International Journal of Mathematical Archive-9(4), 2018, 151-162
Available online through www.ijma.info ISSN 2229 – 5046

STATIC INTERIOR PLANE SYMMETRIC FIVE DIMENSIONAL SOLUTIONS IN *f(R)* GRAVITY

L. S. LADKE
Nilkanthrao Shinde Science and Arts College, Bhandrawati, India.

R. D. MISHRA*
Shriram Commerce and Science College, Kurkheda, India.

S. R. GOMKAR
Janata Mahavidyalaya, Chandrapur, India.

(Received On: 04-02-18; Revised & Accepted On: 09-03-18)

ABSTRACT

In this paper, we investigate the physically acceptable exact solutions in five dimensional space-time in f(R) gravity which may provide some information about the nature of DE responsible for accelerated expansion of the universe. Considering six assumptions, energy density and pressure are evaluated by using conservation law of relativity. Ricci scalar and function of R are also evaluated in this paper.

Keywords: *f(R) theory, conservation law, Interior Solutions, five dimensional plane symmetric solutions.*

1. INTRODUCTION

The geometric properties of space-time are described by the most widely accepted fundamental theory of relativity. The evolution of the universe is described by freidmann equations which came from Einstein field equations. Also general theory of relativity describes the big-bang theory based on radiation. Many observations have confirmed the accelerated expansion of the universe and it is observed that general relativity is not sufficient to explain the problems regarding accelerated expansion of the universe. In order to explain the accelerating expansion of the universe, many modified theories of gravitation has been developed as alternative to Einstein general theory of relativity and *f(R)* gravity is one of the modified theory of gravity proposed during last decade. Weyl [1] in 1919 and Eddington [2] in 1922 studied the *f(R)* actions and later these actions were studied by Buchdhal [3] in the context of non-singular oscillating cosmologies. Many researchers are working in *f(R)* theory of gravity on different issues. Velerio Faraoni [4] studied the success and challenges of *f(R)* gravity. Lorenzo Sebastiani and Ratbay Myrzakulov [5] analyzed *f(R)* gravity and inflation. Stable high curvature limit and well behaved cosmological solutions with a proper era of matter domination are the choices of *f(R)* gravity and are investigated by Hu and Sawicki [6]Dark energy *f(R)* models are cosmologically viable are studied by Amendola *et al.* [7]. P.K. Agrawal [8] studied the vacuum solutions of FRW and axially symmetric space time in *f(R)* theory of gravity. M. Sharif *et al.* [9] explores plane symmetric vacuum solutions in *f(R)* gravity by using the assumption of constant scalar curvature which may be zero or non-zero. Zhang and Noh [10] found a new class of plane symmetric solution in the presence of perfect fluid. Farasat Shamir [11] studied the plane symmetric solution in *f(R)* gravity by using assumption of constant and non-constant scalar curvature. Morteza yavari [12] studied energy distribution for power model of the plane symmetric space time in *f(R)* gravity. Sebastiani, Lorenzo, and Sergio Zerbini [13] worked on Static spherically symmetric solutions in *f(R)* gravity. In addition to this work Sharif and Kausar [14] studied non-vacuum static spherically symmetric solutions by using *f(R)* theory of gravity. Dynamics of spherically symmetric gravitational collapse in *f(R)* theory of gravity are studied by M. Sharif *et al.* [15]. Ali Shojai *et al.* [16] investigated spherically symmetric solutions in *f(R)* theory of gravity and studied the equilibrium star. In metric *f(R)* theory of gravity Azadi *et al.* [17] analyzed cylindrically symmetric vacuum solutions. Momini *et al.* [18] studied constant curvature solutions in cylindrically symmetric metric *f(R)* gravity. The main aim of the cosmology is to study the large scale structure of the universe. The best representation of the large scale structure is Friedmann-Robertson-walker (FRW) models which are isotropic and spatially homogeneous in nature. Recent observation strongly recommended that the universe is in accelerating phase and dark energy is responsible for accelerating expansion of the universe [19, 20, 21, 22].

Corresponding Author: R. D. Mishra*,
Shriram Commerce and Science College, Kurkheda, India.

