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SOLVING INTUITIONISTIC FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM USING DIVISION OPERATION BASED ON SCORE FUNCTION

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ABSTRACT

In this paper, we find the solution of intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP). We define division operation of Triangular Intuitionistic Fuzzy number (TIFN) using α, β – cut and a scoring function to rank TIFNs. The solution of intuitionistic fuzzy multi-objective linear programming problem is obtained by using division operation of Triangular Intuitionistic Fuzzy number and score function. Finally numerical example is provided to verify the proposed method.

Keywords: *Fuzzy set, Intuitionistic fuzzy set (IFS), Triangular Intuitionistic fuzzy numbers (TIFNs), score function, division operation, Intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP), Optimum solution.*

1. INTRODUCTION

Atanassov [20] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set [20]. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by S.Mahapatra& T.K.Roy in [21], by considering the six tuple number itself. Bellman and Zadeh (1970) [1] proposed the concept of decision making in fuzzy environments. After the pioneering work on fuzzy linear programming by Tanaka et al Zimmermann (1976) [2, 3] first introduced fuzzy set theory into the ordinary linear programming and multi-objective linear programming problems with fuzzy goals and fuzzy constraints. A ratio ranking method of TIFN is developed by Deng-Feng-Li in [23]. Ranking methods based on probabilities and hesitations is defined by L.Shen. *et al.* in [24]. Scoring function of a fuzzy number intuitionistic fuzzy value is defined by X.F.Wang in [25]. Fuzzy optimization [4, 5, 6, 7] has to be chosen when we encounter inherent imprecision or vagueness [8, 9]. Therefore, fuzzy mathematical programming representing the uncertainty or ambiguity in decision making situations by fuzzy concepts has attracted attention of many researchers [10, 11, 12, 13]. Fuzzy multi-objective linear programming, first proposed by Zimmermann [14] has been rapidly developed by numerous researchers and it is most frequently applied to the increasing number of real problems. The most common approach to solve fuzzy linear programming problem is to change them to corresponding deterministic linear program. Some methods based on comparison of fuzzy numbers have been suggested by H.R.Maleki [15], Ebrahimnejad, Roubens [16, 17]. Pandian and Jayalakskmi [18] proposed a new method for solving integer linear programming problems with fuzzy variables.

The aim of this paper is to solve intuitionistic fuzzy multi-objective linear programming problem by using division operation of Triangular Intuitionistic Fuzzy number and score function.

2. PRELIMINARIES

Definition 2.1: [19]: Let X is a nonempty set *A*. Fuzzy set *A* in X is characterized by its membership function $\mu_A : x \rightarrow [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element X in fuzzy set A for each $x \in$ X.

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Definition 2.2: [20]: An intuitionistic fuzzy set A in X is defined as an object of the form $A = \{ (x, \mu_A(x), V_A(x)), x \in X \}$ where the function $\mu_A : X \to [0,1]$ and $V_A : X \to [0,1]$ define the degree of membership and the degree of nonmember ship of the element $x \in X$ respectively and for every $x \in X$ in A, $0 \le \mu_A(x) + V_A(x) \le 1$ holds.

Definition 2.3: [21]: A Triangular Intuitionistic fuzzy number (TIFN) A as an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and of nonmember ship function $v_A(x)$.

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & for\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

$$v_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & for\ a_2 \le x \le a_3 \\ 1 & otherwise \end{cases}$$

where $a_1^{'} \le a_1 \le a_2 \le a_3 \le a_3^{'}$ and $\mu_A(x) + V_A(x) \le 1$, or $\mu_A(x) = V_A(x)$ for all $x \in R$. This TIFN is denoted by $A = (a_1, a_2, a_3 ; a_1^{'}, a_2, a_3^{'}) = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'})\}$.

Definition 2.4: [21]: A set of $(\alpha, \beta)-$ cut generated by IFS A, where $(\alpha, \beta) \in [0,1]$ are fixed members such that $\alpha + \beta \le 1$ is defined as $A_{\alpha,\beta} = \{ (x, \mu_A(x), V_A(x)), x \in X \}$, $\mu_A(x) \le \alpha$, $V_A(x) \le \beta$, $(\alpha, \beta) \in [0,1]$. (α, β) - level interval or (α, β) - cut denoted by $A_{\alpha,\beta}$ is denoted as the crisp set of elements of x which belong to A at least to the degree α and which does belong to A at most to the degree β.

Definition 2.5: The support of an IFS A on R is the crisp set of all $x \in R$ such that $c > 0$, $V_A(x) > 0$, and $\mu_A(x) + V_A(x) \le 1$.

