International Journal of Mathematical Archive-9(4), 2018, 168-178
Available online through www.ijma.info ISSN 2229 – 5046

EFFECT OF THERMAL RADIATION ON CONVECTIVE HEAT TRANSFER FLOW OF AL_2O_3-WATER NANO FLUID IN A CONSTRICTED WAVY PIPE WITH THERMAL RADIATION

Dr. D. CHITTI BABU*[1] AND Prof. D.R.V. PRASADA RAO[2]

[1]Department of Mathematics,
Government Degree College, Tadepalligudem, West Godavari, (A.P.), India.

[2]Department of Mathematics,
Sri KrishnadevarayaUniversity, Anantapur, (A.P.), India.

(Received On: 26-03-18; Revised & Accepted On: 17-04-18)

ABSTRACT

An attempt has been to investigate the effect of thermal radiation on free convective heat transfer flow of nanofluid fluid in a non-uniformly heated corrugated pipe in the presence of a constant heat source.The non-linear governing equations have been solved by using perturbation with the slope δ of the boundary of the pipe as perturbation parameter. The effect of thermal radiation and waviness of the boundary on all flow characteristics are discussed graphically. The stress and Nusselt number on the boyndary have been evaluated numerically for different variation. It is found that higher the amplitude of the non-uniform boundary temperature reduces the axial velocity and enhances the secondary velocity and temperature. Higher the dilation of the boundary larger the velocity and temperature in the flow region.

***Key Words**: Non-uniform temperature, Constricted Wavy pipe, Heat source, nanoparticle concentration, Thermal radiation.*

1. INTRODUCTION

Present days, researchers are more concentrating on enhancement of heat transfer. The low thermal conductivity of conventional heat transfer fluids, such as water, is considered a primary limitation in enhancing the heat transfer performance. Maxwell's review [11] demonstrated the possibility of increasing the thermal conductivity of fluid-solid particles. Subsequently, the particles with micrometer or considerably millimeter measurements were utilized. Those particles caused several problems such as abrasion, clogging and pressure losses. During the past decade the technology of producing particles in nanometer dimensions was improved and a new kind of solid-liquid mixture that is called nanofluid was established by Choi [3]. The dispersion of a small amount of solid nanoparticle in conventional fluids such as water or Ethylene glycol changes their thermal conductivity remarkably. In general, in most recent research areas, heat transfer enhancement in forced convection is desirable [7], but there is still a debate on the effect of nano-particles on heat transfer enhancement in natural convection applications. Natural convection of Al_2O_3-water and CuO-water nanofluid inside a cylindrical enclosure heat from one side and cooled from the other side was studied by Putra *et al.*, [13]. Wen and Ding [17] investigated the natural convection of TiO_2-water in a vessel composed of two discs. Jou and Tzeng [7] conducted a numerical study of natural convection heat transfer in rectangular enclosure filled with the stream function-vorticity formulation. Mokhtari Moghari *et al.* [12] quested two phase mixed convection Al_2O_3-water nanofluid flow in an annulus. Abu-Nada *et al.* [1] studied natural convection heat transfer enhancement in horizontal concentric annuli using nanofluid. Abu-Nada [2] investigated the effect of variable viscosity and thermal conductivity of Al_2O_3-water nanofluid on heat transfer enhancement in natural convection. Das *et al*. [4] have studied mixed convective magneto hydrodynamic flow in a vertical channel filled with nanofluids. Sreedevi *et al.* [14] has investigated mixed convective heat and mass transfer flow of nanofluids in concentric annulus with constant heat flux. Sudarsana *et al.*, [16] have analyzed the Soret and Dufour effects on MHD convective flow of Al_2O_3-water and TiO_2-water nanofluids past a stretching sheet in porous media with heat generation with heat generation/absorption. Recently Madhusudhana Reddy *et al.*, [10] have presented Numerical study of Convective Flow of CuO-water and Al_2O_3-water Nanofluids in cylindrical annulus.

Corresponding Author: Dr. D. Chitti Babu[1], [1]Department of Mathematics,
Government Degree College, Tadepalligudem, West Godavari, (A.P.), India.*

In most of the investigations the boundaries are uniform in cross-sections as well as the boundary temperatures. However, there are a few physical situations which warrant the assumption of non-uniformity in either the boundaries or the boundary temperatures. In a convection flow through a channel such a non-uniformity creates a secondary flow. This secondary flow is of vital importance to technological processes. For example, the process of modified chemical vapour deposition (MCVD) has been suggested in drawing optical glass fibres of extremely low and wide band width. The heat transfer in a flow through a pipe in the presence of additional internal heat source has direct application to the modified chemical vapour deposition process. This analysis has been discussed by many researchers [5, 8, 9] in different configurations.

In this paper, we analyse the effect of thermal radiation on combined heat transfer of Al_2O_3-water nanofluid fluid in a non-uniformly heated constricted wavy pipe in the presence of a constant heat source. Using a perturbation method the equations ave been solved to analyse the velocity, temperature, stress, Nusselt number for different parametric variations.

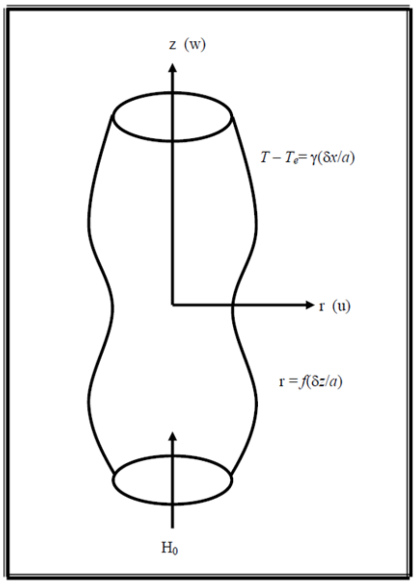
2. FORMULATION OF THE PROBLEM

We consider the steady axisymmetric flow of an incompressible, viscous nanofluid in a vertical pipe of variable cross section maintained at non-uniform temperature γ(δx/a). The Boussinesq approximation is used so that the density variations will be retained only in the buoyancy force. The viscous dissipation is neglected in comparison to the heat flow by convection. The concentration on these walls is taken to be constant. The cylindrical polar system(r, x) is chosen with x-axis along the axis of the pipe. The boundary of the pipe is assumed to be

$$r = af(\delta x / a)$$

where 'a ' is characteristic radial length, f is twice differentiable and 'δ'is a small parameter proportional to the boundary slope. The flow is maintained by a constant flow for which a characteristic velocity U is defined as

$$U = \left(\frac{2}{a^2}\right)\int_0^{af(\delta x/a)} urdr \qquad (1)$$



SCHEMATIC DIAGRAM OF THE CONFIGURATION

The properties of nanofluid such as thermal diffusivity, dynamic viscosity, effective density, specific heat coefficient of thermal expansion, heat capacitance and thermal conductivity are given as follows:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \ \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}},$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s, (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}$$

where the subscripts nf, f and s represent the thermo physical properties of the nanofluid, base fluid and the nanosolid particles respectively and ϕ is the solid volume fraction of the nanoparticles.