One of the most important task of modern science is to understand the physics behind the observed accelerating expansion of the universe. Recently several prominent results are obtained in the development of the superstring theory hence the study of higher dimensional physics is important. To study the hidden knowledge of the universe many researchers are interested to explore higher dimensions. Kaluza and Klein [23, 24] introduced an extra dimension and tried to unify gravity with electromagnetic interaction. Unification of fundamental force with gravity explored by Weinberg [25] which reveals the space time should be different from four therefore the concept of higher dimensions is not unphysical.M. Farasat Shamir *et al.* [26] studied $(n+1)$ dimension for vacuum solutions of static plane symmetry and using constant scalar curvature generalized field equations are studied in *f(R)* gravity. Recently, Ladke, L. S. *et al.* [27] studied higher dimensional plane symmetric solutions in *f(R)* theory of gravitation.

Dalgaty M.S.R. and KayllLake [28] gave the three physical acceptability criteria for *f(R)* gravity as i) isotropy of pressure ii) the monotonic decrease of the energy density and pressure with increasing radius iii) Subluminal sound speed i.e. $v_s{}^2 = \dfrac{dp}{d\rho} < 1$. He also gave the physical acceptability criteria for exact solutionsin general relativity. M. Sharif and Sadia Arif [29] construct some static cylindrically symmetric interior solutions in *f(R)* theory of gravity and discussed them by using physical acceptability criteria. Also M. Sharif and Zuriat Zahra [30] obtain the solutions in plane symmetry and that solutions are checked by using physical acceptability criteria. Motivating with the above work, in this paper, we construct some static interior plane symmetric five dimensional solutions in *f(R)* gravity. We explore these solutions in tabular form by considering six assumptions and applying physical acceptability measure to these solutions, some physical acceptable solutions are obtained.

2. $f(R)$ THEORY OF GRAVITY

The action for $f(R)$ theory of gravity are given by

$$S = \int \left(\frac{1}{16\pi G} f(R) + L_m \right) \sqrt{-g}\; d^5x, \tag{1}$$

where $f(R)$ is general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = kT_{ij}, \quad \text{(i, j=1,2,3,4,5)} \tag{2}$$

where $F(R) \equiv \dfrac{df(R)}{dR}$, $\quad \Box \equiv \nabla^i \nabla_i$

With ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

If we take $f(R) = R$, the field equation (2) in *f(R)* theory of gravity reduce to the field equation of general theory of relativity which is propose by Einstein.

Contracting the above field equations (2), we have

$$F(R)R - \frac{5}{2} f(R) + 4 \Box F(R) = kT \tag{3}$$

From (3), we get

$$f(R) = \frac{2}{5}\left[-KT + 4 \Box F(R) + F(R)R \right]. \tag{4}$$

Using equations (2) and (4), the field equations take the form

$$\frac{F(R)R_{ij} - \nabla_i \nabla_j F(R) - 8\pi T_{ij}}{g_{ij}} = \frac{1}{5}[F(R)R - \Box F(R) - KT]. \tag{5}$$

It follows that the equation (5) is not depend on the index i,

$$F(R)R_{ij} - \frac{1}{5}\left[F(R)R - \Box F(R) \right] g_{ij} - \nabla_i \nabla_j F(R) = K\left[T_{ij} - \frac{1}{5} T g_{ij} \right] \tag{6}$$

3. METRIC & THE FIELD EQUATIONS

The line element of plane symmetric space-Time

$$ds^2 = Adt^2 - B\left(dx^2 + dy^2 + dz^2\right) - Cd\phi^2 \tag{7}$$

Where A, B and C are functions of ϕ.

The Ricci scalar is

$$R = \frac{1}{X}\left[\ddot{A} + 3A\frac{\ddot{B}}{B} - \frac{\dot{X}}{X}\left(\dot{A} + 3A\frac{\dot{B}}{B}\right)\right] \tag{8}$$

Where $X(\phi) = A(\phi)C(\phi)$ and dot denotes derivative with respect to ϕ.

The Stress energy tensor can be written in the simple form

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{9}$$

Where $p = p(\phi)$ $\rho = \rho(\phi)$ and u_i is five velocity

The matter density is given by the scalar function ρ and Pressure p

Equation (6) leads to

$$\frac{2\ddot{F}}{F} + \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} - \frac{\dot{B}}{B}\frac{\dot{X}}{X} - \frac{\dot{F}\dot{X}}{FX} + \frac{KX}{A^2F}(p+\rho) = 0 \tag{10}$$

$$\frac{\ddot{A}}{2A} - \frac{\ddot{B}}{2B} + \left(\frac{\dot{X}}{4X} - \frac{2\dot{F}}{5F}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) - \frac{\dot{B}^2}{4B^2} - \frac{KX}{A^2F}(p+\rho) = 0 \tag{11}$$