3. ARITHMETIC OPERATIONS [21]

Arithmetic Operations of Triangular Intuitionistic fuzzy number based on (α, β) - cut method:

a) If $A = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'})\}$ *and* $B = \{(b_1, b_2, b_3); (b_1^{'}, b_2, b_3^{'})\}$ are two TIFNs, then their sum

$$A + B = \left(a_1 + b_1, a_2 + b_2, a_3 + b_3\right), \left(a_1^{'} + b_1^{'}, a_2 + b_2, a_3^{'} + b_3^{'}\right) \text{is also a TIFN.}$$

b) If $A = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'})\}$ *and* $B = \{(b_1, b_2, b_3); (b_1^{'}, b_2, b_3^{'})\}$ are two TIFNs, then

$$A - B = \left(a_1 - b_3, a_2 - b_2, a_3 - b_1\right), \left(a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'}\right) \text{is also a TIFN.}$$

c) If $A = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'})\}$ *and* $B = \{(b_1, b_2, b_3); (b_1^{'}, b_2, b_3^{'})\}$ are two TIFNs, then

$$A\ X\ B = \left(a_1 b_1, a_2 b_2, a_3 b_3\right), \left(a_1^{'} b_1^{'}, a_2 b_2, a_3^{'} b_3^{'}\right) \text{is also a TIFN.}$$

d) If $A = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'})\}$ and y = ka (with k > 0) then

$$y = ka = \{(ka_1, k a_2, ka_3); (ka_1^{'}, ka_2, ka_3^{'})\} \text{is also a TIFN.}$$

e) If $A=\{(a_1,a_2,\ a_3);\ (a_1^{'},a_2,\ a_3^{'})\}\ and\ B=\{(b_1,b_2,\ b_3);\ (b_1^{'},b_2,\ b_3^{'})\}$ are two positive TIFNs, then $\frac{A}{B}$ is also a TIFN, Where $\frac{A}{B}=\left\{(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}),(\frac{a_1^{'}}{b_3^{'}},\frac{a_2}{b_2},\frac{a_3^{'}}{b_1^{'}})\right.$.

4. PROPOSED DIVISION OF TWO TIFNs BASED ON (α, β) – CUTS METHOD

If $A=\{(a_1,a_2,\ a_3);\ (a_1^{'},a_2,\ a_3^{'})\}\ and\ B=\{(b_1,b_2,\ b_3);\ (b_1^{'},b_2,\ b_3^{'})\}$ are two positive TIFNs, then $\frac{A}{B}$ is also a TIFN, Where $\frac{A}{B}=\left\{(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}),(\frac{a_1^{'}}{b_3^{'}},\frac{a_2}{b_2},\frac{a_3^{'}}{b_1^{'}})\right.$.

Proof: Let $z=\frac{x}{y}$ be the transformation with the membership functions and non

Membership functions of TIFNs. Then $\tilde{z}^I=\frac{\tilde{A}^I}{\tilde{B}^I}$ can be found by (α,β) – cuts Method:

- α-cut for membership function of $\tilde{A}^I$ is $[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)]$, $\forall\alpha\in[0,1]$
  i.e. $x\in(a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2))$.
- α-cut for membership function of $\tilde{B}^I$ is $[b_1+\alpha(b_2-b_1),b_3-\alpha(b_3-b_2)]$, $\forall\alpha\in[0,1]$
  i.e. $x\in(b_1+\alpha(b_2-b_1),b_3-\alpha(b_3-b_2))$

To calculate division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ and $\tilde{B}^I$, we

First divide the α – cuts of $\tilde{A}^I$ and $\tilde{B}^I$ using interval arithmetic.

$$\frac{\tilde{A}^I}{\tilde{B}^I}=\frac{a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)}{b_1+\alpha(b_2-b_1),b_3-\alpha(b_3-b_2)}$$

$$=\frac{a_1+\alpha(a_2-a_1)}{b_3-\alpha(b_3-b_2)},\frac{a_3-\alpha(a_3-a_2)}{b_1+\alpha(b_2-b_1)}$$

To find the membership function $\mu_{\frac{\tilde{A}^I}{\tilde{B}^I}}(x),$ we equate to x both the first and second component, which gives

$$x=\frac{a_1+\alpha(a_2-a_1)}{b_3-\alpha(b_3-b_2)}\qquad x=\frac{a_3-\alpha(a_3-a_2)}{b_1+\alpha(b_2-b_1)}$$

Now expressing α in terms of x and setting α = 0 and α = 1, we get

$$\mu_{\frac{\tilde{A}^I}{\tilde{B}^I}}(x)=\begin{cases}\dfrac{a_3-b_1x}{(a_3-a_2)(b_2-b_1)x^{'}} & \dfrac{a_2}{b_2}\le x\le\dfrac{a_3}{b_1}\\ \dfrac{b_3x-a_1}{(a_2-a_1)(b_3-b_2)x^{'}} & \dfrac{a_1}{b_3}\le x\le\dfrac{a_2}{b_2}\end{cases}$$

iii) β-cut for non - membership function of $\tilde{A}^I$ is $(a_2-\beta(a_2-a_1^{'}),a_2+\beta(a_3^{'}-a_2))$