Table-1: The thermo physical properties of the nanofluid are given in Table 1.

Physical properties	Fluid phase	CuO (Copper)	Al_2O_3 (Alumina)	TiO_2 (Titanium dioxide)
C_p(j/kg K)	4179	385	765	686.2
ρ(kg m^3)	997.1	8933	3970	4250
k(W/m K)	0.613	400	40	8.9538
β$x10^{-5}$ 1/k)	21	1.67	0.63	0.85

The equations of flow and heat transfer with Rosseland approximation are

$$A_1A_3R_e(\bar{\zeta}x\bar{q}) = A_1\nabla\left(p + \frac{1}{2q^2}\right) + \nabla^2 q + GA_1A_4(\theta) \qquad (2)$$

$$\nabla.\bar{q} = 0 \qquad (3)$$

$$A_5 P_e(\overline{q}.\nabla)\theta = A_2\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial x^2}\right)+\frac{4}{3N_1}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right)+\alpha \tag{4}$$

where

$R_e = \dfrac{Ua}{\nu}$ (Reynolds number), $M = aB_0(\dfrac{\sigma}{\rho\nu})^{1/2}$ (Hartmann number)

$G = \dfrac{\beta_1 g\Delta T_e a^3}{\nu^2}$ (Grashof number), $P_e = \dfrac{\mu_e U C_p a}{\lambda\nu}$ (Peclet number),

$N_1 = \dfrac{\beta_R T_e^3}{4\sigma^\bullet}$ (Radiation parameter), $\alpha = \dfrac{Qa^2}{\lambda C_p}$ (Heat source parameter)

$A_1 = (1-\varphi)^{2.5}$, $A_2 = \dfrac{k_{nf}}{k_f}$, $A_3 = (1-\varphi)+\varphi(\dfrac{\rho_s}{\rho_f})$,

$A_4 = (1-\varphi)+\varphi(\dfrac{(\rho\beta)_s}{(\rho\beta)_f})$, $A_5 = (1-\varphi)+\varphi(\dfrac{(\rho C_p)_s}{(\rho C_p)_f})$

The boundary conditions relevant to the problem are

$$v(r,x)=0,\frac{\partial v}{\partial r}=0,\frac{\partial\theta}{\partial r}=0 \qquad on\ r=0$$

$$u(r,x)=0, T-T_e=\gamma(\delta x) \qquad on\ r=a \tag{5}$$

Equations (2)-(4) constitute a system of three equations for the three unknowns u, v and θ. These may be reduced to three equations for the Stoke's stream function ψ(r,x) is given by

$$u=-\frac{1}{r}\frac{\partial\psi}{\partial r},\quad v=\frac{1}{r}\frac{\partial\psi}{\partial x}$$

(The subscripts x and r denote the respective partial derivatives).

Taking the curl of the former to eliminate the pressure, equation (2) and (3) reduce to

$$A_1A_3R_e\left(\frac{1}{r}\frac{\partial\psi}{\partial x}\frac{\partial(E^2\psi)}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial(E^2\psi)}{\partial x}-\frac{2}{r^2}\frac{\partial\psi}{\partial x}E^2\psi\right)=E^2(E^2\psi)-(GA_1A_5/R_e)\left(r\left(\frac{\partial\theta}{\partial r}\right)\right) \tag{6}$$

$$A_5P_1(\theta_r\psi_x-\theta_x\varphi_r)=\left(A_2+\frac{4}{3N_1}\right)(\theta_{rr}+1/r)\theta_r+A_2N_2\theta_{xx}+\alpha \tag{7}$$

These coupled equations (6) & (7) are to be solved subject to non dimensional boundary conditions.

$$\psi(r,x)=0,\quad \frac{\partial}{\partial r}((1/r)\frac{\partial\psi}{\partial r}=0 \tag{8}$$

$$\frac{\partial\theta}{\partial r}=0 \quad on\ r=0 \tag{9}$$

$$\psi(r,x)=-1/2,\quad \frac{\partial\psi}{\partial r}=0 \tag{10}$$

$$\theta(r,x)=\gamma(\delta x) \quad on\ r=f \tag{11}$$

The value of ψ on the boundary assures the constant volumetric flow in consistence with the hypothesis (1) and conditions (8) & (9) corresponds to axial symmetry of the flow.

3. ANALYSIS OF THE FLOW

Introducing the transformations

$$\overline{x}=\delta x$$

we assume $\frac{\partial}{\partial x} \approx O(\delta)$ *such that* $\frac{\partial}{\partial \overline{x}} \approx O(1)$ for small values of δ ,the flow develops slowly along the axial direction with gradient $O(\delta)$. Making use of the above transformation the equations (6) & (7) reduce to

$$(A_1 A_3 \delta R_e)\left(\frac{1}{r}\frac{\partial \psi}{\partial \overline{x}}\frac{\partial (E_1^2\psi)-}{\partial r}\frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial (E_1^2\psi)}{\partial \overline{x}}-\frac{2}{r^2}\frac{\partial \psi}{\partial \overline{x}}E_1^2\psi\right)-E_1^4\psi-(GA_1A_4/R_e)\left(r\left(\frac{\partial \theta}{\partial r}\right)\right) \quad (12)$$

$$(A_5 \delta P_1)(\theta_r\psi_{\overline{x}}-\theta_x\varphi_r)=\left(A_2+\frac{4}{3N_1}\right)(\theta_{rr}+(1/r)\theta_r)+\delta^2A_2\theta_{xx}+\alpha \quad (13)$$

where $E_1^2 = r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r})+\delta^2\frac{\partial^2}{\partial \overline{x}^2}$