By using conservation equation $T_{i\ ;j}^{\ j} = 0$ we get

$$\frac{\dot{A}}{A} = \frac{-2\dot{p}}{\rho + p} \tag{12}$$

We are using six types of assumptions to solve the field equations

No.	Assumptions	No.	Assumptions
1	$X = X_0\phi^l$, $A = A_0$, $F = F_0$	4	$X = X_0\phi^l$, $A = A_0\phi^m$, $F = F_0$
2	$X = X_0$, $A = A_0\phi^m$, $F = F_0$	5	$X = X_0\phi^l$, $A = A_0$, $F = F_0\phi^n$
3	$X = X_0$, $A = A_0$, $F = F_0\phi^n$	6	$X = X_0$, $A = A_0\phi^m$, $F = F_0\phi^n$

Where l, m, n, X_0, A_0 and F_0 are arbitrary constant

1. Solutions of Type I: $X = X_0\phi^l$, $A = A_0$, $F = F_0$

Using the values of case-I in equation (10) and equation (11), we get

$$-\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{2B^2} + \frac{\dot{B}l}{2B\phi} = \frac{KX_0\phi^l(p+\rho)}{A_0^{\ 2}F_0} \tag{13}$$

$$-\frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} + \frac{\dot{B}l}{4B\phi} = \frac{KX_0\phi^l(p+\rho)}{A_0^{\ 2}F_0} \tag{14}$$

From equation (13) and (14), we get

$$\frac{\ddot{B}}{2B}-\frac{3\dot{B}^2}{4B^2}-\frac{\dot{B}l}{4B\phi}=0 \tag{15}$$

Now taking $B=B_0\phi^m$ (16)

Substituting the value of equation (16) in equation (15), we get

$$m=-(l+2) \tag{17}$$

From the values of case-I, as $\frac{\dot{A}}{A}=0$ then equation (12), gives

$$p=p_0 \ \& \ \rho=-p_0 \tag{18}$$

where p_0 is constant

Using thevalue of equation (16) in equation (8), we get

$$R=\frac{3A_0 m}{X_0}\left[m-1-\frac{l}{2}\right]\phi^{-l-2} \tag{19}$$

Table-I (Type I solutions): $X=X_0\phi^l, A=A_0, F=F_0$

No.	l	m	$B(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s^{\ 2}$
i	0	-2	$B_0\phi^{-2}$	p_0	$-p_0$	$\frac{18A_0}{X_0}\phi^{-2}$	F_0R+f_0	0
ii	1	-3	$B_0\phi^{-3}$	p_0	$-p_0$	$\frac{81A_0}{2X_0}\phi^{-3}$	F_0R+f_0	0
iii	2	-4	$B_0\phi^{-4}$	p_0	$-p_0$	$\frac{72A_0}{X_0}\phi^{-4}$	F_0R+f_0	0
iv	-2	0	B_0	p_0	$-p_0$	0	f_0	0

All four solutions in Table-I of Type-I assumption have constant pressure and density and hence not physically acceptable

2. Solutions of Type II: $X=X_0, A=A_0\varphi^m, F=F_0$

Using the values of case-II in equation (10) and equation (11), we get

$$\frac{-\ddot{B}}{B}+\frac{\dot{B}^2}{2B^2}=\frac{KX(p+\rho)}{A^2F} \tag{20}$$

$$\frac{m(m-1)}{2\phi^2}\frac{-\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}=\frac{KX(p+\rho)}{A^2F} \tag{21}$$

From equations (20) and (21), we get

$$\frac{m(m-1)}{2\phi^2}+\frac{\ddot{B}}{2B}-\frac{3\dot{B}^2}{4B^2}=0 \tag{22}$$

Consider, $B=B_0\phi^n$ (23)

Substituting the value of equation (23) in equation (22), we get

$$n=m-1 \tag{24}$$

From the values of case-II, as $\frac{\dot{A}}{A}=\frac{m}{\phi}$ then equation (12), becomes