$\forall\beta\in[0,1]$ i.e. $x\in(a_2-\beta(a_2-a_1^{'}),a_2+\beta(a_3^{'}-a_2))$

iii) β-cut for non - membership function of $\tilde{B}^I$ is $(b_2-\beta(b_2-b_1^{'}),b_2+\beta(b_3^{'}-b_2))$

$\forall\beta\in[0,1]$ i.e. $x\in(b_2-\beta(b_2-b_1^{'}),b_2+\beta(b_3^{'}-b_2))$

To calculate division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ and $\tilde{B}^I$, we

First divide the β – cuts of $\tilde{A}^I$ and $\tilde{B}^I$ using interval arithmetic

$$\frac{\tilde{A}^I}{\tilde{B}^I}=\frac{a_2 - \beta(a_2-a'_1), a_2 + \beta(a'_3-a_2)}{b_2 - \beta(b_2-b'_1), b_2 + \beta(b'_3-b_2)}$$

$$= \frac{a_2 - \beta(a_2-a'_1)}{b_2 + \beta(b'_3-b_2)}, \frac{a_2 + \beta(a'_3-a_2)}{b_2 - \beta(b_2-b'_1)}$$

$$x=\frac{a_2 - \beta(a_2-a'_1)}{b_2 + \beta(b'_3-b_2)} \qquad x=\frac{a_2 + \beta(a'_3-a_2)}{b_2 - \beta(b_2-b'_1)}$$

Now expressing β in terms of x and setting β = 0, β = 1, we get

$$v_{\frac{\tilde{A}^I}{\tilde{B}^I}}(x) = \begin{cases} \frac{a_2-b_2x}{(a_2-a'_1)(b'_3-b_2)x'} & \frac{a'_1}{b'_3} \le x \le \frac{a_2}{b_2} \\ \frac{b_2x-a_2}{(a'_3-a_2)(b_2-b'_1)x'} & \frac{a_2}{b_2} \le x \le \frac{a'_3}{b'_1} \end{cases}$$

Hence the division rule is proved for membership and non – membership functions

Thus $\frac{A}{B}=\left\{ \left(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}\right),\left(\frac{a'_1}{b'_3},\frac{a_2}{b_2},\frac{a'_3}{b'_1}\right)\right.$ is also a TIFN.

5. INTUITIONISTIC FUZZY LINEAR PROGRAMMING

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined as

(IFLP) $$Maximize\ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I\, \tilde{x}_j^I, \qquad (1)$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I\, \tilde{x}_j^I \le \tilde{b}_i^I, \quad i=1,2.....................m$$

$$\tilde{x}^I{}_j \ge 0, \; j=1,2,......................n \qquad (2)$$

where $\tilde{A}^I = \tilde{a}_{ij}^I$, $\tilde{c}^I$, $\tilde{b}^I$, $\tilde{x}^I$ are (m X n) , (1 X n), (m X 1), (n X 1) Intuitionistic fuzzy matrices consisting of Triangular Intuitionistic Fuzzy Numbers (TIFN).

6. MULTI-OBJECTIVE INTUITIONISTIC FUZZY LINEAR PROGRAMMING

A mathematical model can be stated as:

Find $X = (x_1\ x_2............x_n)^T$

So as to

$$Maximize\ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^{Ik}\, \tilde{x}_j^I, \qquad (3)$$

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I\, \tilde{x}_j^I \le \tilde{b}_i^I, \quad i=1,2.....................m \qquad (4)$$

$$\tilde{x}^I{}_j \ge 0, \; j=1,2,......................n \qquad (5)$$

6.1 Standard Form [22]

The objective function should be of maximization form (IFLP)

$$Maximize\ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I\, \tilde{x}_j^I, \qquad (6)$$

Subject to

$$\tilde{a}^I{}_{11}\tilde{x}_1{}^I + \tilde{a}^I{}_{12}\,\tilde{x}_2{}^I + \dots\dots + \tilde{a}^I{}_{1n}\tilde{x}_n{}^I + \tilde{1}^I\ \tilde{x}_{n+1}{}^I + \tilde{A}^I{}_1\tilde{1}^I = \tilde{b}_1{}^I$$

$$\tilde{a}^I{}_{21}\tilde{x}_1{}^I + \tilde{a}^I{}_{22}\,\tilde{x}_2{}^I + \dots\dots + \tilde{a}^I{}_{2n}\tilde{x}_n{}^I + \tilde{1}^I\ \tilde{x}_{n+2}{}^I + \tilde{A}^I{}_2\,\tilde{1}^I = \tilde{b}_2{}^I \qquad (7)$$

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$$\tilde{a}^I{}_{m1}\tilde{x}_1{}^I + \tilde{a}^I{}_{m2}\,\tilde{x}_2{}^I + \dots\dots + \tilde{a}^I{}_{mn}\tilde{x}_n{}^I + \tilde{1}^I\ \tilde{x}_{n+m}{}^I + \tilde{A}^I{}_m\,\tilde{1}^I = \tilde{b}_m{}^I$$