Taking the transformation $\eta = \frac{r}{f(\overline{x})}$ the above equations reduce to

$$(A_1 A_3 \delta\, f\, R_e)\left(\frac{1}{\eta}\frac{\partial \psi}{\partial \overline{x}}\frac{\partial (F^2\psi)-}{\partial \eta}\frac{1}{\eta}\frac{\partial \psi}{\partial \eta}\frac{\partial (F^2\psi)}{\partial \overline{x}}-\frac{2}{\eta^2}\frac{\partial \psi}{\partial \overline{x}}F^2\psi\right)-F^4\psi-(Gf^4A_1A_4/R_e)\left(\eta\left(\frac{\partial \theta}{\partial \eta}\right)\right) \quad (14)$$

$$(A_5 \delta\, f\, P_1)(\theta_\eta\psi_{\overline{x}}-\theta_x\varphi_\eta)=\left(A_2+\frac{4}{3N_1}\right)\theta_{\eta\eta}+(1/\eta)\theta_\eta+\delta^2f^2A_2\theta_{xx}+\alpha f^2 \quad (15)$$

where $F^2 = \eta\frac{\partial}{\partial \eta}\left(\frac{1}{\eta}\frac{\partial}{\partial \eta}\right)$

We use the asymptotic expansions

$$\psi(\eta,\overline{x}) = \psi_0(\eta,\overline{x})+\delta\psi_1(\eta,\overline{x})+\delta^2\psi_2(\eta,\overline{x})+ \ldots\ldots\ldots\ldots\ldots$$
$$\theta(\eta,\overline{x}) = \theta_0(\eta,\overline{x})+\delta\theta_1(\eta,\overline{x})+\delta^2\theta_2(\eta,\overline{x})+ \ldots\ldots\ldots\ldots\ldots$$
$$\phi(\eta,\overline{x}) = \phi_0(\eta,\overline{x})+\delta\phi_1(\eta,\overline{x})+\delta^2\phi_2(\eta,\overline{x})+ \ldots\ldots\ldots\ldots\ldots \quad (16)$$

Substituting (16) in equations (14) & (15) and separating the like powers of δ , the equations corresponding to the zeroth order are

$$\left(A_2+\frac{4}{3N_1}\right)\left(\theta_{0,\eta\eta}+\frac{1}{\eta}\theta_{0,\eta}\right)+\alpha f^2 = 0 \quad (17)$$

$$F^4\psi_0 - h^2F^2\psi_0+\frac{GA_1A_4f^4}{R_e}(\eta(\theta_{0,\eta}+NC_{0,\eta}))=0 \quad (18)$$

The corresponding conditions on ψ_0, θ_0, and ϕ_0 are

$$\psi_0(1,\overline{x}) = -1/2, \qquad (\psi_{0,\eta})_{\eta=1} = 0$$
$$(\eta\psi_{0,\eta\eta}-\psi_{0,\eta})_{\eta=0} = 0, \qquad \psi_0(0,\overline{x}) = 0 \quad (19a)$$
$$\theta_0(1,\overline{x}) = \gamma(\overline{x}), \qquad (\theta_{0,\eta})_{\eta=0} = 0 \quad (19b)$$
$$\phi_{0,x}(0,\overline{x}) = 0, \quad \lim_{r\to\infty}(\frac{1}{\eta}\phi_{0,\eta}) = -1 \quad (19d)$$

The equations to the first order are

$$\left(A_2+\frac{4}{3N_1}\right)(\eta\theta_{1.,\eta\eta}\quad +\theta_{1,\eta}) = P_1A_5(\psi_{0,\overline{x}}\theta_{0,\eta}-\psi_{0,\eta}\theta_{0,\overline{x}}) \quad (20)$$

$$F^2(F^2-h^2)\psi_1 = A_1A_3 f^4 R_e(\psi_{0,\bar{x}}(F^2\psi_0)_\eta - \psi_{0,\eta}(F^2\psi_0)_{\bar{x}} - \\ -\left(\frac{2}{r^2}\right)\psi_{0,\bar{x}}(F^2\psi_0)) + \frac{GA_1A_4 f^4}{R_e}(\eta(\theta_{1,\eta})) \tag{21}$$

The corresponding conditions on ψ_1, θ_1 are

$$\psi_1(1,\bar{x}) = -0, \qquad (\psi_{1,\eta})_{\eta=1} = 0$$

$$(\eta\psi_{1,\eta\eta} - \psi_{1,\eta})_{\eta=0} = 0, \quad \psi_1(0,\bar{x}) = 0 \tag{22}$$

$$\theta_1(1,\bar{x}) = \gamma(\bar{x}), \qquad (\theta_{1,\eta})_{\eta=0} = 0 \tag{23}$$

The equations to the second order are

$$(\mathrm{A}\left(A_2 + \frac{4}{3N_1}\right)(\eta\theta_{2.,\eta\eta} + \theta_{2,\eta}) = P_1A_5(\psi_{0,\bar{x}}\theta_{1,\eta} - \psi_{0,\eta}\theta_{1,\bar{x}} + \psi_{1,\bar{x}}\theta_{0,\eta} - \psi_{1,\eta}\theta_{0,\bar{x}}) \tag{24}$$

$$F^2(F^2-h^2)\psi_2 = A_1A_3 f^4 R_e(\psi_{1,\bar{x}}(F^2\psi_0)_\eta - \psi_{0,\eta}(F^2\psi_1)_{\bar{x}} - \\ -\left(\frac{2}{r^2}\right)(\psi_{0,\bar{x}}(F^2\psi_1) + \psi_{1,\bar{x}}(F^2\psi_0)) - \psi_{0,\bar{x}}(F^2\psi_1)_\eta \\ -\psi_{1,\eta}(F^2\psi_0)_{\bar{x}} + \frac{GA_1A_4 f^4}{R_e}(\eta(\theta_{1,\eta})) \tag{25}$$

The corresponding conditions on ψ_2, θ_2, and ϕ_2 are

$$\psi_2(1,\bar{x}) = -0, \qquad (\psi_{2,\eta})_{\eta=1} = 0$$

$$(\eta\psi_{2,\eta\eta} - \psi_{2\eta})_{\eta=0} = 0, \qquad \psi_2(0,\bar{x}) = 0 \tag{26}$$

$$\theta_2(1,\bar{x}) = 0, \qquad (\theta_{2,\eta})_{\eta=0} = 0 \tag{27}$$

where $F^2 = \dfrac{\partial^2}{\partial\eta^2} - \dfrac{1}{\eta}\dfrac{\partial}{\partial\eta}$

