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ONE POINT UNION OF TAIL GRAPHS FOR CORDIAL LABELING AND INVARIANCE

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ABSTRACT

One point union of k copies of graph G i.e. $G^{(k)}$ are obtained by fusing k copies of G at the same fixed point on G. If we change this point of fusion we may get different structures up to isomorphism. We have taken G = tail (C_5, P_2), tail (C_5,P_3), tail(C_5,$2p_2$), tail(C_5,P_4), tail(C_5, P_2, P_3), tail(C_5,$3P_2$) and also have split tail P_k in sub tails of shorter length whose sum will be k-1 edges. We obtained all possible structures on $G^{(k)}$. We show that all these structures are cordial.

Key words: *tail, cordial, one point union, fusion of vertex, structures, isomorphism.*

Subject Classification: *05C78.*

1. INTRODUCTION

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West. [9]. I.Cahit introduced the concept of cordial labeling [5]. f:V(G)$\rightarrow${0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e. $v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e. $e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that: every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian[8].

2. PRELIMINARIES

2.1 Fusion of vertex. Let G be a (p, q) graph. Let $u \neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. [9]

2.2 A tail graph (also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail (G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail (G, tp_k). If G is a (p, q) graph and a tail P_k is attached to it then tail (G, P_k) has p + k - 1 vertices and q + k - 1 edges

2.3 $G^{(k)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1) + 1$ and $|E(G)| = k.q$

3. THEOREMS PROVED

3.1 Theorem: $G^{(k)}$ is cordial where G = tail(C_5, P_2)

Proof: Define a function f: V(G)$\rightarrow${0,1} as follows. It introduces three types of labeling units as given below. Type C and Type B are not cordial while Type A is cordial. We combine them suitably to obtain a labeled copy of $G^{(k)}$.

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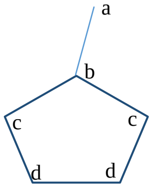

Fig 3.1 G = tail(C_5,P_2)

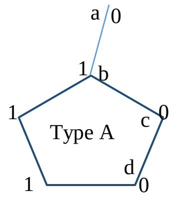

Fig 3.2 $v_f(0,1)$ =(3,3), $e_f(0,1)$=(3,3)

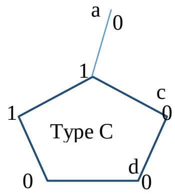

Fig 3.3 $v_f(0,1)$ =(4,2), $e_f(0,1)$=(3,3)

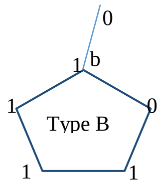

Fig 3.4 $v_f(0,1)$ =(2,4), $e_f(0,1)$=(3,3)

To construct one point union we can use any of the points ‘a’, ‘b’, ‘c’, or ‘d’ .Depending on it structure 1 through structure 4 are obtained.

To obtain **structure 1** we fuse vertex ‘a’ from type A with vertex ‘a’ from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1$ (mod 2) and type C if $i \equiv 0$ (mod 2), i = 1, 2,..k.

To obtain **structure 3** we fuse vertex ‘c’ from type A with vertex ‘c’ from type C. In $G^{(k)}$ the i^{th} copy is of type A if $i \equiv 1$ (mod 2) and type C if $i \equiv 0$ (mod 2), i = 1, 2,..k.

To obtain **structure 4** we fuse vertex ‘d’ from type A with vertex ‘d’ from type C. In $G^{(k)}$ the i^{th} copy is of type A if $i \equiv 1$ (mod 2) and type C if $i \equiv 0$ (mod 2), i = 1, 2,..k.

To obtain **structure 2** we fuse vertex ‘b’ from type A with vertex ‘b’ from type B. In $G^{(k)}$ the i^{th} copy is of type A if $i \equiv 1$ (mod 2) and type B if $i \equiv 0$ (mod 2), i = 1, 2,..k.

The resultant label numbers for **structure 1, structure 3** and **structure 4** are : on vertices $v_f(0, 1) = (3 + 5x, 3 + 5x)$ and $e_f(0, 1) = (3k, 3k)$ when k is of type 2x+1, x = 0, 1, 2,…and $v_f(0, 1) = (1 + 5x, 5x)$ and $e_f(0, 1) = (3k, 3k)$ when k is of type 2x, x = 1, 2, ,,.

For **structure 2** we have labelnumbers, on vertices $v_f(0, 1) = (3 + 5x, 3 + 5x)$ and $e_f(0, 1) = (3k,3k)$ when k is of type 2x + 1, x = 0, 1, 2,…and $v_f(0, 1) = (5x, 5x + 1)$ and $e_f(0, 1) = (3k, 3k)$ when k is of type 2x, x= 1, 2, ,,.

Theorem 3.2: $G^{(k)}$ is cordial where G = tail(C_5, P_3).

Proof: Define a function f: V(G)→{0, 1} as follows. It introduces five types of labeling units as given below. All are cordial. We combine them suitably to obtain a labeled copy of $G^{(k)}$.