Where

$$\tilde{x}_1{}^I,\ \tilde{x}_2{}^I + \dots\dots \tilde{x}_n{}^I\tilde{x}_{n+1}{}^I \dots\dots \tilde{x}_{n+m}{}^I \geq 0 \qquad (8)$$

6.2 Intuitionistic fuzzy optimum feasible solution [22]

Let X is the set of all intuitionistic fuzzy feasible solutions of (6). An intuitionistic fuzzy feasible solution $\tilde{x}^I{}_0 \in X$ is said to be an intuitionistic fuzzy optimum solution to (6), if $\tilde{c}^I\ \tilde{x}^I{}_0 \in \tilde{c}^I\ \tilde{x}^I$ for all $\tilde{x}^I{}_0 \in X$,
where $\tilde{c}^I = (\tilde{c}_1{}^I, \tilde{c}_2{}^I, \dots\dots \tilde{c}_n{}^I)$, and $\tilde{c}^I\tilde{x}^I = \tilde{c}_1{}^I\ \tilde{x}^I{}_1 + \tilde{c}_2{}^I\tilde{x}^I{}_2 + \dots\dots \tilde{c}_n{}^I\tilde{x}^I{}_n$.

7. PROPOSED SCORE FUNCTION AND ACCURACY FUNCTION

$Let\ \ A = \{(a_1, a_2,\ a_3);\ (a_1^{'}, a_2,\ a_3^{'})\}$ be a TIFN, then we define a Score function for membership and non-membership values respectively as

$S(\tilde{A}^{I\alpha}) = \dfrac{a_1 + 2a_2 + a_3}{4}$ and $S(\tilde{B}^{I\beta}) = \dfrac{a^{'}{}_1 + 2a_2 + a^{'}{}_3}{4}$ $Let\ \ A = \{(a_1, a_2,\ a_3);\ (a_1^{'}, a_2,\ a_3^{'})\}$ be a TIFN, then we define

$\tilde{A}^I = \dfrac{(a_1 + 2a_2 + a_3)(a^{'}{}_1 + 2a_2 + a^{'}{}_3)}{8}$, an accuracy function of $\tilde{A}^I$ to defuzzify the given number.

7.1 Ranking using Score function:

Let $A = \{(a_1, a_2,\ a_3);\ (a_1^{'}, a_2,\ a_3^{'})\}\ and\ B = \{(b_1, b_2,\ b_3);\ (b_1^{'}, b_2,\ b_3^{'})\}$ be two TIFNs and $S(\tilde{A}^{I\alpha})$, $S(\tilde{A}^{I\beta})$ and $S(\tilde{B}^{I\alpha})$, $S(\tilde{B}^{I\beta})$ be the scores of respectively $\tilde{A}^I$ and $\tilde{B}^I$.

Then i) if $S(\tilde{A}^{I\alpha}) \leq S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) \leq S(\tilde{B}^{I\beta})$, then $\tilde{A}^I \leq \tilde{B}^I$

ii) if $S(\tilde{A}^{I\alpha}) \geq S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) \geq S(\tilde{B}^{I\beta})$, then $\tilde{A}^I \geq \tilde{B}^I$

iii) if $S(\tilde{A}^{I\alpha}) = S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) = S(\tilde{B}^{I\beta})$, then $\tilde{A}^I = \tilde{B}^I$

8. NUMERICAL ILLUSTRATION

Solve $Max\ \tilde{z}^I{}_1 = \tilde{4}^I\,\tilde{x}^I{}_1 + \tilde{3}^I\,\tilde{x}^I{}_2$

$Max\ \tilde{z}^I{}_2 = \tilde{2}^I\,\tilde{x}^I{}_1 + \tilde{4}^I\,\tilde{x}^I{}_2$

Subject to

$\tilde{4}^I\tilde{x}_1^I + \tilde{3}^I\ \tilde{x}_2^I \leq 1\tilde{2}^I$

$\tilde{1}^I\tilde{x}_1^I + 3\tilde{x}_2^I \leq \tilde{6}^I$

And $\tilde{x}^I{}_1, \tilde{x}^I{}_2 \geq 0$

$\tilde{c}^I{}_1 = \tilde{4}^I = \{\ (\ 3,\ 4,\ 5\ ); (3, 4, 5.1)\}$ $\qquad \tilde{c}^I{}_2 = \tilde{3}^I = \{\ (\ 2.5,\ 3,\ 3.2\ ); (2, 3,\ 3.5)\}$

$\tilde{a}^I{}_{12} = \tilde{3}^I = \{(\ 2.5,\ 3,\ 3.5\ ); (2.4, 3,\ 3.6)\}$ $\qquad \tilde{a}^I{}_{22} = \tilde{3}^I = \{\ (\ 2.8,\ 3,\ 3.2\ ); (2.5, 3,\ 3.2)\}$

$\tilde{a}^I_{11} = \tilde{4}^I = \{\ (\ 3.5,\ 4,\ 4.1\ ); (3, 4, 5)\}$ $\qquad \tilde{a}^I{}_{21} = \tilde{1}^I = \{\ (\ 0.8,\ 1,\ 2\ ); (0.5, 1,\ 2.1)\}$