$$p+\rho=\frac{-2\dot{p}\phi}{m} \tag{25}$$

Using values of equation (23) and (25) in equation (10), we get

$$p=\frac{mn(n-2)A_0{}^2F_0}{4KX_0(2m-2)}\phi^{2m-2}+p_0 \tag{26}$$

$$\rho=-p_0-\frac{n(n-2)(5m-4)A_0{}^2F_0}{4KX_0(2m-2)}\phi^{2m-2} \tag{27}$$

where p_0 is constant

Using the value of equation (23) in equation (8), we get

$$R=\frac{A_0}{X_0}\left(m^2+3n^2-m-3n\right)\phi^{m-2} \tag{28}$$

Table-II (Type-II solutions): $X=X_0, A=A_0\phi^m, F=F_0$

No.	m	n	$A(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s{}^2$
i	2	1	$B_0\phi$	$p_0-\frac{1}{4}\frac{A_0{}^2F_0\varphi^2}{KX_0}$	$-p_0+\frac{3}{4}\frac{A_0{}^2F_0\varphi^2}{KX_0}$	$\frac{2A_0}{X_0}$	F_0R+f_0	$-\frac{1}{3}$
ii	-2	-3	$B_0\phi^{-3}$	$p_0+\frac{5}{4}\frac{A_0{}^2F_0\varphi^{-6}}{KX_0}$	$-p_0+\frac{35}{4}\frac{A_0{}^2F_0\varphi^{-6}}{KX_0}$	$\frac{42A_0}{2X_0}\phi^{-4}$	F_0R+f_0	$\frac{1}{7}$
iii	1	0	B_0	p_0	$-p_0$	0	f_0	0
iv	3	2	$B_0\phi^2$	p_0	$-p_0$	$\frac{12A_0}{X_0}\phi$	F_0R+f_0	0
v	-1	-2	$B_0\phi^{-2}$	$p_0+\frac{1}{2}\frac{A_0{}^2F_0\varphi^{-4}}{KX_0}$	$-p_0-\frac{9}{2}\frac{A_0{}^2F_0\varphi^{-4}}{KX_0}$	$\frac{20A_0}{X_0}\phi^{-3}$	F_0R+f_0	$-\frac{1}{9}$
vi	$\frac{1}{2}$	$-\frac{1}{2}$	$B_0\phi^{\frac{-1}{2}}$	$p_0-\frac{5}{32}\frac{A_0{}^2F_0\varphi}{KX_0}$	$-p_0-\frac{15}{32}\frac{A_0{}^2F_0\varphi}{KX_0}$	$\frac{2A_0}{X_0}\phi^{\frac{-3}{2}}$	F_0R+f_0	$\frac{1}{3}$

In Table-II of Type-II assumption,

It is found that the solutions (i) & (v) are not physically acceptable as the speed of sound should not be subluminal due to negative squared sound speed.

For solutions (iii) & (iv) the pressure and density are constant & hence not acceptable.

For Solutions of (ii) & (vi), pressure and density are decreasing with increasing ϕ and $0<v_s{}^2<1$ and hence physically acceptable

3. Solutions of Type III: $X=X_0, A=A_0, F=F_0\phi^n$

Using the values of case-III in equation (10) and equation (11), we get

$$\frac{-2n(n-1)}{\phi^2}-\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{2B^2}=\frac{KX(p+\rho)}{A^2F} \tag{29}$$

$$\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{2\dot{B}n}{5B\phi}=\frac{KX\,(p+\rho)}{A^2F} \tag{30}$$

From equation (29) and (30), we get

$$-2n\frac{(n-1)}{\phi^2}-\frac{\ddot{B}}{2B}+\frac{3\dot{B}^2}{4B^2}+\frac{2}{5}\frac{\dot{B}}{B}\frac{n}{\phi}=0 \tag{31}$$

Assuming, $B=B_0\phi^n$ (32)

Substituting the values of equation (32) in equation (31), we get

$$m=(-1-n)\pm\sqrt{1-n^2} \tag{33}$$

From the values of case-III, as $\frac{\dot{A}}{A}=0$ then equation (12), gives

$$p=p_0\;\;\rho=-p_0 \tag{34}$$

Where p_0 is constant

Using the value of equation (32) in equation (8), we get

$$R=\frac{3A_0m}{X_0}(m-1)\phi^{-2} \tag{35}$$

Table-III (Type III solutions): $X=X_0$, $A=A_0$, $F=F_0\varphi^n$

No.	m	n	$B(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s{}^2$
i	0	0	B_0	p_0	$-p_0$	0	f_0	0
ii	0	-2	$B_0\phi^{-2}$	p_0	$-p_0$	$\frac{18A_0}{2X_0}\phi^{-2}$	$F_0\varphi^{-2}R+f_0$	0
iii	1	-2	$B_0\phi^{-2}$	p_0	$-p_0$	$\frac{18A_0}{2X_0}\phi^{-2}$	$F_0\varphi^{-2}R+f_0$	0
iv	-1	0	B_0	p_0	$-p_0$	0	f_0	0
v	2	$-3\pm i\sqrt{3}$	$B_0\varphi^{-3\pm i\sqrt{3}}$	p_0	$-p_0$	$\frac{3A_0\left(-3\pm i\sqrt{3}\right)\left(-4\pm i\sqrt{3}\right)}{X_0\varphi^2}$	$F_0\varphi^{-3\pm i\sqrt{3}}R+f_0$	0

Pressure and density are constant for all solutions in Table-III of the Type-III assumption and hence not physically acceptable for the values of m and n.

