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COMPLETE SYNCRONIZATION OF THE PHOTOGRAVITATIONAL RESTRICTED THREE BODY PROBLEM WHEN BIGGER PRIMARY IS OBLATE SPHEROID AND SMALLER PRIMARY IS ELLIPSOID

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***ABSTRACT*.**

In this article we have investigated the complete synchronization behavior of the planar restricted three body problem by taking into consideration the bigger primary is an oblate spheroids and source of radiation and smaller is ellipsoid evolving from deferent initial conditions using active control technique based on the Lyapunov-stability theory and Routh-Hurwitz criteria. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control techniques.

Key words: *Space dynamics, restricted three body problem, complete synchronization, hybrid synchronization, Lyapunov stability theory and Routh- Hurwitz criteria.*

1. INTRODUCTION

Chaos control and synchronization are especially noteworthy and important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pec-ora and Carroll [5] introduced a method to synchronize two identical chaotic systems with deferent initial conditions in (1990) and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems.

Chaos synchronization using active control has recently been widely accepted as an efficient technique for synchronizing chaotic systems. This method has been applied to many practical systems such as Non-linear Bloch equations modeling "jerk" equation Ucar *et al.*[9], Rikitake two-disc dynamo-a geographical system Vincent [8], Chua's circuits Tang & Wang [7], Qi system Lei *et al.* [2], parametrically excited systems Ucar [10] and magnetic binary problem Mohd. Arif [4].

Photogravitational restricted three body problem have been discussed by Yu. A Chernikov [11], Bhatnagar and Chawala [1], Sharma *et al.* [6]. In (2013) M. javid Idrisi and Z. A. Taqvi [3] have been studied the restricted three body problem when one of the primaries is an ellipsoid.

Being motivated by the above discussion, in this article we have discussed the complete synchronization behavior of the planar restricted three body problem by taking into consideration the bigger primary is an oblate spheroids and source of radiation and smaller is ellipsoid. The paper is organized as follows. In section 2 we derive the equations of motion when the primaries moving in a circular orbit around their center of mass. Section 3 deals with the complete synchronization of the problem. Finally, we conclude the paper in section 4.

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2. EQUATION OF MOTION

In formulating the problem we shall assume that the two primaries one is in the shape of ellipsoid and other is oblate spheroid and source of radiation participate in the circular motion around their centre of mass O Fig.(1). The motion of a particle P of mass m defined by its radius vector $\boldsymbol{r}$ will be referred to a frame of reference $Oxyz$ that rotates in the same direction and the same angular velocity $\boldsymbol{\omega}$ as the primaries, which in this frame are taken to stay at rest on x-axis. Here we assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is (μ, 0, 0) then the other is ($\mu-1$, 0, 0). We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is ($1-\mu$). The unit of time in such a way that the gravitational constant G has the value unity.

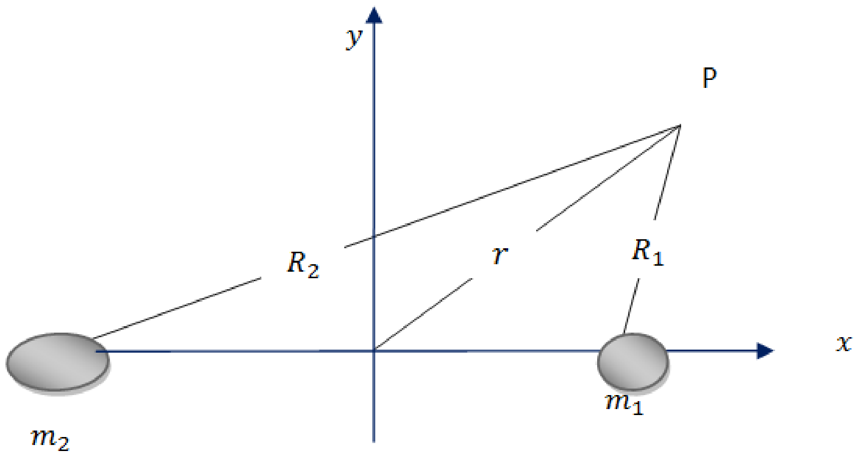

Figure-1

The equation of motion of the particle P may be written as:

$$\ddot{x} - 2\omega\,\dot{y} = U_x \tag{1}$$

$$\ddot{y} + 2\omega\,\dot{x} = U_y \tag{2}$$

Where

$$U = \frac{\omega^2}{2}(x^2+y^2) + p\left\{\frac{1-\mu}{R_1} + \frac{I}{2R_1^3}\right\} + V \tag{3}$$