$\tilde{b}_1^I = 1\tilde{2}^I = \{\ (\ 11,\ 12,\ 13\ ); (11, 12, 14)\}$ $\qquad \tilde{b}^I{}_2 = \tilde{6}^I = \{\ (\ 5.5, 6, 7.5); (5, 6, 8.1)\}$

Solution: Now, we take the first objective function to solve rewriting the problem in standard form:

$$Max\ \tilde{z}^I{}_1 = \tilde{4}^I\tilde{x}^I{}_1 + \tilde{3}^I\tilde{x}^I{}_2$$

Subject to

$$\tilde{4}^I\tilde{x}_1^I + \tilde{3}^I\ \tilde{x}_2^I\ + \tilde{S}^I{}_1 = \tilde{12}^I$$

$$\tilde{1}^I\tilde{x}_1^I + 3\tilde{x}_2^I + \tilde{S}^I{}_2 = \tilde{6}^I$$

and

$$\tilde{x}^I{}_1, \tilde{x}^I{}_2, \tilde{S}^I{}_1, \tilde{S}^I{}_2 \geq 0$$

Here the co-efficient of $\tilde{S}^I{}_1, \tilde{S}^I{}_2$ are given by $\tilde{1}^I = \{\ (1, 1, 1\ ); (1,1,1)\}$ and $\tilde{0}^I = \{\ (\ 0,\ 0,\ 0\ ); (0,\ 0,\ 0)\}$

Initial Iteration: Basic variables are $\tilde{S}^I{}_1 = 12, \tilde{S}^I{}_2 = 6$

			4	3	0	0	
C_B	BV	$\tilde{b}^I$	$\tilde{x}^I{}_1$	$\tilde{x}^I{}_2$	$\tilde{S}^I{}_1$	$\tilde{S}^I{}_2$	Ratio
0	$\tilde{S}^I{}_1$	12	4	3	1	0	3
0	$\tilde{S}^I{}_2$	6	1	3	0	1	6
	$\tilde{Z}^I{}_j$	0	0	0	0	0	
	$\tilde{C}_j^I - Z_j^I$	0	4	3	0	0	

Since all $\tilde{C}_j^I - Z_j^I \geq 0$, the solution is not optimal. $\tilde{x}^I{}_1$ is the entering variable, since the most positive value corresponds to the $\tilde{x}^I{}_1$ column. Then the ratio is calculated. Using the division procedure and Scoring function as defined in the sections (3 & 4) of this paper, we get the following results:

i) $\dfrac{\tilde{12}^I}{\tilde{4}^I}$ {(2.68, 3, 3.71); (2.2, 3, 4.67) } = $\tilde{3}^I$

ii) $\dfrac{\tilde{6}^I}{\tilde{1}^I}$ { (2.75, 6, 9.37); (2.38, 6, 16.2) } = $\tilde{1}^I$

iii) Score function $S(\tilde{3}^{I\alpha}) =$ 3.0975 & $S(\tilde{3}^{I\beta}) =$ 3.2175

iv) Score function $S(\tilde{6}^{I\alpha}) =$ 6.03125 & $S(\tilde{6}^{I\beta}) =$ 7.645

Since $S(\tilde{3}^{I\alpha}) \leq S(\tilde{6}^{I\alpha})$ and $S(\tilde{3}^{I\beta}) \leq S(\tilde{6}^{I\beta})$. We get $\tilde{3}^I \leq \tilde{6}^I$. $\tilde{S}^I{}_1$ is the leaving variable.

First Iteration: Basic variables are $\tilde{x}^I{}_1 = \tilde{3}^I$, $\tilde{S}^I{}_1 = \tilde{3}^I$

			4	3	0	0	
C_B	BV	$\tilde{b}^I$	$\tilde{x}^I{}_1$	$\tilde{x}^I{}_2$	$\tilde{S}^I{}_1$	$\tilde{S}^I{}_2$	Ratio
4	$\tilde{x}^I{}_1$	3	1	0.75	0.25	0	
0	$\tilde{S}^I{}_2$	3	0	2.25	-0.25	1	
	$\tilde{Z}^I{}_j$	12	4	3	1	0	
	$\tilde{C}_j^I - Z_j^I$		0	0	-1	0	

Where the Triangular intuitionistic representation for each element based on arithmetic operations is listed below:

$\tilde{c}^I{}_1 = \tilde{4}^I = \{\ (\ 3,\ 4,\ 5\ ); (3,4,5.1)\}$

$\tilde{c}^I{}_2 = \tilde{3}^I = \{\ (\ 2.5,\ 3,\ 3.2\ ); (2,3,\ 3.5)\}$

$\tilde{a}^I{}_{11} = \tilde{1}^I = \{\ (\ .85,\ 1,\ 1.17\ ); (.6,1,\ 1.67)\}$

$\tilde{a}^I{}_{12} = .7\tilde{5}^I = \{\ (\ .61,\ .75,\ 1\ ); (.46,75,\ 1.2)\}$

$\tilde{a}^I{}_{13} = 0.2\tilde{5}^I = \{(0.24,0.25,0.29); (0.2, 0.25,0.67)\}$ $\tilde{a}^I{}_{14} = \tilde{0}^I = \{(0,0,0); (0,0,0)\}$