4. Solutions of Type IV: $X=X_0\phi^l$, $A=A_0\phi^m$, $F=F_0$

Using the values of case-IV in equation (10) and equation (11), we get

$$\frac{-\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\dot{B}l}{2B\phi}=\frac{KX\left(p+\rho\right)}{A^2F} \tag{36}$$

$$\frac{m(m-1)}{\phi^2}-\frac{\ddot{B}}{2B}+\frac{l}{4\phi}\left(\frac{\dot{B}}{B}-\frac{m}{\phi}\right)-\frac{\dot{B}^2}{4B^2}=\frac{KX\left(p+\rho\right)}{A^2F} \tag{37}$$

From equation (36) and (37), we get

$$\frac{1}{2}\frac{m(m-1)}{\phi^2}+\frac{\ddot{B}}{2B}-\frac{l\dot{B}}{\phi B}-\frac{3\dot{B}^2}{4B^2}-\frac{1}{4}\frac{l}{\phi}\frac{m}{\phi}=0 \tag{38}$$

Now taking $B = B_0\phi^n$ (39)

Substituting the value of equation (39) in equation (38), we get

$$n = (1+l) \pm \sqrt{(l+1)^2 - m^2 + 2m(l+1)} \tag{40}$$

From the values case-IV, as $\frac{\dot{A}}{A} = \frac{m}{\phi}$ then equation (12), becomes

$$p + \rho = \frac{-2\dot{P}\phi}{m} \tag{41}$$

Using the value of equations (39) and (40) in equation (10), we get

$$p = \frac{(n^2 - 2n - nl)A_0^{\,2} mF_0}{4KX_0(2m-l-2)}\phi^{2m-l-2} + p_0 \tag{42}$$

$$\rho = -p_0 - \frac{n(n-2-l)(5m-2l-4)A_0^{\,2}F_0}{4KX_0(2m-l-2)}\phi^{2m-l-2} \tag{43}$$

Where p_0 is constant

Using the value of equation (39) in equation (8), we get

$$R = \frac{a_0}{X_0}\left(m^2 - m + 3n^2 - 3n - \frac{1}{2}l(m+3n)\right)\phi^{m-l-2} \tag{44}$$

Table-IV (Type IV solutions): $X = X_0\phi^l$, $A = A_0\phi^m$, $F = F_0$

No.	l	m	n	$B(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s^{\,2}$
i	0	0	2	$B_0\phi^2$	p_0	$-p_0$	$\frac{6A_0}{X_0}\phi^{-2}$	$F_0R + f_0$	0
ii	0	2	2	$B_0\phi^2$	p_0	$-p_0$	$\frac{8A_0}{X_0}$	$F_0R + f_0$	0
iii	0	2	0	B_0	p_0	$-p_0$	$\frac{2A_0}{X_0}$	$F_0R + f_0$	0
iv	1	$2\pm\sqrt{7}$	3	$B_0\phi^3$	p_0	$-p_0$	$\frac{A_0}{X_0}\left(\frac{(43\pm5\sqrt{7})}{2}\right)\varphi^{-1\pm\sqrt{7}}$	$F_0R + f_0$	0
v	1	4	4	$B_0\phi^4$	$p_0 + \frac{4}{5}\frac{A_0^{\,2}F_0}{KX_0}\varphi^5$	$-p_0 - \frac{14}{5}\frac{A_0^{\,2}F_0}{KX_0}\varphi^5$	$\frac{40A_0}{X_0}\phi$	$F_0R + f_0$	$-\frac{4}{9}$

Solutions of (i), (ii), (iii)&(iv)of Table-IV of Type-IV assumption having constant pressure and density hence not physically acceptable.

Solution (v) not physically acceptable due to negative squared sound speed.