$R_1^2 = (x-\mu)^2 + y^2$, $R_2^2 = (x+1-\mu)^2 + y^2$

p is the radiation factor due to bigger primary

$$V = \frac{3\mu}{2}\left[\left\{1 + \frac{y^2-(x+1-\mu)^2}{(a^2-b^2)}\right\}\frac{F(\varphi,k)}{\sqrt{(a^2-c^2)}} + \left\{\frac{(x+1-\mu)^2}{(a^2-b^2)} + \frac{(c^2-a^2)y^2}{(a^2-b^2)(b^2-c^2)}\right\}\frac{E(\varphi,k)}{\sqrt{(a^2-c^2)}} + \frac{\sqrt{(\gamma+c^2)}\,y^2}{(b^2-c^2)\sqrt{(\gamma+a^2)(\gamma+b^2)}}\right] \tag{4}$$

$F(\varphi,k) = \int_0^{\varphi}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$ Elliptic integral of first kind

$E(\varphi,k) = \int_0^{\varphi}\sqrt{1-k^2\sin^2\theta}\,d\theta$ Elliptic integral of second kind

$k = \sqrt{\frac{(a^2-b^2)}{(a^2-c^2)}}$ $\quad 0 \le k^2 \le 1,\quad \varphi = \sin^{-1}\sqrt{\frac{(a^2-c^2)}{(\gamma+c^2)}}\quad 0 \le \varphi \le \frac{\pi}{2},$

$\gamma = \frac{1}{2}\left[(x+1-\mu)^2 + y^2 - p_1 + \sqrt{\{(x+1-\mu)^2+y^2-p_1\}^2 + 4\{p_3(x+1-\mu)^2 + p_4y^2 - p_2\}}\right],$

$p_1 = a^2+b^2+c^2,\ p_2 = a^2b^2+b^2c^2+a^2c^2,\ p_3 = b^2+c^2,\ p_4 = a^2+c^2,$

$p_5 = a^2+b^2$. a, b and c are the axes of the ellipsoid. I is the moment of inertia of oblate body. ω is the mean motion of the primaries.

$$\omega = 1 + \frac{3}{10}\frac{\mu}{(1-\mu)}[2a^2 - b^2 - c^2] + \frac{3I}{2\mu}$$

3. COMPLETE SYNCHRONIZATION

Let

$$x = x_1,\ \dot{x} = x_2,\ y = x_3,\ \dot{y} = x_4$$

Then the equation (1) and (2) can be written as:

$$\dot{x_1} = x_2 \tag{5}$$

$$\dot{x_2} = 2\omega x_4 + \omega^2 x_1 + A_1 \tag{6}$$

$$\dot{x_3} = x_4 \tag{7}$$

$$\dot{x_4} = -2\omega x_2 + \omega^2 x_3 + A_2 \tag{8}$$

where

$$A_1 = -(x_1 - \mu)\, p\left\{\frac{1-\mu}{r_1^3} + \frac{3I}{2r_1^5}\right\} - 3\mu\,(x_1 + 1 - \mu) \times$$
$$\times \left[\frac{E(\varphi,k) - F(\varphi,k)}{p_6 p_8} - \left\{1 - k^2 sin^2\varphi \frac{(x_1+1-\mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 sin^2\varphi}{p_9}\right) {x_3}^2\right\}\right.$$
$$\times \frac{\gamma_1 + p_3}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 sin^2\varphi}}$$
$$\left. - \frac{(2c^2\gamma_1 + p_{11} + {\gamma_2}^2)(\gamma_1 + p_3){x_3}^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + {\gamma_1}^2)^{\frac{3}{2}}}\right]$$

$r_1^2 = (x_1 - \mu)^2 + {x_3}^2$, $r_2^2 = (x_1 + 1 - \mu)^2 + {x_3}^2$, $p_6 = a^2 - b^2$, $p_7 = b^2 - c^2$, $p_8 = \sqrt{a^2 - c^2}$,
$p_9 = \frac{(a^2 - b^2)(b^2 - c^2)}{(c^2 - a^2)}$, $p_{10} = b^2a^2$, $p_{11} = p_{10} - c^2p_5$,

$$\gamma_1 = \frac{1}{2}\left[(x_1 + 1 - \mu)^2 + {x_3}^2 - p_1 + \sqrt{\{(x_1 + 1 - \mu)^2 + {x_3}^2 - p_1\}^2 + 4\{p_3(x_1 + 1 - \mu)^2 + p_4{x_3}^2 - p_2\}}\right]$$