4. SOLUTION OF THE PROBLEM

Solving the coupled equations (17) & (18) subject to the corresponding boundary conditions (19a)-(19d), we get the expressions for zeroth order

$$\theta_0(\bar{x},\eta) = \gamma(\bar{x}) + \alpha_1 f^2(1-\eta^2)/4$$

$$\psi_0 = \frac{a_4}{2}\eta^2 + \frac{a_3}{16}\eta^4 + \frac{a_2}{192}\eta^6$$

Solving the coupled equations (20) & (21) subject to the corresponding conditions (23)-(24), the solution for θ_1, ψ_1 *and* ϕ_1 are

$$\theta_1 = \frac{a_5}{64}(\eta^8-1) + \frac{a_6}{49}(\eta^7-1) + \frac{a_7}{36}(\eta^6-1) + \frac{a_8}{25}(\eta^5-1) + \\ + \frac{a_9}{16}(\eta^4-1) + \frac{a_{10}}{9}(\eta^3-1) + \frac{a_{11}}{4}(\eta^2-1)$$

$$\psi_1 = \frac{a_{61}}{9}\eta^3 + \frac{a_{62}}{180}\eta^5 + \frac{a_{63}}{576}\eta^6 + \frac{a_{64}}{1225}\eta^7 + \frac{a_{65}}{(36x64)}\eta^8 + \frac{a_{66}}{(49x81)}\eta^9 \\ + \frac{a_{67}}{6400}\eta^{10} + \frac{a_{68}}{(81x21)}\eta^{11} + \frac{a_{69}}{14400}\eta^{12} + \frac{a_{70}}{(160x121)}\eta^{13} + \frac{a_{71}}{(144x189)}\eta^{14} + \frac{B_1}{2}\eta^2 + B_2$$

where $a_1, a_2, \ldots\ldots, a_{71}$, B_1, B_2 are constants involving parameters.

5. SHEAR STRESS, NUSSELT NUMBER

The stress tensor for the motion on the pipe

$$\sigma_{ij} = -p\delta_{ij} + 2\rho v e_{ij}, e_{xx} = \frac{\partial u}{\partial x}, e_{rr} = \frac{\partial v}{\partial r}, e_{rx} = 0.5(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x})$$

The shear stress on the pipe r = f(x), in the non-dimensional form is given by

$$\tau = (\sigma_{rx}(1 - f'^2) + (\sigma_{rr} - \sigma_{xx})f'/(1 + f'^2)$$

In terms of non-dimensional variables, we obtain the non-dimensional shear stress is

$$\tau = \left(\left(\frac{P_{11}}{2f^3}\left(\begin{array}{c}\left(P_2\left(\frac{1}{\eta^2}\right)\psi_{0,\eta} - \frac{1}{\eta}\psi_{0,\eta\eta}\right) + \in (P_{12}(\left(\frac{1}{\eta^2}\right)\psi_{1,\eta} - \frac{1}{\eta}\psi_{1,\eta\eta} \\ + 2ff'\left(\left(\frac{2}{\eta}\right)\psi_{0,\eta x} - \left(\frac{1}{\eta^2}\right)\psi_{0,x}\right)\end{array}\right)\right)\right)$$

where $P_{11} = \frac{1}{1 + f'^2}$, $P_{12} = 1 - f'^2$

and the corresponding expression is $\tau = -\frac{P_{11}P_{12}}{2f^3}(B_8 + \delta(B_{11} + \frac{2f'}{f}B_{10}))$

The local rate of heat transfer coefficient (Nusselt number) on the boundary of the pipe is calculated using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}, \quad \theta_m = 2\int_0^1 \theta d\eta$$ and the corresponding expression is

$$Nu = \frac{B_5 + \delta B_6}{f(B_3 + \delta B_4 - \gamma(\overline{x}))}$$

where B_3, B_4, B_5, B_6 ....B_{11}, P_{11}, P_{12} are constants involving governing parameters.

6. DISCUSSION OF THE NUMERICAL RESULTS

The aim of the analysis is to study of the effect of radiation on convective heat transfer flow of Al_2O_3-water nanofluids in a non-uniform pipe which is maintained at non-uniform temperature in the presence of a constant heat source/sink. The coupled equations governing the flow and heat transfer have been solved using a perturbation technique. The velocity and the temperature distributions in the fluid region are analytically evaluated and their behaviour with reference to variations in the governing parameters G, R, α_1, β, ϕ, N_1, Pr, x has been analyzed numerically. For computational purpose the geometry of the pipe wall in the non-dimensional form is assumed to be $\eta = f(\overline{x}) = 1 + \beta \exp(-x^2)$ and the prescribed wall temperature $\gamma(\overline{x})$ is chosen to be $\alpha_1 \sin(\overline{x})$. $\beta > 0$ corresponds to dilation and $\beta < 0$ corresponds to constriction of the pipe. In this analysis we confine our study in a constricted pipe. An increase in the Grashof number (G) /Reynolds number (R) reduces the primary velocity and enhances the secondary velocity and temperature in the flow region (figs.2, 10, 18). The effect of heat sources on the velocity components and temperatrure can be seen from the figs.3, 11, 19. The primary velocity and temperature enhances, the secondary velocity reduces with increase in the strength of the heat source while in the case of heat sink ($\alpha < 0$), the primary velocity enhances and the temperature reduces in the flow region. The secondary velocity reduces with $\alpha < -4$, the secondary velocity reduces and enhances with higher $\alpha \geq -6$ (fig.11).