5. Solutions of Type V: $X = X_0\phi^l$, $A = A_0$, $F = F_0\phi^n$

Using the values of case-V in equation (10) and equation (11), we get

$$\frac{-2n(n-1)}{\phi^2} - \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}l}{2B\phi} + \frac{nl}{\phi^2} = \frac{KX(p+\rho)}{A^2F} \tag{45}$$

$$\frac{\ddot{B}}{2B}+\frac{l\dot{B}}{4\phi B}-\frac{2n\dot{B}}{5\phi B}-\frac{\dot{B}^2}{4B^2}=\frac{KX\left(p+\rho\right)}{A^2F} \tag{46}$$

From equation (45) and (46), we get

$$\frac{2n(n-1)}{2\phi^2}+\frac{\ddot{B}}{2B}-\frac{3\dot{B}^2}{4B^2}-\frac{l\dot{B}}{4\phi B}-\frac{l}{\phi}\frac{n}{\phi}-\frac{2n\dot{B}}{5\phi B}=0 \tag{47}$$

By Assuming $B=B_0\phi^m$ (48)

Substituting the value of equation (48) in equation (47), we get

$$m=\frac{1}{2}\left[\left(\frac{l}{\phi}-4n-2\right)\pm\sqrt{-16n^2+16n+8nl+\left(2+4n-\frac{l}{2}\right)^2}\right] \tag{49}$$

From the values of case-V, as $\frac{\dot{A}}{A}=0$ then equation (12), gives

$p=p_0$ and $\rho=-p_0$ (50)

Where p_0 is constant

Using the value of equation (48) in equation (8), we get

$$R=\frac{3A_0m}{X_0}\left(m-1-\frac{l}{2}\right)\phi^{-l-2} \tag{51}$$

Table-V (Type V solutions): $X=X_0\phi^l$, $A=A_0$, $F=F_0\phi^n$

No.	l	m	n	$B(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s{}^2$
i	-8	2	-1	$B_0\phi^2$	p_0	$-p_0$	$\frac{30A_0}{X_0}\phi^6$	$F_0\phi^{-1}R+f_0$	0
ii	-8	-4	-1	$\frac{B_0}{\phi^4}$	p_0	$-p_0$	$\frac{12A_0}{X_0}\phi^6$	$F_0\phi^{-1}R+f_0$	0
iii	-4	0	-1	B_0	p_0	$-p_0$	0	f_0	0
iv	-2	0	0	B_0	p_0	$-p_0$	0	f_0	0
v	-2	-3	0	$\frac{B_0}{\phi^3}$	p_0	$-p_0$	$\frac{27A_0}{X_0}$	F_0R+f_0	0

All five solutions in Table-V of Type-V assumption not following the physical acceptability criteria as pressure and density are constant.

6. Solutions of Type VI: $X=X_0$, $A=A_0\varphi^m$, $F=F_0\varphi^n$

Using the values of case-VI in equation (10) and equation (11), we get

$$\frac{-2n(n-1)}{\phi^2}-\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{2B^2}=\frac{KX\,(p+\rho)}{A^2F} \tag{52}$$

$$\frac{-m(m-1)}{\phi^2}-\frac{\ddot{B}}{2B}-\frac{2n\dot{B}}{5\phi B}+\frac{2mn}{5\phi^2}-\frac{\dot{B}^2}{4B^2}=\frac{KX\,(p+\rho)}{A^2F} \tag{53}$$

From equation (52) and (53), we get

$$\frac{1}{2}\frac{m(m-1)}{\phi^2}+\frac{2n(n-1)}{\phi^2}+\frac{\ddot{B}}{2B}-\frac{3\dot{B}^2}{4B^2}-\frac{2\dot{B}n}{5B\phi}+\frac{2mn}{5\phi^2}=0 \tag{54}$$

Now taking $B = B_0\phi^l$ (55)

Substituting the value of equation (55) in equation (54), we get

$$l=-1-4n\pm\sqrt{2m^2-2m+8n^2-8n+8mn+(1+4n)^2} \tag{56}$$

From the values of case-VI as $\frac{\dot{A}}{A}=\frac{m}{\phi}$ then equation (12), becomes

$$p+\rho=\frac{-2\dot{p}\phi}{m} \tag{57}$$

Put the value of (55) and (57) in equation (10), we get

$$p=\frac{m\left[4n(n-1)+2l(l-1)-l^2\right]A_0^{\,2}F_0}{4KX_0(2m+n-2)}\phi^{2m+n-2}+p_0 \tag{58}$$

$$\rho=-p_0-\left[\frac{4n(n-1)+2l(l-1)-l^2}{4KX_0}\right]\left(\frac{5m+2n-4}{2m+n-2}\right)A_0^{\,2}F_0\phi^{2m+n-2} \tag{59}$$