$$A_2 = -x_3\, p\left\{\frac{1-\mu}{r_1^3} + \frac{3I}{2r_1^5}\right\} - 3\mu x_3$$
$$\times \left[\frac{E(\varphi,k)}{p_9 p_8} + \frac{F(\varphi,k)}{p_6 p_8} - \left\{1 - k^2 sin^2\varphi \frac{(x_1+1-\mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 sin^2\varphi}{p_9}\right) {x_3}^2\right\}\right.$$
$$\times \frac{\gamma_1 + p_4}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 sin^2\varphi}}$$
$$\left. - \frac{(2c^2\gamma_1 + p_{11} + {\gamma_1}^2)(\gamma_1 + p_4){x_3}^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + {\gamma_1}^2)^{\frac{3}{2}}} + \frac{\sqrt{(\gamma_1 + c^2)}}{p_7\sqrt{(p_{10} + p_5\gamma_1 + {\gamma_1}^2)}}\right]$$

The system (5, 6, 7 and 8) is the master system. The state orbits of this master system are shown in Figure (2) and this figure shows that the system is chaotic.

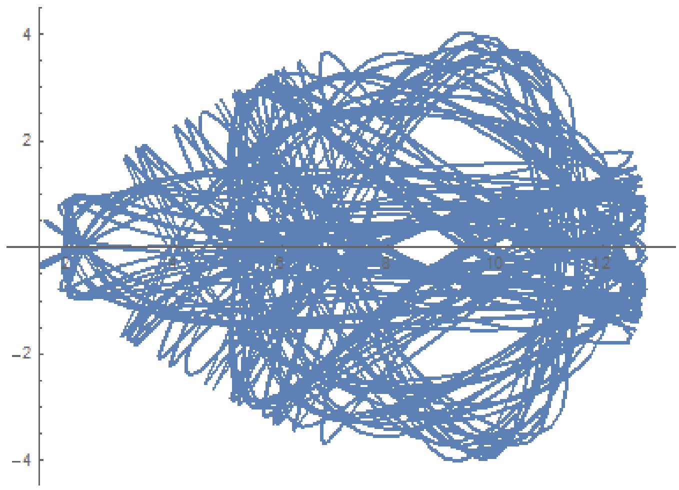

Figure-2

Corresponding to master system (5, 6, 7 and 8), the identical slave system is

$$\dot{y_1} = y_2 + u_1(t) \tag{9}$$
$$\dot{y_2} = 2\omega y_4 + \omega^2 y_1 + A_3 + u_2(t) \tag{10}$$
$$\dot{y_3} = y_4 + u_3(t) \tag{11}$$
$$\dot{y_4} = -2\omega y_2 + \omega^2 y_3 + A_4 + u_4(t) \tag{12}$$

Where

$r_{11}^2 = (y_1 - \mu)^2 + {y_3}^2$, $r_{21}^2 = (y_1 + 1 - \mu)^2 + {y_3}^2$

$$A_3 = -(y_1 - \mu)\, p\left\{\frac{1-\mu}{r_{11}^3} + \frac{3I}{2r_{11}^5}\right\} - 3\mu(y_1 + 1 - \mu) \times$$

$$\left[\frac{\mathrm{E}(\varphi,\mathrm{k})-\mathrm{F}(\varphi,\mathrm{k})}{p_6 p_8}-\left\{1-k^2 sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2 sin^2\varphi}{p_9}\right){y_3}^2\right\}\right.$$
$$\times\frac{\gamma_2+p_3}{2(\gamma_2+a^2)(2\gamma_2+p_1-\mathrm{r}_{21}^2)\sqrt{(\gamma_2+c^2)}\sqrt{1-k^2 sin^2\varphi}}$$
$$\left.-\frac{(2c^2\gamma_2+p_{11}+{\gamma_2}^2)(\gamma_2+p_3){y_3}^2}{2p_7(2\gamma_2+p_1-\mathrm{r}_{21}^2)\sqrt{(\gamma_2+c^2)}(p_{10}+p_5\gamma_2+{\gamma_2}^2)^{\frac{3}{2}}}\right]$$

$$A_4=-y_3 p\left\{\frac{1-\mu}{\mathrm{r}_{11}^3}+\frac{3\mathrm{I}}{2\mathrm{r}_{11}^5}\right\}-3\mu y_3\times$$
$$\left[\frac{\mathrm{E}(\varphi,\mathrm{k})}{p_9 p_8}+\frac{\mathrm{F}(\varphi,\mathrm{k})}{p_6 p_8}-\left\{1-k^2 sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2 sin^2\varphi}{p_9}\right){y_3}^2\right\}\right.$$
$$\times\frac{\gamma+p_4}{2(\gamma_2+a^2)(2\gamma_2+p_1-\mathrm{r}_{21}^2)\sqrt{(\gamma_2+c^2)}\sqrt{1-k^2 sin^2\varphi}}$$
$$\left.-\frac{(2c^2\gamma_2+p_{11}+{\gamma_2}^2)(\gamma_2+p_4){y_3}^2}{2p_7(2\gamma_2+p_1-\mathrm{r}_{21}^2)\sqrt{(\gamma_2+c^2)}(p_{10}+p_5\gamma_2+{\gamma_2}^2)^{\frac{3}{2}}}+\frac{\sqrt{(\gamma_2+c^2)}}{p_7\sqrt{(p_{10}+p_5\gamma_2+{\gamma_2}^2)}}\right]$$

