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A NEW CLASS OF GENERALIZED CLOSED SETS IN KHALIMSKY TOPOLOGY

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ABSTRACT

In the present paper we introduce the concept of rg-closed sets in digital space. We analyze the characteristics of rg-closed sets and their relationships in the digital plane. Further we develop the notions to derive some properties of singletons in khalimsky topology or digital topology.

INTRODUCTION:

Digital topology is a term that has arisen for the study of geometric and topological properties of digital images. In our increasingly digital technology, such questions arise in a wide range of diverse applications, including computer graphics, computer tomography, pattern analysis and robotic design, to mention few of the areas of current interest.

The study of images begins with Jordan Curve Theorem which states that simple closed curve separates the real plane RXR in to exactly two connected components. We want to analog for the plane ZxZ. For this we require connected topology on Z regarded as an ordered set. Hence we are primarily interested in T_0 spaces defined on the intervals of Z. Khalimsky et al.[5] utilized a connected topology on a finite ordered set to computer graphics. Khalimsky introduced a topology on Z called digital topology generated by the triplets {2n-1, 2n, 2n+1} as sub base. As this topology was introduced by khalimsky it is also called khalimsky topology.

1. PRELIMINARIES:

In this paper we study the concept of rg-closed sets,rg-homeomorphism in $rgT_{1/2}$ space, rgc-homeomorphism in the digital line. Throughout this paper (Z,k) represents a digital topological space, Where Z is the set of all integers and K denotes the khalimsky topology.

Definition: 1.1
A subset A of (X,τ) is called regular generalized closed (briefly rg-closed) if $A \subseteq U$, where U is regular open then $cl(A) \subseteq U$.

Definition: 1.2
A function $f:(X,\tau)\to(Y,\sigma)$ is called,
(1) rg-continuous if $f^{-1}(V)$ is rg-closed in (X,τ) for every closed set V of (Y,σ),
(2) rg-irresolute if $f^{-1}(V)$ is rg-closed in (X,τ) for every rg-closed set V of (Y,σ),
(3) rg-homeomorphism if f is bijective, f and f^{-1} are both rg-continuous
(4) rgc-homeomorphism if f is bijective, f and f^{-1} are both rg-irresolute maps.

Note: 1.3 For a space (X,τ) and a subset H of X,
(1) $rgch_{(0)}$ (X,X/H) is a group,
(2) $rgch_{(0)}$ (X,X/H) is a subgroup of rgch(X,H)
(3) $h_{(0)}$ (X,X/H) is a subgroup of the group $rgch_{(0)}$ (X,X/H)

Definition: 1.4 Let (Z, k) be the digital line,
(1) If m is even, then cl(m) = {m} and int (m) = ϕ
(2) If m is odd, then cl(m) = {m-1,m,m+1} and int (m) = m.

Definition: 1.5 The digital line (Z, k) contains both odd and even points. In general $Z = Z_E \cup Z_O$,
Z_E = { x/ x is even in Z}, Z_O = {x / x is odd in Z}

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Definition: 1.6 Let U(x) be the neighbourhood of the point x. If x is odd then U(x) = x. If x is even then U(x) = {x-1, x, x+1}.

Definition: 1.7 In the digital line (Z, k), if the corner points are odd then it is called open. If the corner points are even then it is called closed.

2. rg-CLOSED SETS IN DIGITAL LINE:

In this section we derive some basic properties of rg-closed sets in digital line so called khalimsky topology.

Theorem: 2.1 Every closed set in (Z, k) is rg-closed in (Z, k).

Proof: Suppose U is regular open of the form,U = { 2n , 2n $\pm$1 , 2n$\pm$2,……… 2n$\pm$q ,2n$\pm$q$\pm$1 } for any integer n and q is an even integer .

Case (i) A contains all even points ($Z_E \subseteq$ A) ,where A = { 2n , 2n $\pm$1 , 2n$\pm$2,……… 2n$\pm$q } $\subseteq$ U . Then cl(A) $\subseteq$ U. Therefore A is rg-closed.

Case (ii) A contains both odd and even points ($Z_O \cup Z_E$) $\subseteq$ A, Assume that A $\subseteq$ U where U is regular open, then cl(A) $\subseteq$ U. Therefore, A is rg closed in (Z,k).