$\tilde{a}^I{}_{22} = \overline{.2.25}^I = \{(1.8, 2.25, 2.59); (1.3, 2.25, 2.74)\}$ $\tilde{a}^I{}_{21} = \{(0,0,0); (0,0,0)\} = \tilde{0}^I$

$\tilde{a}^I{}_{23} = -0.2\tilde{5}^I = -\{(0.24, 0.25, 0.29); (0.2, 0.25,0.67)\}$ $\tilde{a}^I{}_{24} = \tilde{1}^I = \{(1, 1, 1); (1,1, 1)\}$

$\tilde{b}^I{}_1 = \tilde{3}^I = \{(2.68, 3, 3.71); (2.2, 3, 4.67)\}$ $\tilde{b}^I{}_2 = \tilde{3}^I = \{(1.79, 3, 4.82); (0.33, 3, 5.9)\}$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\tilde{Z}^I$

$\tilde{a}_{31}{}^I = \tilde{4}^I = \{(2.4, 4, 6.02); (1.4, 4, 8.181)\}$ $\tilde{a}_{32}{}^I = \tilde{3}^I = \{(2, 3, 6); (1, 3, 7.3)\}$

$\tilde{a}^I{}_{33} = \tilde{1}^I = \{(.76, 1, 1.25); (.7, 1, 4)\}$ $\tilde{a}^I{}_{34} = \{(0,0,0); (0,0,0)\} = \tilde{0}^I$

$\tilde{Z}^I{}_j = \{(8.32, 12, 19.25); (6.8, 12, 25.4)\}$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\tilde{C}_j^I - Z_j^I$

$\tilde{a}_{41}{}^I = \tilde{0}^I = \{(-3.02, 0, 2.6); (-6.87, 0, 8, 3.7)\}$ $\tilde{a}^I{}_{42} = \{(0,0,0); (0,0,0)\} = \tilde{0}^I$

$\tilde{a}^I{}_{43} = -\tilde{1}^I = \{(.85, 1, 1.17); (.8, 1, 1.67)\}$ $\tilde{a}^I{}_{44} = \{(0,0,0); (0,0,0)\} = \tilde{0}^I$

Since all $\tilde{C}_j^I - Z_j^I \leq 0$, the elements in the row are less than or equal to zero, the solution is optimal

i.e. $Max\,\tilde{z}^I = 1\tilde{2}^I$ when $\tilde{x}^I{}_1 = 3$, $\tilde{S}^I{}_2 = 3$, $\tilde{x}^I{}_2 = 0$, $\tilde{S}^I{}_1 = 0$

Next, we take the first objective function to solve,

$$Max\ \tilde{z}^I{}_2 = \tilde{2}^I \tilde{x}^I{}_1 + \tilde{4}^I \tilde{x}^I{}_2$$

Subject to

$$\tilde{4}^I \tilde{x}_1^I + \tilde{3}^I \tilde{x}_2^I \leq 1\tilde{2}^I$$
$$\tilde{1}^I \tilde{x}_1^I + 3\tilde{x}_2^I \leq \tilde{6}^I$$

And $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$

$\tilde{c}^I{}_1 = \tilde{2}^I = \{(1.8, 2, 3); (1.5, 2, 3.1)\}$ $\tilde{c}^I{}_2 = \tilde{4}^I = \{(3.1, 4, 5); (3, 4, 5.4)\}$

$\tilde{a}^I{}_{11} = \tilde{4}^I = \{(3.5, 4, 4.1); (3, 4, 5)\}$ $\tilde{a}^I{}_{12} = \tilde{3}^I = \{(2.5, 3, 3.5); (2.4, 3, 3.6)\}$

$\tilde{a}^I{}_{22} = \tilde{3}^I = \{(2.8, 3, 3.2); (2.5, 3, 3.2)\}$ $\tilde{a}^I{}_{21} = \tilde{1}^I = \{(0.8, 1, 2); (0.5, 1, 2.1)\}$

$\tilde{b}_1^I = 1\tilde{2}^I = \{(11, 12, 13); (11, 12, 14)\}$ $\tilde{b}^I{}_2 = \tilde{6}^I = \{(5.5, 6, 7.5); (5, 6, 8.1)\}$

Rewriting the problem in standard form:

$$Max\ \tilde{z}^I{}_2 = \tilde{2}^I \tilde{x}^I{}_1 + \tilde{4}^I \tilde{x}^I{}_2$$