$$\gamma_2=\frac{1}{2}\left[(y_1+1-\mu)^2+{y_3}^2-p_1+\sqrt{\{(y_1+1-\mu)^2+{y_3}^2-p_1\}^2+4\{p_3(y_1+1-\mu)^2+p_4{y_3}^2-p_2\}}\right]$$

where $u_i(t)$; i =1, 2, 3, 4 are control functions to be determined. Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization.

From (5) to (12), we obtain the error dynamics as follows:

$$\dot{e_1}=e_2+u_1(t) \tag{13}$$
$$\dot{e_2}=2\omega e_4+\omega^2 e_1+A_3-A_1+u_2(t) \tag{14}$$
$$\dot{e_3}=e_4+u_3(t) \tag{15}$$
$$\dot{e_4}=-2\omega e_2+\omega^2 e_3+A_4-A_2+u_4(t) \tag{16}$$

This above error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (13) to (16) which cannot be expressed as linear terms in e_i 's are eliminated:

$$u_1(t)=v_1(t)$$
$$u_2(t)=-A_3+A_1+v_2(t)$$
$$u_3(t)=v_3(t)$$
$$u_4(t)=-A_4+A_2+v_4(t)$$

The new error system can be expressed as:

$$\left.\begin{aligned}\dot{e_1}&=e_2+v_1(t)\\ \dot{e_2}&=2\omega e_4+\omega^2 e_1+v_2(t)\\ \dot{e_3}&=e_4+v_3(t)\\ \dot{e_4}&=-2\omega e_2+\omega^2 e_3+v_4(t)\end{aligned}\right\} \tag{17}$$

The error system (17) to be controlled is a linear system with a control input $v_i(t)$ $(i=1,\dots 4)$ as function of the error states e_i ($i=1,\dots 4$). As long as these feedbacks stabilize the system e_i $(i=1,\dots 4)$ converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control. There are many possible choice for the control $v_i(t)$ ($i=1,\dots 4$). We choose.

$$\begin{bmatrix}v_1(t)\\ v_2(t)\\ v_3(t)\\ v_4(t)\end{bmatrix}=A\begin{bmatrix}e_1\\ e_2\\ e_3\\ e_4\end{bmatrix} \tag{18}$$

Here A is a 4×4 constant matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (18) stable, proper choice of elements of A has to be made so that the system (18)

must have all eigen values with negative real parts. Choosing

$$A=\begin{bmatrix}-1 & -1 & 0 & 0\\ -\omega^2 & -1 & 0 & -2\omega\\ 0 & 0 & -1 & -1\\ 0 & 2\omega & -\omega^2 & -1\end{bmatrix} \tag{19}$$

and, defining a matrix B as

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{bmatrix} = B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{20}$$

Where B is

$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{21}$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t\to\infty}|e_i| = 0;\ i = 1, 2, 3, 4$ and hence, complete synchronization is achieved between the master and slave systems

3.1. Numerical Simulation

We select the parameters $\mu = .00230437$ and $\lambda = 1$ with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures.3, 5, 7, 9 and that of controlled system are shown in figures.4, 6, 8 and 10 for respectively.