Remark: 2.2 Converse of the above need not be true as seen by the following example.

Let A = {5, 6} and U = {3, 4, 5, 6, 7}. Assume that A $\subseteq$ U and U is regular open in (Z, k).Then cl(A) = {4,5,6} $\subseteq$ U

Therefore A is rg-closed set in (Z, k).But cl(A) $\neq$ A .Hence A is not closed in (Z, k).

Theorem: 2.3 If A set A is open in (Z, k), then it is rg-open in (Z, k).

Proof:
Case (i): Suppose A contains all odd points ($Z_O \subseteq$ A). Let A = {2n, 2n $\pm$1, 2n$\pm$2 … 2n$\pm$q} for any n and q is an odd integer and U = ϕ. Assume that U $\subseteq$ A and U is rg-closed in (Z, k). Then U $\subseteq$ A = int A. Therefore A is regular open in (Z, k).

Case (ii): A contains both odd and even points. [$Z_E \cup Z_O \subseteq$ A] . Let A = (2n, 2n $\pm$1 , 2n$\pm$2,……… 2n$\pm$q) and U = (2n, 2n $\pm$1 , 2n$\pm$2,……… 2n$\pm$(q-1)) for any n and q is an odd integer .Assume that U $\subseteq$ A and U is regular closed. U $\subseteq$ int (A) since A is open. Therefore A is rg-open in (Z, k).

Remark: 2.4 By the example given below it can be shown that the converse of the above need not be true.

Let A = {11, 12, 13, 14} and U= {12}.Assume that U $\subseteq$ A and rg-closed in (Z, k). Then {12} $\subseteq$ int A = {11, 12, 13}.Therefore A is rg-open set in (Z, k) .But int (A) $\neq$ A.Hence A is not open in (Z, k).

Theorem: 2.5 A non empty proper subset A of (Z, k) is rg-closed if its complement Z~A contains only odd integers.(i.e., A contains all even integers).

Proof: Let $Z_E \subseteq$ A. If U is a regular open super set of A, then $Z_E \subseteq$ A $\subseteq$ U. Thus $Z_E \subseteq$ U. Since U is regular open subset of the digital topology, then for every e $\in$U, we have {e-1, e+1} $\subseteq$ U. Thus U =Z .Thus in Z the closure of any set A exists. Trivially A is a rg-closed set in (Z, k) .

Remark: 2.6 A nonempty proper subset A of (Z, k) is rg-closed it does not imply that its complement Z~A contains only odd integers. We can easily prove this remark by the following example.

Example: 2.7 Let A = {2,5} and U= {1,2,3,4,5,6,7,8,9} where U is regular open. Also the cl(A) $\subseteq$ U implies that A is rg-open but its complement does not contain all even points.

Note: 2.8 A space is said to $rgT_{1/2}$ space if every rg-closed set is closed in (Z, k)

Theorem: 2.9 A bijective translation map $f_a : (Z_1 , k_1) \to (Z_2 , k_2)$ is defined by f_a (x) = x+a for any x $\in$ Z where a is an integer and Z_1,Z_2 are $rgT_{1/2}$ spaces.

(1) The function f_a is rg-continuous iff the integer a is even .
(2) f_a is a rg-homeomorphism iff the integer a is even .

Proof: (1) Sufficiency Part: Let V = (2n+q+a) be a closed set in (Z_2, k_2) .Since a is an even integer for any n and q where q is also an even integer .Then f_a^{-1} (V) = f^{-1} (2n+q+a) = {2n+q} is a rg-closed set in (Z_1 , k_1). Then fa is rg-continuous.

Necessity Part: Since f_a is rg-continuous, f_a^{-1} (V) = {2n+q} is a rg-closed set in (Z_1 ,k_1).{2n+q} is in (Z_1 , k_1) . Since q is even, {2n+q} is closed. Thus {2n+q} is rg-closed.

Therefore f_a [f_a^{-1}] (V) =V = {2n+q+a} it should be a closed set .Thus a should be an even number .

(2) Sufficiency Part: This part contains two cases. In the first case we prove f_a is rg-continuous and in the second case f_a^{-1} is rg-continuous .

Case I: The same as the sufficiency of (1).