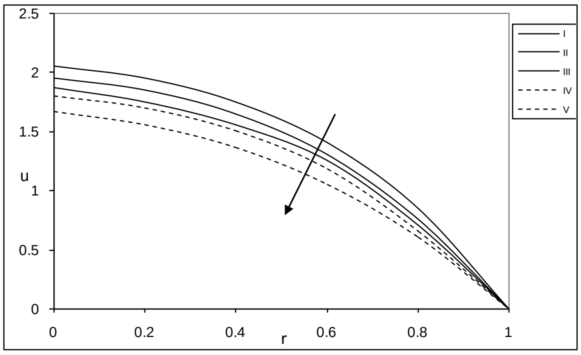


FIG.2. VARIATION OF u WITH G&R

	I	II	III	IV	V
G	100	300	500	100	100
R	10	10	10	30	50

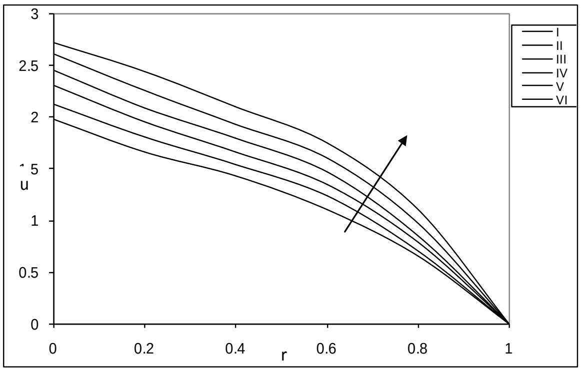


FIG. 3. VARIATION OF u WITH α

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

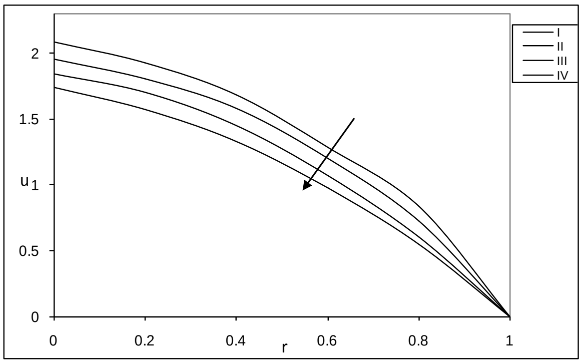


FIG. 4. VARIATION OF u WITH α1

	I	II	III	IV
α1	0.3	0.5	0.7	0.9

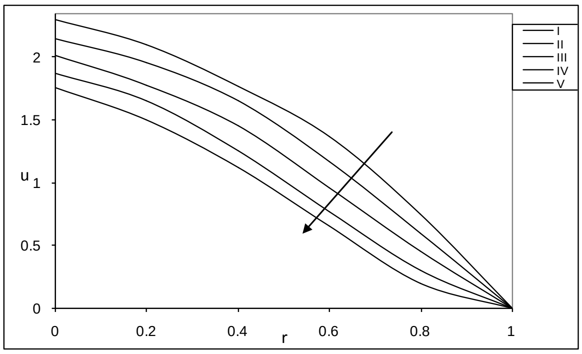


FIG. 5. VARIATION OF u WITH β

	I	II	III	IV	V
β	-0.3	-0.5	-0.7	-0.9	-1.20

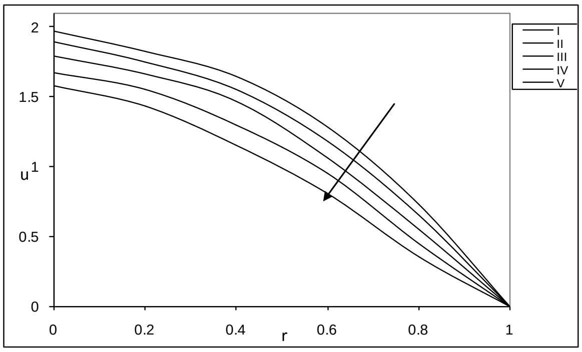


FIG.6. VARIATION OF u WITH ϕ

	I	II	III	IV	V
ϕ	0.1	0.3	0.5	0.7	0.9

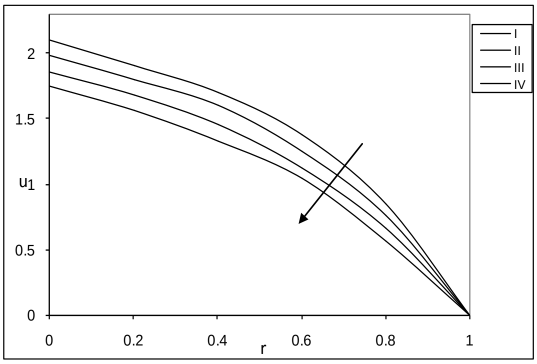


FIG.7. VARIATION OF u WITH N1

	I	II	III	IV	V
N1	0.5	1.5	3.5	5	10

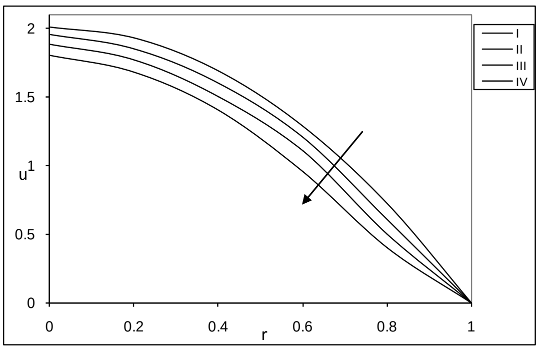


FIG.8. VARIATION OF u WITH Pr

	I	II	III	IV
Pr	0.71	1.71	3.71	6.2

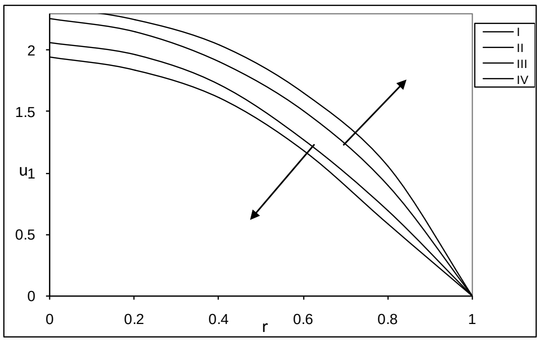


FIG.9. VARIATION OF u WITH X

	I	II	III	IV
X	π/4	π/2	π	2π

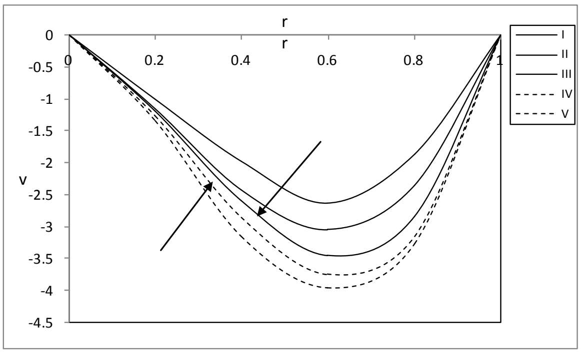


FIG.10. VARIATION OF V WITH G&R

	I	II	III	IV	V
G	500	100	300	100	100
R	10	10	10	30	50

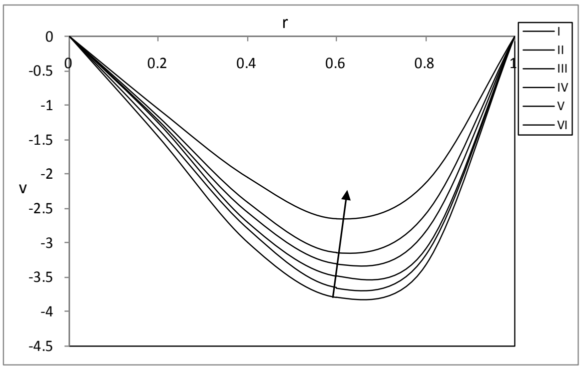


FIG. 11. VARIATION OF V WITH α