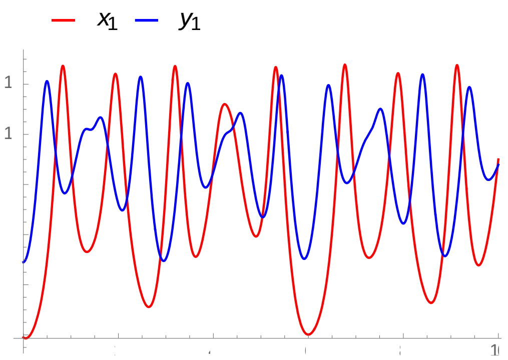

Figure-3

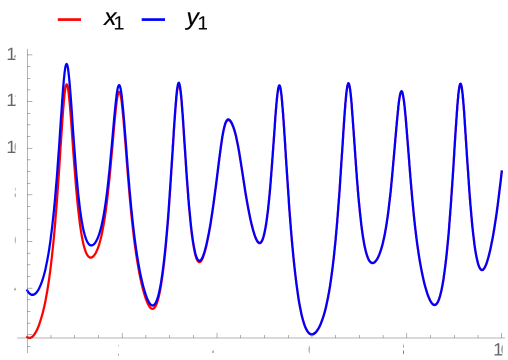

Figure-4

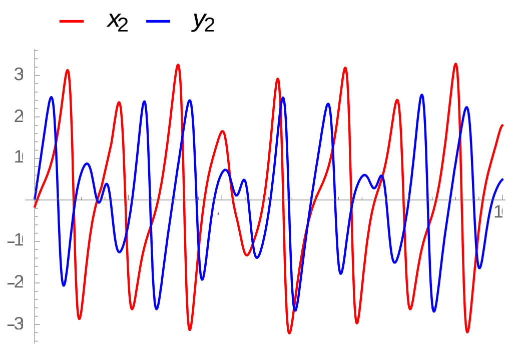

Figure-5

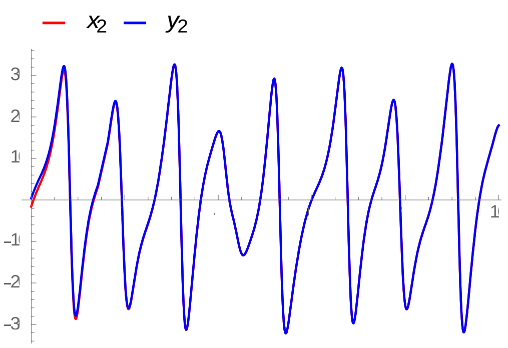

Figure-6

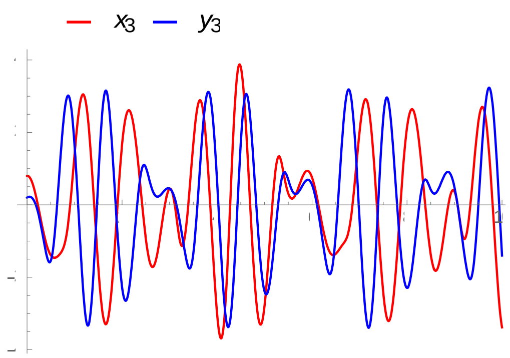

Figure-7

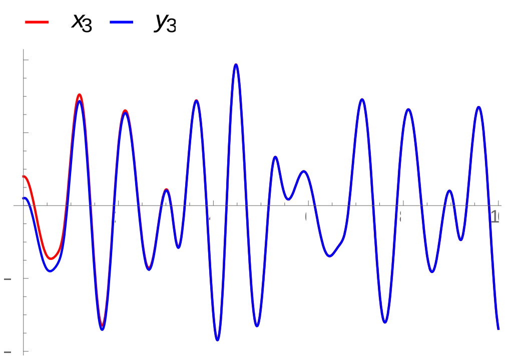

Figure-8

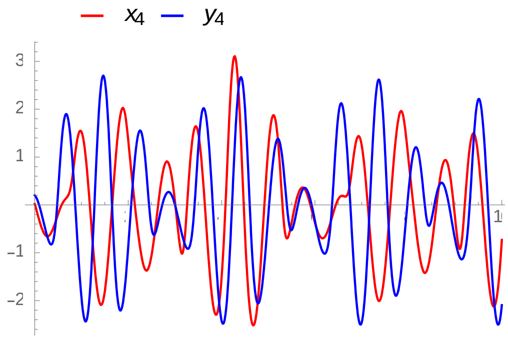

Figure-9

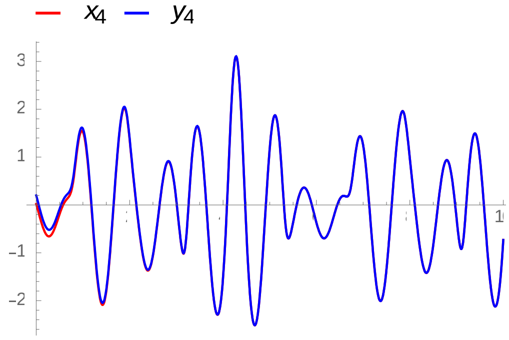

Figure-10

4. CONCLUSION

An investigation on complete synchronization in the planar restricted three problem by taking into consideration the small primary is ellipsoid and bigger primary an oblate spheroid and source of radiation, via active control technique based on Lyapunov stability theory and Routh-Hurwitz criteria have been made. Here two systems (master and slave) are compete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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