Case II: Assume f_a^{-1} : (Z_2 ,k_2) $\rightarrow$ (Z_1 ,k_1). Let V = {2n+q} be a closed set in (Z_1 ,k_1) for any n and q is an even integer. Then [f_a^{-1} $]^{-1}$ (V) = f_a (V) = {2n+q +a } is rg-closed in (Z_2 ,k_2) ,since a is even integer.

Therefore f_a^{-1} is a rg-continuous .Therefore f_a is a rg-homeomorphism.

Necessity Part: Similar Proof for both the cases f_a and f_a^{-1} are rg-continuous .

Note: 2.10 A bijective map f: $(Z_1,k_1) \rightarrow (Z_2,k_2)$ is a homeomorphism then it is rg-homeomorphism.

Remark: 2.11
(1) A bijective map f_a : (Z_1 ,k_1) $\rightarrow$ (Z_2 ,k_2) is defined by f_a (x) = x+a for any $x \in Z$ where a is an integer.
(i) The function f_a is contra rg continuous iff the integer a is odd .
(ii) f_a is a contra rg-homeomorphism iff the integer a is odd .
(2) If we are taking Z_1 and Z_2 as $rgT_{1/2}$ spaces then the continuity will turn as contra rg-irresolute function.

Theorem: 2.12 Let H be a nonempty subset of the digital line (Z, k) and suppose that H = {o_1,o_2 , ………..o_{n_o} ,e_1,e_2,e_3, ….e_{no}} where o_i and e_j are odd and even for i = 1,2, … n_o and j =1,2, …m_o. A function f : (Z ,k) $\rightarrow$ (Z ,k) is bijective and f(x) = x for any $x \in Z \sim H$. H is a non empty finite set then $f \in rgch_{(o)}$ (Z , Z ~ H) iff $f^{-1}(o_i)$ and $f(o_i) \in \{o_1, o_2, o_3, \ldots o_{n_0}\}$ hold for each i = 1,2,3…. n_o

Proof: Necessity Part: Let V = Z ~ {o_1,o_2 , …..o_{n_o} } be a rg-closed set in (Z,k) .Since f is rgc homeomorphism then f^{-1} (V) is rg-closed set in (Z, k)

Hence we have f^{-1}(V) = Z ~ f^{-1}{o_1,o_2 , …..o_{n_o} } and f^{-1}{o_1,o_2,….o_{n_o} } = {o_1,o_2 , ………..o_{n_o} }(then only f^{-1} (V) will be rg-closed set) Therefore f^{-1} (o_i) $\in$ {o_1,o_2 , ………..o_{n_o} } hold for each i = 1, 2,….n_0 .

Let V = Z ~ {o_1, o_2, ...o_{n_0}} be a rg-closed set in (Z, k). Since f is rg-homeomorphism then $[f^{-1}]^{-1}$ (V) is rg-closed set in (Z, k). Hence we have f(V) = Z ~ $[f^{-1}]^{-1}$ {o_1 , o_2, ...o_{n_0}} and f {o_1 , o_2, ...o_{n_0}} = {o_1 , o_2, ...o_{n_0}}. Therefore f(o_i) $\in$ {o_1,o_2 , ………..o_{n_o} } hold for each i = 1,2,….n_0

Sufficiency Part: Let V be a rg-closed set in (Z, k) .First we prove that f^{-1} (V) is rg-closed set. That is $Z_E \subseteq f^{-1}(V)$. Let $x \notin f^{-1}$ (V) be an integer .Since $x \notin f^{-1}$ (Z ~ V) then there exists a y$\in$(Z ~ V) such that f(x) = y .Since $Z_E \subseteq V$. we have $y \in Z_o$.

Case I: y $\in$ {o_1,o_2 , ….o_{n_o} } say y = o_i .Then $O_i \in$ Z ~ V and so x = f^{-1}(y) f^{-1}(O_i)$\in${o_1,o_2,…..o_{n_o} }$\subseteq Z_o$ by the assumption.

Case II: y $\notin$ {o_1,o_2 , …o_{n_o} } that is y $\in$ Z ~ H. It follows from the assumption that x = f^{-1}(y) = y and so x $\notin Z_E$. In both the cases we assume that for every x$\notin$ f^{-1}(V) and x $\notin Z_E$ Thus $Z_E \subseteq f^{-1}$(V). Therefore f^{-1} (V) is a rg-closed set.