Where p_0 is constant

Using the value of equation (55) in equation (8), we get

$$R=\frac{A_0}{X_0}\left[m(m-1)+3l(l-1)\right]\phi^{m-2} \tag{60}$$

Table-VI (Type VI solutions): $X = X_0, A = A_0\phi^m, F = F_0\phi^n$

No.	l	m	n	$B(\phi)$	$p(\phi)$	$\rho(\phi)$	$R(\phi)$	f	$v_s^{\,2}$
i	-2	0	-1	$B_0\phi^{-2}$	p_0	$-p_0-\frac{8A_0^{\,2}F_0}{KX_0}\varphi^{-3}$	$\frac{18A_0}{X_0}\phi^{-2}$	$F_0R^{\frac{3}{2}}\left(\frac{X_0}{18a_0}\right)^{\frac{1}{2}}+f_0$	0
ii	-2	5	-1	$B_0\phi^{-2}$	$p_0+\frac{20A_0^{\,2}F_0}{KX_0}\varphi^{7}$	$-p_0-\frac{76A_0^{\,2}F_0}{7KX_0}\varphi^{7}$	$\frac{38A_0}{X_0}\phi^{3}$	$F_0R^{\frac{2}{3}}\left(\frac{38a_0}{18X_0}\right)^{\frac{1}{2}}+f_0$	$\frac{-35}{19}$
iii	-2	0	0	$B_0\phi^{-2}$	p_0	$-p_0-\frac{4A_0^{\,2}F_0}{KX_0}\varphi^{-2}$	$\frac{18A_0}{X_0}\phi^{-2}$	$F_0R^{\frac{3}{2}}\left(\frac{X_0}{18a_0}\right)^{\frac{1}{2}}+f_0$	0
iv	-2	-1	0	$B_0\phi^{-2}$	$p_0+\frac{A_0^{\,2}F_0}{2KX_0}\varphi^{-4}$	$-p_0+\frac{9A_0^{\,2}F_0}{2KX_0}\varphi^{-4}$	$\frac{20A_0}{X_0}\phi^{-3}$	$F_0\left(\frac{20a_0}{X_0}\right)\varphi^{-3}+f_0$	$\frac{4}{9}$

In Table-VI of Type-VI assumption,

Solutions (i) & (iii) are not physically acceptable as pressure and density not decreasing with increasing ϕ .Solution (ii) having negative squared sound speed hence not acceptable. Solution (iv) is physically acceptable as satisfying the physical acceptability criteria such as pressure and density are decreasing with increasing ϕ and $0<v_s^{\,2}<1$

SUMMARY OF RESULT AND DISCUSSION

This paper is devoted to study some static interior plane symmetric five dimensional solutions in *f(R)* theory of gravity. Here we used the matter as perfect fluid and equations obtained here are highly nonlinear which can not be solved analytically without using any technique. Here by taking six assumptions we have found some solutions and physical acceptability measure is applied to obtain some physical acceptable solutions. Also we have evaluated energy density and pressure.

The result is as follows.

i) In this assumption, four solutions are given but none of them are physically acceptable as pressure and density are constant

ii) In table II, Out of six solutions, two are physically acceptable.

iii) For the solutions of the type III, all five solutions are not physically acceptable.

iv) In table IV, None of the solutions satisfies physical acceptable measure.

v) In these solutions, pressure and density are constant and therefore solutions are not physically acceptable.

vi) Out of four solution of sixth type, solution number (iv) is physically acceptable

Here we have found 29 solutions out of which only three solutions are physically acceptable. These three physical acceptable solutions may be useful to explore the nature of the dark energy which is responsible for accelerated expansion of the nature.