	I	II	III	IV	V	VI
α	6	4	2	-4	-6	-2

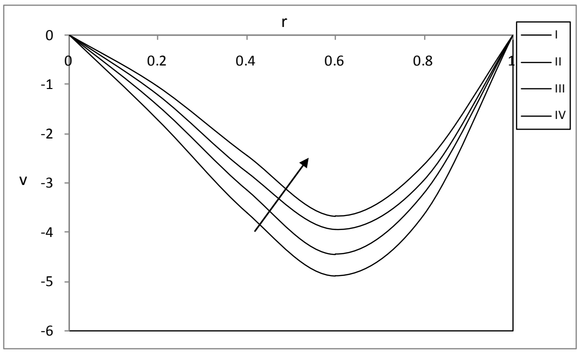


FIG. 12. VARIATION OF u WITH α1

	I	II	III	IV
α1	0.3	0.5	0.7	0.9

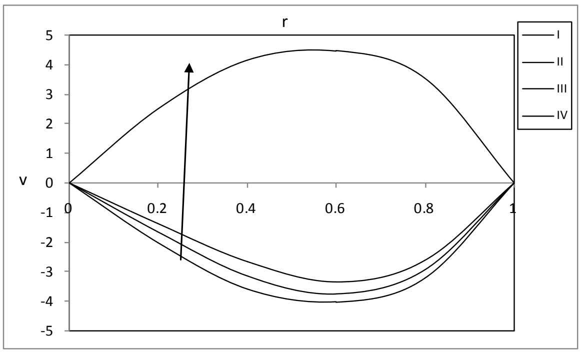


FIG. 13. VARIATION OF V WITH β

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

An increase in the amplitude α1 reduces the primary velocity and enhances the secondary velocity and temperature (figs.4, 12, 20) .higher the constriction of the wavy pipe smaller the velocity components and temperature (figs.5,13,21). The secondary velocity and temperature enhance while the primary velocity reduces with increase in radiation parameter N_1 (figs.7, 15, 23).The secondary velocity and temperature enhances while the primary velocity reduces with increase in the nanoparticle concentration ϕ (figs.6, 14, 22). Lesser the thermal diffusivity smaller the primary velocity, larger the secondary velocity and temperature in the flow region (figs.8, 16, 24). Moving along the axial direction the primary velocity and temperature enhance while the secondary velocity reduces in the flow region (figs.9, 17, 25).