Hence f is rg-irresolute. Similarly we can prove that f^{-1} is also irresolute. Therefore f is rgc homeomorphism.
$f \in rgch_{(o)}$ (Z , Z ~ H)

Theorem: 2.13 Let H be a non empty finite subset of Z containing exactly o (H) odd integers and e (H) even integers, where o(H) and e (H) are some positive integers .Then $rgch_{(o)}$ (Z , Z ~ H) is isomorphic to the product group of the o(H) – symmetric group $S_{o(H)}$ and e(H) –symmetric group $S_{e(H)}$.
(i.e.,) $rgch_{(o)}$ (Z , Z / H) $\equiv S_{o(H)} \times S_{e(H)}$

Proof: Let Z_E be the set of all even integers and Z_o be the set of all odd integers**.**
Let $H = H_o \cup H_E$ where $H_o = H \cap Z_o$ say $\{ x_1 , x_2 ,\ldots\ldots x_{o(H)} \}$ and $H_E = H \cap Z_E$, say $\{ y_1, y_2 , \ldots y_{e(H)} \}$.
Let f ∈ $rgch_{(o)}$ (Z , Z /H) , then by definition f is bijective and f(x) = x for any x ∈ Z ~ H . By the theorem 1.5, f ∈ $rgch_{(o)}$ (Z , Z ~ H) iff f is bijection satisfying the following,

(a) $f (H_o) = H_o$

(b) $f (H_E) = H_E$ and

(c) f(x) = x for any x ∈ Z ~ H. First, there exists o (H) ! bijections from H_o onto itself satisfying (a), say $f_1, f_2,\ldots .f_{o(H)!}$ we note that

(d) $\{f_i$ / i=1, 2,… .o(H)!} = S (H_o) is such that set of all bijections from H_o onto itself. Second, we can construct e(H) ! bijections from H_E onto itself satisfying the property (b), say $g_1,g_2,\ldots\ldots g_{e(H)\,!}$, we note that,

(e) { g_i / i = 1,2,3,…e(H)! } = S(H_E) where S(H_E) is set of all bijections onto itself. Then for each i and j with 1 ≤ i ≤ o(H)! , 1 ≤ i ≤ e(H)!, we can define $S_{ij} : Z \to Z$ by $S_{ij} (x) = f_i(x)$ for any x ∈ H_o , $S_{ij}(x) = g_j(x)$ for any x ∈ H_E, also $S_{ij}(x) = x$ for any x∈ Z ~ H. Since these e(H) ! × o(H)! bijections S_{ij} satisfying (a) – (c) conditions we have $rgch_{(o)}$ (Z , Z ~ H) = { S_{ij} / 1 ≤ i ≤ o(H)! , 1 ≤ i ≤ e(H)! }

We note that,

(f) $S_{ij} / H_o = f_i \in S (H_o)$

(g) $S_{ij} / H_E = f_i \in S (H_E)$ and

(h) if $S_{cd} = S_{ab} \circ S_{ij} : H \to H$ then

$$S_{cd} \setminus H_o = (S_{ab} \setminus H_o) \circ (S_{ij}\setminus H_o) = f_a \circ f_i : H_o \to H_o \quad \text{and}$$
$$S_{cd} \setminus H_E = (S_{ab} \setminus H_E) \circ (S_{ij}\setminus H_E) = g_b \circ g_j : H_E \to H_E .$$

Let f : $rgch_{(o)}$ (Z , Z ~ H) → S(H_o) ×S(H_E) Be a function defined as F (S_{ij}) = ($S_{ij} \setminus H_o$, $S_{ij} \setminus H_E$) for any S_{ij} ∈ $rgch_{(o)}$ (Z , Z ~ H) where S(H_o) ×S(H_E) is the direct product group of S(H_o) and S(H_E) . We claim that F is an homomorphism. Let S_{ij} and S_{ab} ∈ $rgch_{(o)}$ (Z , Z ~ H) and $\mu(S_{ij} , S_{ab}) = S_{ab} \circ S_{ij}$

Then by (f) – (h),
$$\begin{aligned} F (\mu(S_{ij} , S_{ab})) &= F (S_{ab} \circ S_{ij}) \\ &= ((S_{ab} \circ S_{ij}) \setminus H_o , (S_{ab} \circ S_{ij})\setminus H_E) \\ &= (f_a \circ f_i , g_b \circ g_j) \\ &= (f_i \circ f_a , g_i \circ g_b) \\ &= (f_i , g_i) . (f_a , g_b) \\ &= F (S_{ij}).F (S_{ab}) \end{aligned}$$
Where . is the product in S(H_o) × S(H_E) .By (f) and (g) , $f_i = f_a$ and $g_i = g_b$ that is $S_{ij} = S_{ab}$ Thus F is injective .