REFERENCES

1. Weyl, Hermann. "Eineneueerweiterung der relativitaetstheorie."*Annalen der Physik*364.10 (1919): 101-133.
2. Eddington, A.S.: The Mathematical Theory of Relativity (Cambridge University Press, Cambridge, 1923).
3. Buchdahl, Hans A. "Non-linear Lagrangians and cosmological theory." *Monthly Notices of the Royal Astronomical Society* 150.1(1970): 1-8.
4. Valerio Faraoni.: “f(R) gravity: successes and challenges”, arXiv:0810.2602v1 [gr-qc] 15 Oct 2008**.**
5. Lorenzo Sebastiani and RatbayMyrzakulov, “F(R)-gravity and inflation”, arXiv:1506.05330v3 [gr-qc] 16 Nov 2015
6. 6)Sawicki, Ignacy, and Wayne Hu. "Stability of cosmological solutions in f (R) models of gravity." *Physical Review D* 75.12 (2007): 127502.
7. Amendola, Luca, et al. "Conditions for the cosmological viability of f (R) dark energy models." *Physical Review D* 75.8 (2007): 083504.
8. P. K. Agrawal, D. D. Pawar.: “Vacuum Solutions of FRW and Axially Symmetric space-time in f(R) Theory of Gravity”, arXiv:1703.06258v1 [gr-qc] 18 Mar 2017
9. Sharif, M., and M. Farasat Shamir. "plane symmetric solutions in f (R) gravity." *Modern Physics Letters A* 25.15 (2010): 1281-1288.
10. Zhang, Hongsheng, and Hyerim Noh. "n-dimensional plane symmetric solution with perfect fluid source." *Physics Letters B* 671.4-5 (2009): 428-434.
11. M. Farasat Shamir. “Exploring Plane Symmetric Solutions in f(R) Gravity.” arXiv:1706.04435v1 [gr-qc] 13 Jun 2017
12. Yavari, Morteza. "Energy Distribution for a Power Model of the Plane Symmetric Spacetime in f(R) Gravity. " *Communications in Theoretical Physics* 60.3 (2013): 323.
13. Sebastiani, Lorenzo, and Sergio Zerbini. "Static spherically symmetric solutions in F (R) gravity." *The European Physical Journal C* 71.3 (2011): 1591.
14. Sharif, Muhammad, and HafizaRizwanaKausar. "Dust static spherically symmetric solution in f(R) gravity." *Journal of the Physical Society of Japan* 80.4 (2011): 044004.
15. Sharif, M., and H. RizwanaKausar. "Dynamics of spherically symmetric gravitational collapse in f(R) theory." *Journal of Physics: Conference Series*. Vol. 354. No. 1. IOP Publishing, 2012.
16. Shojai, Ali, and Fatimah Shojai. "Some static spherically symmetric interior solutions off(R) gravity." *General Relativity and Gravitation* 44.1 (2012): 211-223.
17. Azadi, A., D. Momeni, and M. Nouri-Zonoz. "Cylindrical solutions in metric f (R) gravity." *Physics Letters B* 670.3 (2008): 210-214.
18. Momeni, D., and H. Gholizade. "A note on constant curvature solutions in cylindrically symmetric metric f (R) Gravity." *International Journal of Modern Physics D* 18.11 (2009): 1719-1729.
19. Perlmutter, S., *et al.* (1997) Measurement of the cosmological parameters Ω and Λ from the first seven supernovae at $z \geq 0.35$.The Astrophysical Journal, 483, 565.
20. Perlmutter, S., et al. (1998) Discovery of Supernovae Explosion at Half the Age of the Universe. Nature, 391, 51-54.http://dx.doi.org/10.1038/34124.

21. Reiss, A.G., et al. (1998) Observational Evidence from super-novae for an Accelerating Universe and a Cosmological Constant. The Astrophysical Journal, 116, 1009-1038.
22. Perlmutter, S. et al. (1999) Measurement of and 42 high-Redshift Supernovae. The Astrophysical Journal, 517, 565-586.http://dx.doi.org/10.1086/307221
23. Kaluza, Theodor. "Zumunitätsproblem der physik." *Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math.Phys.)* 1921. 966972 (1921): 45.
24. Klein, Oskar. "Quantum theory and five-dimensional relativity theory." *The Oskar Klein Memorial Lectures: 1988–1999* (2014): 69.
25. Piran, Tsvi, and Steven Weinberg. *Physics in higher dimensions*. No. CONF-8412132--.Teaneck, NJ (USA); World Scientific Pub.Co., 1986.
26. Shamir, M. Farasat, and AdilJhangeer. "A Note on Plane Symmetric Solutions in f (R) Gravity." *International Journal of Theoretical Physics* 52.7 (2013): 2326-2328.
27. Ladke, L. S., and R. D. Mishra. "Higher Dimensional Plane Symmetric Solutions in f (R) Theory of Gravitation." *Prespacetime Journal* 8.5 (2017).
28. Delgaty, M. S. R., and Kayll Lake. "Physical acceptability of isolated, static, spherically symmetric, perfect fluid solutions of Einstein's equations." *Computer Physics Communications* 115.2-3 (1998): 395-415.
29. Sharif, M., and SadiaArif."Non-vacuum static cylindrically symmetric solution and energy distribution in f (R) gravity."*Astrophysics and Space Science* 342.1 (2012): 237-243.
30. M. Sharif and Zuriat Zahra.: “static plane symmetric interior solutions in f(R) theory of gravity”, Modern Physics Letters A, Vol. 28, No. 10 (2013) 1350041 (15 pages) c World Scientific Publishing Company, DOI: 10.1142/S0217732313500417

Source of support: Nil, Conflict of interest: None Declared.