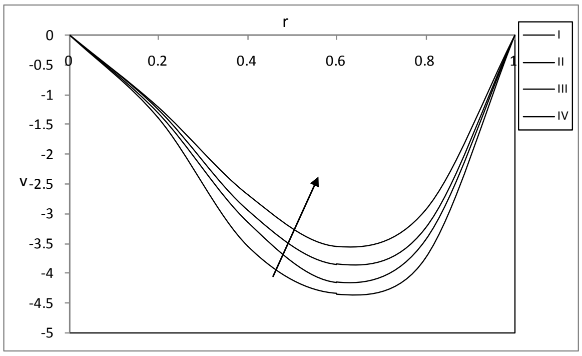


FIG. 14. VARIATION OF V WITH ϕ

	I	II	III	IV	V
ϕ	0.1	0.3	0.5	0.7	0.9

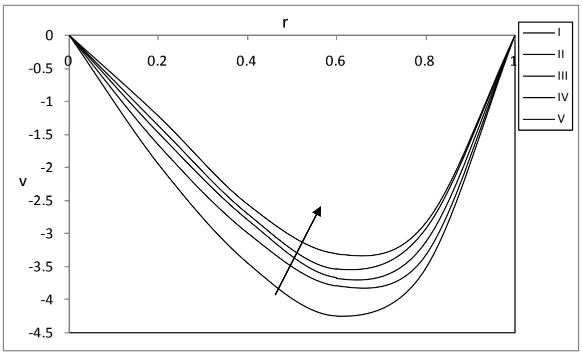


FIG. 15. VARIATION OF V WITH N1

	I	II	III	IV	V
N1	0.5	1.5	3.5	5	10

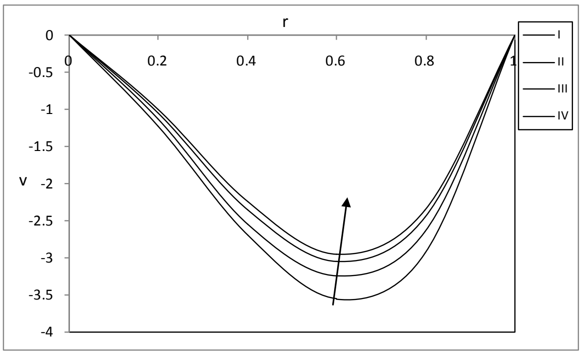


FIG. 16. VARIATION OF V WITH Pr

	I	II	III	IV
Pr	0.71	1.71	3.71	6.2

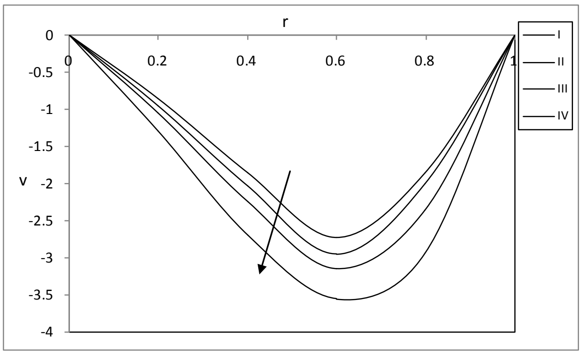


FIG. 17. VARIATION OF V WITH X

	I	II	III	IV
X	π/4	π/2	π	2π

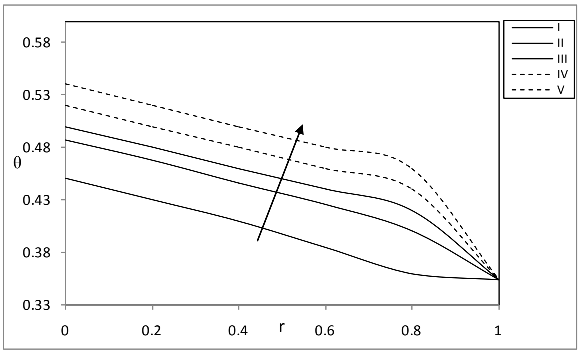


FIG. 18. VARIATION OF θ WITH G&R

	I	II	III	IV	V
G	100	300	500	100	100
R	10	10	10	30	50

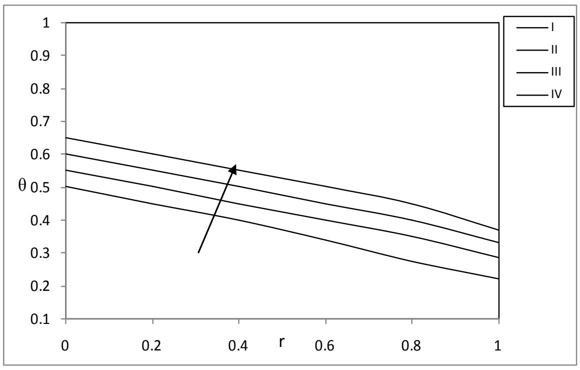


FIG. 20. VARIATION OF θ WITH α1

	I	II	III	IV
α1	0.3	0.5	0.7	0.9

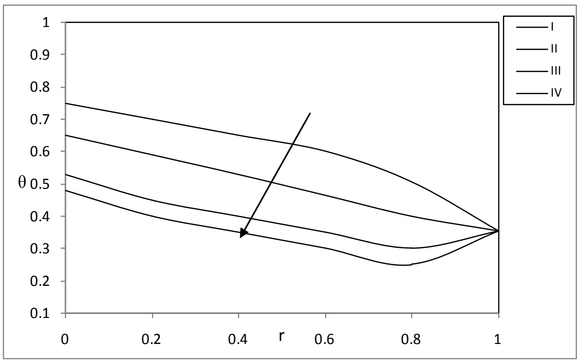


FIG. 21. VARIATION OF θ WITH β

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

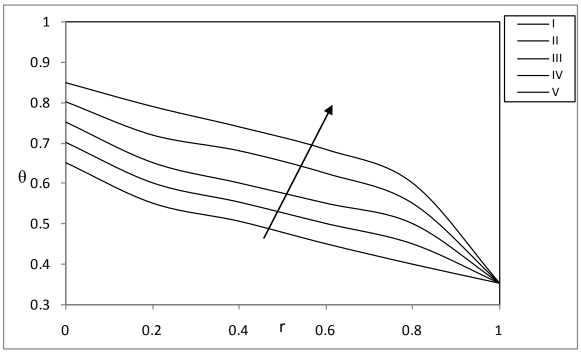


FIG. 22. VARIATION OF θ WITH ϕ

	I	II	III	IV	V
ϕ	0.1	0.3	0.5	0.7	0.9

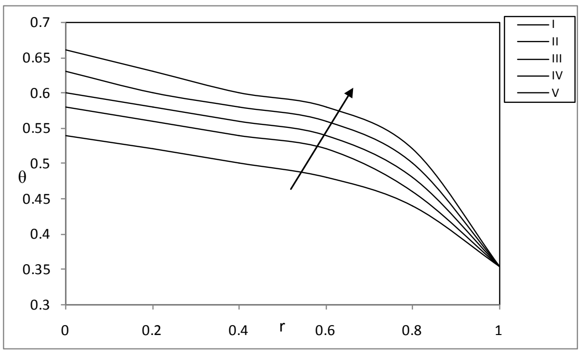


FIG. 23. VARIATION OF θ WITH N1

	I	II	III	IV	V
N1	0.5	1.5	3.5	5	10

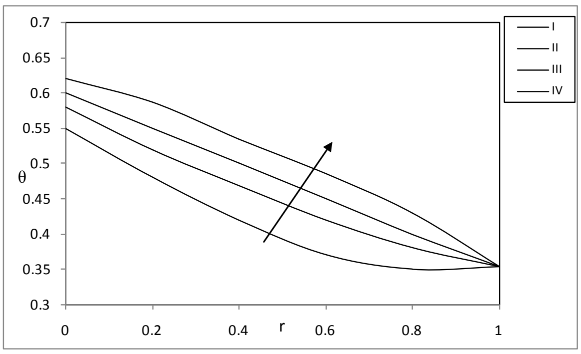


FIG. 24. VARIATION OF θ WITH Pr

	I	II	III	IV
Pr	0.71	1.71	3.71	6.2

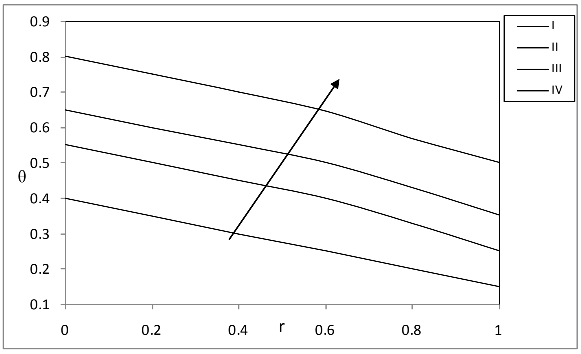


FIG. 25. VARIATION OF θ WITH X

	I	II	III	IV
X	π	2π	π/4	π/2

An increase in the amplitude α1 increases the stress and Nusselt number on the surface η=1. Higher the constriction of the pipe/constriction of the wavy pipe, larger the stress and the Nusselt number. The variation of stress with nanoparticle volume faction ϕ shows that the stress and the Nusselt number experiences a depreciationon the pipe with increase in ϕ. Higher the radiative heat flux smaller the stress and Nusselt number on the surface of the pipe. Lesser the thermal diffusivity (Pr ≤ 3.71) larger the stress and Nusselt number and for further lowering of the thermal diffusivity (Pr ≥ 7.0) smaller the stress and nusselt Number at the surface of the wavy pipe.