To prove the surjectivity of F , let (f, g) ∈ S(H_o) ×S(H_E) . Then by (d) and (e) assumptions there exists bijections,
$$\begin{aligned} F (S_{ib}) &= (S_{ib} / H_o , S_{ib} /H_E) \\ &= (f_i , g_b) = (f ,g) \end{aligned}$$

Thus F is onto. Therefore F is an isomorphism. Since S (H_o) (respectively S(H_E)) is isomorphic to $S_{o(H)}$(respectively $S_{e(H)}$), the group $rgch_{(o)}$ (Z , Z/ H) is isomorphic to the direct product group $S_{o(H)} \times S_{e(H)}$.

3. In this section we deal some results about the singletons in digital line. Here we introduce the ideas of rg-kernal, rg-interior and rg-closure, also we move on to some more results about these concepts.

Theorem: 3.1 Let A be a subset of (Z,k) and x be a point of (Z, k) .If A is rg-closed set of (Z , k) and x ∈ A_o then cl ({x}) –{x} ⊂ A and hence cl({x}) ⊂ A in (Z ,k) .

Proof: Since $x \in A_o$, x = (2n+1) for any $n \in Z$. Then cl ({x}) = {2n, 2n+1, 2n+2}

Therefore cl ({x}) – {x}={2n ,2n+1,2n+2}- {2n+1}={2n,2n+2}.It is noted that {2n} and {2n+2}are rg closed points of (Z, k). Suppose that $\{2n\} \notin A$ or $\{2n+2\} \notin A$, Let $2n = x^-$ and $2n+2 = x^+$. If $x^+ \notin A$ than $x^+ \in cl\{x\} \subset cl(A)$ and so $x \in cl(A) - A$. Then cl (A) – A contains rg-closed point $\{x^+\}$. This is a contradiction to the fact that for a topological space (X,τ), A subset A of X is rg-closed iff cl(A) –A does not contain nonempty rg-closed subset of (X,τ). Similarly we can also prove the result for x. Therefore $x^+ \in A$ and $x^- \in A$ and hence $cl(\{x\}) - \{x\} \subset A$ and we have
$cl(\{x\}) \subset A$ because $x \in A_0 \subset A$.

Theorem: 3.2 Let B be a non empty subset of (Z,k). If $B_E = \phi$, then B is rg-open set of (Z,k).

Proof: Let F be a regular closed set such that $F \subset B$. since $B_E = \phi$, we have obviously that $B = B_0$ and so $F \subset B_0$. It is obtained that $F = \phi$, because F is regular closed and $F \subset B$, $F = \phi \subset$ int (B). Therefore B is rg-open set of (Z, k).

Theorem: 3.3 Let B be a non-empty subset of (Z,k). For a subset B such that $B_E = \phi$, if a subset B is a rg-open set of (Z, k) then $(\cup(\{x\}))_0 \subset B$ holds for Each point $x \in B_E$.

Proof: Let $x \in B_E$. Since{x}is closed, then{x}is a regular closed set and$\{x\} \subset B$. By assumption $\{x\} \subset$ int {B}. Therefore {x} is one of the interior points of the set B. Thus, we have that for the smallest open set U (x) $\subset$ int (B).

We can set x = (2n) for any $n \in Z$, because $x \in B_E$. Then U{x}=U ({2n}) ={2n+1,2n,2n+2}. It is shown that $(U(\{x\}))_0 = \{y \in U(x) / y \text{ is odd}\} = \{2n-1, 2n+1\}$.Thus $\{2n-1,2n+1\} \in B$ and so $\{2n-1,2n+1\} \cap B \neq \phi$.

Therefore $(U\{x\}) \subset B$.