Subject to

$$\tilde{4}^I \tilde{x}_1^I + \tilde{3}^I \tilde{x}_2^I + \tilde{S}^I{}_1 = 1\tilde{2}^I$$
$$\tilde{1}^I \tilde{x}_1^I + 3\tilde{x}_2^I + \tilde{S}^I{}_2 = \tilde{6}^I$$

and

$$\tilde{x}^I{}_1, \tilde{x}^I{}_2, \tilde{S}^I{}_1, \tilde{S}^I{}_2 \geq 0.$$

Here the co-efficient of $\tilde{S}^I{}_1, \tilde{S}^I{}_2$ are given by $\tilde{1}^I = \{(1, 1, 1); (1,1,1)\}$ and $\tilde{0}^I = \{(0, 0, 0); (0, 0, 0)\}$

Initial Iteration: Basic variables are $\tilde{S}^{I}{}_{1} = 12,\ \tilde{S}^{I}{}_{2} = 6$

			2	4	0	0	
C_B	BV	$\tilde{b}^{I}$	$\tilde{x}^{I}{}_{1}$	$\tilde{x}^{I}{}_{2}$	$\tilde{S}^{I}{}_{1}$	$\tilde{S}^{I}{}_{2}$	Ratio
0	$\tilde{S}^{I}{}_{1}$	12	4	3	1	0	4
0	$\tilde{S}^{I}{}_{2}$	6	1	3	0	1	2
	$\tilde{Z}^{I}{}_{j}$		0	0	0	0	
	$\tilde{C}_j^I - Z_j^I$		2	4	0	0	

Since all $\tilde{C}_j^I - Z_j^I \geq 0$, the solution is not optimal. $\tilde{x}^{I}{}_{2}$ is the entering variable, since the most positive value corresponds to the $\tilde{x}^{I}{}_{2}$ column. Then the ratio is calculated. Using the division procedure and scoring function as defined in the sections (3 & 4) of this paper, we get the following results:

i) $\dfrac{1\tilde{2}^{I}}{\tilde{3}^{I}}$ = {(3.68, 4, 4.71); (3.2,4, 5.67) } = $\tilde{4}^{I}$

ii) $\dfrac{\tilde{6}^{I}}{\tilde{3}^{I}}$ = {(1.5, 2 , 2.6); (1.2, 2, 3.5) } = $\tilde{2}^{I}$

iii) Score function $S(\tilde{4}^{I\alpha}) =$ 4.0975 & $S(\tilde{4}^{I\beta}) =$ 4.2

iv) Score function $S(\tilde{2}^{I\alpha}) =$ 2.025 & $S(\tilde{2}^{I\beta}) =$ 2.175

Since $S(\tilde{2}^{I\alpha}) \leq S(\tilde{4}^{I\alpha})$ and $S(\tilde{2}^{I\beta}) \leq S(\tilde{4}^{I\beta})$. We get $\tilde{2}^{I} \leq \tilde{4}^{I}$. $\tilde{S}^{I}{}_{2}$ is the leaving variable.

First Iteration: Basic variables are $\tilde{x}^{I}{}_{2} = \tilde{2}^{I}$, $\tilde{S}^{I}{}_{1} = \tilde{6}^{I}$

			2	4	0	0	
C_B	BV	$\tilde{b}^{I}$	$\tilde{x}^{I}{}_{1}$	$\tilde{x}^{I}{}_{2}$	$\tilde{S}^{I}{}_{1}$	$\tilde{S}^{I}{}_{2}$	Ratio
0	$\tilde{S}^{I}{}_{1}$	6	3.1	0	1	2	1.9
4	$\tilde{x}^{I}{}_{2}$	2	0.3	1	0	0.3	6.66
	$\tilde{Z}^{I}{}_{j}$	8	1.2	4	0	1.2	
	$\tilde{C}_j^I - Z_j^I$		0.8	0	1	1.2	

Since all $\tilde{C}_j^I - Z_j^I \geq 0$, the solution is not optimal. $\tilde{x}^{I}{}_{1}$ is the entering variable, since the most positive value corresponds to the $\tilde{x}^{I}{}_{1}$ column. Then the ratio is calculated. Using the division procedure and scoring function as defined in the sections (3 & 4) of this paper, we get the following results:

i) $\dfrac{\tilde{6}^{I}}{3.\tilde{1}^{I}}$ = {(1.1, 1.9, 2.71); (1, 1.9, 2.9)} = $1.\tilde{9}^{I}$

ii) $\dfrac{\tilde{2}^{I}}{0.\tilde{3}^{I}}$ = {(2.95, 6.6, 9.975); (2.28, 6.6, 16)} = $6.6\tilde{6}^{I}$

iii) Score function $S(1.\tilde{9}^{I\alpha}) =$ 1.9025 & $S(1.\tilde{9}^{I\beta}) =$ 1.925

iv) Score function $S(6.6\tilde{6}^{I\alpha}) =$ 6.53125 & $S(6.6\tilde{6}^{I\beta}) =$ 7. 87

Since $S(1.\tilde{9}^{I\alpha}) \le S(6.6\tilde{6}^{I\alpha})$ and $S(1.\tilde{9}^{I\beta}) \le S(6.6\tilde{6}^{I\beta})$. We get $1.\tilde{9}^{I} \le 6.6\tilde{6}^{I}$. $\tilde{S}_1^I$ is the leaving variable.

