuzzy-ρ-pre continuous map and $g: Y \to Z$ is a fuzzy-ρ-continuous map. Let C be a fuzzy-ρ-open set in Z so that $\delta(C) \geq \rho$, then $\sigma\left(g^{-1}(C)\right) \geq \delta(C) \geq \rho$, because g is a fuzzy-ρ-continuous mapping. Thus $g^{-1}(C)$ is a fuzzy-ρ-open set in Y.

Since f is a fuzzy-ρ-pre continuous map, we get that $f^{-1}(g^{-1}(C))$ is a fuzzy-ρ-pre open set in X. Now $f^{-1}\left(g^{-1}(C)\right) = (g \circ f)^{-1}(C)$. Hence $(g \circ f)^{-1}(C)$ is a fuzzy-ρ-pre open set in X,

Now $g \circ f: (X,\tau) \to (Z,\delta)$ is a map and we have derived that for any fuzzyρ-open set C in Z, fuzzy set $(g \circ f)^{-1}(C)$ is a fuzzy-ρ-pre open set in X. Hence $(g \circ f)$ is a fuzzy-ρ-pre continuous map.

CONCLUSION

In the present paper, we have defined fuzzy pre open (closed) sets, fuzzy pre-closure (interior) operators and fuzzy pre-continuity in Sostak fuzzy topological space. The concepts is introduced as extensions of concepts of fuzzy preopen sets introduced in [7]. Several significant results have been obtained.

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