Theorem: 3.4 Let A be a rg-closed set in (Z, k).
Then (1) $(cl(A))_0 = \phi$ if $A = A_E$
(2) $(cl(A))_0 \neq \phi$ if $A = A_0 \cup A_E$

Proof:
(1) Let $A = A_E = (2n)$ for any $n \in Z$ and $\{2n\} \subseteq \{2n-1,2n,2n+1\} = U$, Where U is regular open set . By assumption cl {2n}={2n} $\subseteq$ U. Then $cl(\{2n\})_0 = \phi$. Thus $(cl(A))_0 = \phi$.

(2) Let $A = A_0 \cup A_E = \{2n, 2n+1, 2n+2\}$ for any $n \in Z$, and{2n, 2n+1, 2n+2} $\subseteq$ {2n-1, 2n, 2n+1, 2n+2, 2n+3} = U . where U is a regular open set.

By Assumption Cl(A) = { 2n, 2n+1, 2n+2} $\subseteq$ U. Then $(cl(A))_0 = \{2n+1\}$, Thus $(cl(A))_0 \neq \phi$.

Definition: 3.5 For any subset A of Z, we define the rg-closure of A, rg-interior of A and rgO-kernel of A as follows,
1. $\text{rg-cl}(A) = \cap \{V / V \in rgC(Z, k) , A \subset V\}$
2. $\text{rg-int}(A) = \cup \{V / V \in rgO(Z, k) , V \subset A\}$
3. $\text{rgO-ker}(A) = \cap \{V / V \in rgO(Z, k) , A \subset V\}$

Proposition: 3.5(a) For a topological space (Z, k), we have the properties on the singletons as follows. Let x be a point of Z and $n \in Z$.
1. If $x \in (Z)_0$, then rgO-ker({x}) = {x} and rgO- ker({x}) $\in$ rgO(Z, k)
2. If $x \in (Z)_E$, then rgO-ker({x}) = $\{x\} \cup (U\{x\})_0 = \{2n\} \cup \{2n-1 , 2n+1\}$, where x={2n} and rgO-ker{x}$\in$ rgO(Z, k)
3. If $x \in (Z)_0$, then rg-cl{x} = { 2n, 2n+1, 2n+2} , where x = {2n+1}
4. If $x \in (Z)_E$, then rg-cl({x}) = {x}
5. If $x \in (Z)_0$, then rg-int({x}) = {x}
6. If $x \in (Z)_E$, then rg-int({x}) = Φ

Proof:
1. For a point $x \in (Z)_0$, then by theorem 2.3, {x} is rg-open in (Z,k). Thus we have that rgO- ker({x}) = {x} and rg-ker({x}) $\in$ rgO(Z, k)

2. Let B be any rg-open set of (Z, k) containing the point $x = (2n) \in (Z)_E$ $\{x\} \cup \{U\{x\}\}_0 \subset B$ holds and$\{x\} \cup \{U\{x\}\}_0 \in$ rgO(Z, k). Thus we have rg-ker({x}) = $\cap \{\{x\}/\{x\} \subset V, V \in rgO(Z, k)\} = \{x\} \cup (U\{x\})_0 = \{2n-1,2n,2n+1\}$

Then $\{x\} \cup \{U\{x\}\}_0$ is open in (Z, k). Therefore $\{x\} \cup \{U\{x\}\}_0$ is rg-open in(Z, k). The kernel, rgO-ker{x} $\in$ rgO(Z, k)}

3. For a point $x \in (Z)_0$, we can put $x = (2n+1)$, $y\in$ rg-cl($\{x\}$) holds ((i.e.,) $y\in$ rg-cl$(\{x\})_0$) iff $x\in$ rg-cl($\{y\}$) holds (i.e.,) $x = y$. Thus we have, that rg-cl$(\{x\})_E = W_x$ Where $W_x = \{2n , 2n+2\}$ and $x = \{2n+1\}$. We obtain cl$\{x\}=\{x\}\cup W_x$ because $A = A_0 \cup A_E$

4. For a point $x\in (Z)_E$, rg-cl($\{x\}$) = $\{x\}$
5. For a point $x\in (Z)_0$, rg-int($\{x\}$) = $\{x\}$
6. For a point $x \in (Z)_E$, rg- int($\{x\}$) = Φ

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