Skin friction(τ), Nusselt Number(Nu) on r=1-(Al2O3-water nanofluid)

G	R	α	α 1	β	N1	ϕ	Pr	τ(1)	Nu(1)
100	3.5	2	0.3	-0.3	0.5	0.1	0.71	-3.58124	3.25325
200	35	2	0.3	-0.3	0.5	0.1	0.71	-3.59345	3.31081
300	35	2	0.3	-0.3	0.5	0.1	0.71	-3.60625	3.36920
100	70	2	0.3	-0.3	0.5	0.1	0.71	-3.56601	2.05165
100	140	2	0.3	-0.3	0.5	0.1	0.71	-3.54646	1.24704
100	35	4	0.3	-0.3	0.5	0.1	0.71	-3.65569	-3.47398
100	35	-2	0.3	-0.3	0.5	0.1	0.71	-3.98718	-3.60409
100	35	-4	0.3	-0.3	0.5	0.1	0.71	-3.60645	-3.44528
100	35	2	0.5	-0.3	0.5	0.1	0.71	-3.58124	3.25325
100	35	2	0.7	-0.3	0.5	0.1	0.71	-3.60540	3.83567
100	35	2	0.3	-0.5	0.5	0.1	0.71	-3.58124	3.25325
100	35	2	0.3	-0.7	0.5	0.1	0.71	-4.01482	4.44077
100	35	2	0.3	0.9	0.5	0.1	0.71	-3.95658	5.86664
100	35	2	0.3	-0.3	1.5	0.1	0.71	-3.63894	3.44470
100	35	2	0.3	-0.3	5.0	0.1	0.71	-3.5777	-3.40342
100	35	2	0.3	-0.3	0.5	0.3	0.71	-3.59360	3.30548
100	35	2	1	-0.3	0.5	0.5	0.71	-3.58844	3.25838
100	35	0.2	1	-0.3	0.5	0.1	3.71	-3.60712	4.09873
10	35	0.2	1	-0.3	0.5	0.1	7.00	-3.58124	3.25325

7. CONCLUSIONS

We analyse the effect of thermal radiation on combined heat transfer of Al_2O_3-water nanofluid fluid in a non-uniformly heated constricted wavy pipe in the presence of a constant heat source. Using a perturbation method the equations ave been solved to analyse the velocity, temperature, stress, Nusselt number for different parametric variations.

- An increase in the amplitude α_1 reduces the primary velocity and enhances the secondary velocity and temperature.
- Higher the constriction of the wavy pipe smaller the velocity components and temperature.
- The secondary velocity and temperature enhance while the primary velocity reduces with increase in radiation parameter N_1.
- The secondary velocity and temperature enhances while the primary velocity reduces with increase in the nanoparticle concentration ϕ.
- An increase in the amplitude α_1 increases the stress and Nusselt number on the surface η=1.
- Higher the constriction of the pipe/constriction of the wavy pipe, larger the stress and the Nusselt number.
- The variation of stress with nanoparticle volume faction ϕ shows that the stress and the Nusselt number experiences a depreciation on the pipe with increase in ϕ.
- Higher the radiative heat flux smaller the stress and Nusselt number on the surface.

8. REFERENCES

1. Abu-Nada, E., Masoud, Z., Hijazi, A: Natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids, Int Commun Heat Mass Transf, V.35, 657-665(2008),
2. Abu-Nada, E: Effect of variable viscosity and thermal conductivity of Al_2O_3-water nanofluid on heat transfer enhancement in natural convection, Int J heat Fluid Flow, V.30, 679-690(2009).
3. Choi, S.U.S: Enhancing thermal conductivity of fluid with nanoparticles, developments and applications of non-Newtonian flow, ASME FED 231, 99-105(1995).
4. Das, S., Jana, R.N., and Makinde, O.D : Mixed convective magneto hydrodynamic flow in a vertical channel filled with nanofluids, Engineering science and technology an International Journal, V.18, pp.244-255(2015).
5. Costa, V.A.F.: Thermodynamics of natural convection in enclosures with viscous dissipation,Int.Jour.Heat Mass Transfer,V.48,pp.2333-2341(2005).
6. Gill, W.M. and Casal, A.D., A theoretical investigation of natural convection effects in forced horizontal flows. Amer.Inst.Chem. Engg.Jour.8,513-520(1962).
7. Jou, R.Y., Tzeng, S. C: Numerical research of nature convective heat transfer enhancement filled with nanofluids in rectangular enclosures, Int Commun Heat Mass Transf, V.33, 727-736(2006).
8. Krishna D.V. Kalposchikov, V.L, Martynenko, O.G, Shabunya, S.I., Shnip, A.I, Luikov, A.V, “Reports from Lukov Heat and Mass Transport Institue of Minsk, U.S.S.R(1985).
9. Krishna D.V, Prasada Rao, D.R.V and Sarojamma.G: Int. Comm.Heat and Mass Transfer, V.16, P.237 (1988).
10. Madhusudhana Reddy, Y., Rama Krishna, G.N. and Prasada Rao, D.R.V: Numerical study of Convective Flow of CuO-water and Al_2O_3-water nanofluids in cylindrical annulus, Int. Journal of Research & Development in Tech., Vol. 7, Issue 3, pp. 142-148 (2017).
11. Maxwell, J.C: Electricity and magnetism, Clarendon Press, Oxford, UK (1873).
12. Mokhtari Moghari, R., Akbarinia, A., Shariat, M., Talebi, F., Laur, R: Two phase mixed convection Al_2O_3-water nanofluid flow in an annulus, Int J Multiph Flow, V.37(6), 585-595(2011).
13. Putra, N., Roetzel W, Das, S.K: Natural convection of nanofluids, Heat and Mass Transfer, V.39 (8-9), 775-784 (2003).
14. Sree devi G., Raghavendra Rao, R., Chamka, A.J and Prasada Rao, D.R.V: Mixed convective heat and mass transfer flow of nanofluids in concentric annulus with constant heat flux, Procedia Engineering, 127, 1048-1055(2015).
15. Sree Ramanchandra Murthy,A., Buoyancy induced hydromagnetic flows through a porous medium –A study, Ph.D thesis submitted to S.K.University, Anantapur, India (1992).
16. Sudarsana Reddy, P., Chamka, A.J: Soret and Dufour effects on MHD convective flow of Al2O3-water and TiO2- water nanofluids past a stretching sheet in porous media with heat generation/absorption, J. Advanced Powder Technology, V. 27, 1207-1218(2016).
17. Wen, D and Ding, Y: Formulation of nanofluids for natural convective heat transfer applications, Int J Heat Fluid Flow, V. 26(6), 855-864(2005).

Source of support: Nil, Conflict of interest: None Declared.