Second Iteration: Basic variables are $\tilde{x}^I{}_1 = 1.\tilde{9}^I$, $\tilde{x}^I{}_2 = 6.6\tilde{6}^I$

			2	4	0	0	
C_B	BV	$\tilde{b}^I$	$\tilde{x}^I{}_1$	$\tilde{x}^I{}_2$	$\tilde{S}^I{}_1$	$\tilde{S}^I{}_2$	Ratio
2	$\tilde{x}^I{}_1$	1.9	1	0	0.3	0.6	
4	$\tilde{x}^I{}_2$	1.43	0	1	-0.09	0.12	
	$\tilde{Z}^I{}_j$	9.52	2	4	.24	1.68	
	$\tilde{C}_j^I - Z_j^I$		0	0	.24	1.68	

Where the Triangular intuitionistic representation for each element based on arithmetic operations is listed below:

$\tilde{c}^I{}_1 = \tilde{2}^I = \{(1.8, 2, 3); (1.5, 2, 3.1)\}$ $\tilde{c}^I{}_2 = \tilde{4}^I = \{(3.1, 4, 5); (3, 4, 5.4)\}$

$\tilde{a}^I{}_{11} = \tilde{1}^I = \{(.85, 1, 1.17); (.6, 1, 1.67)\}$ $\tilde{a}^I{}_{13} = 0.\tilde{3}^I = \{(0.28, 0.3, 0.34); (0.25, 0.3, 0.38)\}$

$\tilde{a}^I{}_{21} = \tilde{0}^I = \{(0,0,0); (0,0,0)\}$ $\tilde{a}^I{}_{14} = 0.\tilde{6}^I = \{(.57, 0.6, 0.62); (0.55, 0.6, 0.68)\}$

$\tilde{a}^I{}_{22} = \tilde{1}^I = \{(.85, 1, 1.17); (.6, 1, 1.67)\}$ $\tilde{a}_{23} = -0.0\tilde{9}^I = -\{(0.01, 0.09, -0.11); (0.2, 0.25, 0.67)\}$

$\tilde{a}^I{}_{12} = .\tilde{0}^I = \{(0,0,0); (0,0,0)\}$ $\tilde{a}^I{}_{24} = 0.1\tilde{2}^I = \{(0.09, 0.12, 0.19); (0.05, 0.12, 0.20)\}$

$\tilde{b}^I{}_2 = 1.4\tilde{3}^I = \{(1.1, 1.43, 1.80); (1.3, 1.43, 2.1)\}$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\tilde{Z}^I$

$\tilde{a}_{31}{}^I = \tilde{2}^I = \{(1.8, 2, 3); (1.5, 2, 3.1)\}$ $\tilde{a}_{32}{}^I = \tilde{4}^I = \{(3.1, 4, 5); (3, 4, 5.4)\}$

$\tilde{a}^I{}_{33} = 0.2\tilde{4}^I = \{(.23, 0.24, 0.29); (.2, 0.25, 0.69)\}$

$\tilde{a}^I{}_{34} = \{(1.1, 1.68, 2); (1.5, 1.68, 1.90)\} = 1.6\tilde{8}^I$

$\tilde{Z}^I{}_j = 9.5\tilde{2}^I = \{(7.4, 9.52, 11.25); (8.4, 9.52, 16.1)\}$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\tilde{C}_j^I - Z_j^I$

$\tilde{a}_{41}{}^I = \tilde{0}^I = \{(-3.02, 0, 2.6); (-6.87, 0, 8, 3.7)\}$ $\tilde{a}^I{}_{42} = \tilde{0}^I = (0,0,0); (0,0,0)\}$

$\tilde{a}^I{}_{43} = 0.2\tilde{4}^I = \{(.23, 0.24, 0.29); (.2, 0.25, 0.69)\}$

$\tilde{a}^I{}_{44} = 1.6\tilde{8}^I = \{(1.1, 1.68, 2); (1.5, 1.68, 1.90)\}$

Since all $\tilde{C}_j^I - Z_j^I \ge 0$, the elements in the row are less than or equal to zero, the solution is optimal

i.e. $Max\,\tilde{z}^I = 9.5\tilde{2}^I$ when $\tilde{x}^I{}_1 = 1.9$, $\tilde{S}^I{}_2 = 0$, $\tilde{x}^I{}_2 = 1.43$, $\tilde{S}^I{}_1 = 0$.

9. CONCLUSION

In this paper, we considered fuzzy multi-objective linear programming problem. We have defined a more general division of TIFN. Thereafter, a ranking function based on Score function has been proposed. We conclude that the above refinement gives the better solution. It is also helps to defuzzify TIFNs. The solution methodology is illustrated through an example. In future it is proposed to obtain a better method to defuzzify TIFNs.

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