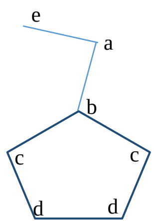

Fig 3.5 G = tail(C_5,P_3)

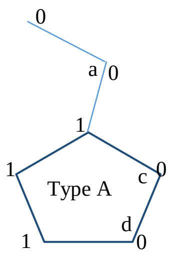

Fig 3.6 $v_f(0,1)$ =(4,3), $e_f(0,1)$=(4,3)

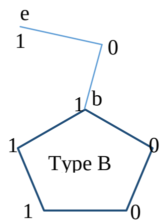

Fig 3.7 $v_f(0,1)$ =(3,4), $e_f(0,1)$=(3,4)

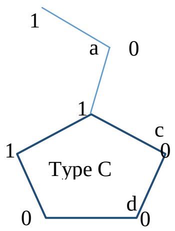

Fig 3.8 $v_f(0,1)$ =(4,3), $e_f(0,1)$=(3,4)

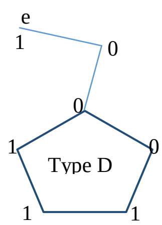

Fig 3.9 $v_f(0,1)$ =(3,4), $e_f(0,1)$=(4,3)

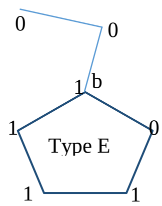

Fig 3.10 $v_f(0,1)$ = (3,4), $e_f(0,1)$=(4,3)

To obtain **structure 1** we fuse vertex 'e' from type B with vertex 'e' from type D. In $G^{(k)}$ the i^{th} copy is type B if $i \equiv 1 \pmod 2$ and type D if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 3** we fuse vertex 'b' from type B with vertex 'b' from type E. In $G^{(k)}$ the i^{th} copy is type B if $i \equiv 1 \pmod 2$ and type E if $i \equiv 0 \pmod 2$, i = 1, 2,..k. For above two structures the label numbers are: on vertices $v_f(0, 1) = (3 + 6x, 4 + 6x)$ and $e_f(0, 1) = (3 + 7x, 4 + 7x)$ when k is of type 2x + 1, x= 0, 1, 2,…and $v_f(0, 1) = (6x, 6x+1)$ and $e_f(0,1) = (7x, 7x)$ when k is of type 2x, x = 1, 2, ,,.

To obtain **structure 2** we fuse vertex 'a' from type C with vertex 'a' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 4** we fuse vertex 'c' from type C with vertex 'c' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 5** we fuse vertex 'd' from type C with vertex 'd' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above three structures the label numbers are: on vertices $v_f(0, 1) = (4 + 6x, 3 + 6x)$ and $e_f(0, 1) = (3 + 7x, 4 + 7x)$ when k is of type 2x+1, x= 0, 1, 2,…and $v_f(0, 1) = (6x+1, 6x)$ and $e_f(0, 1) = (7x, 7x)$ when k is of type 2x, x= 1, 2, ,,. The graph is cordial

Theorem 3.3: Let G = (C_5, p_2, p_2), a two tailed graph with each tail a p_2 copy, then all structures of $G^{(k)}$ are cordial.

Proof: Define f:V(G)→{0,1} that introduces four types of labelings as given below. All are cordial but differ in labeling pattern.

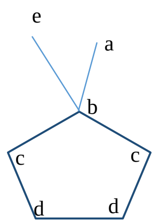

Fig 3.11 G = tail(C_5,P_3)

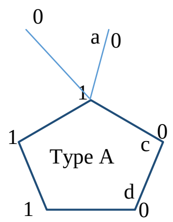

Fig 3.12 $v_f(0,1) = (4,3)$, $e_f(0,1)=(3,4)$

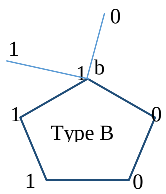

Fig 3.13 $v_f(0,1) = (3,4)$, $e_f(0,1)=(3,4)$

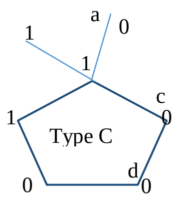

Fig 3.14 $v_f(0,1) = (4,3)$, $e_f(0,1)=(4,3)$

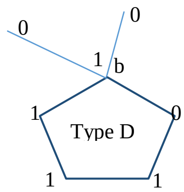

Fig 3.15 $v_f(0,1)$ = (3,4), $e_f(0,1)=(4,3)$

We obtain different structures on $G^{(k)}$ depending on which point on G is used to fuse to obtain one point union. We can take one point union at 'e' or 'a', 'b', 'c', or 'd'(refer fig 4.5). This will produce four different structures given by structure 1, structure2, structure3, structure 4 respectively.

To obtain **structure 1** we fuse vertex 'a' from type C with vertex 'a' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k

To obtain **structure 3** we fuse vertex 'c' from type C with vertex 'c' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 4** we fuse vertex 'd' from type C with vertex 'd' from type A. In $G^{(k)}$ the i^{th} copy is type C if $i \equiv 1 \pmod 2$ and type A if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above three structures the label numbers are: on vertices $v_f(0, 1) = (4 + 6x, 3 + 6x)$ and $e_f(0, 1) = (4 + 7x, 3 + 7x)$ when k is of type 2x + 1, x = 0, 1, 2,…and $v_f(0, 1) = (6x + 1, 6x)$ and $e_f(0, 1) = (7x, 7x)$ when k is of type 2x, x = 1, 2, ,,.

To obtain **structure 2** we fuse vertex 'b' from type B with vertex 'b' from type D. In $G^{(k)}$ the i^{th} copy is type B if $i \equiv 1 \pmod 2$ and type D if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For structures 2 the label distribution is given by : on vertices $v_f(0, 1) = (3 + 7x, 4 + 7x)$ and on edges $e_f(0, 1) = (3x, 4x)$ when $k = 2x + 1$, $x= 0, 1, 2..$ and when k is of type $k = 2x$, on vertices we have $v_f(0, 1) = (7x, 7x+1)$ and on edges $e_f(0, 1) = (7x, 7x)$. The graph is cordial.

Theorem 3.4: For $G =(C_5, p_4)$ all structures of $G^{(k)}$ are cordial.

Proof: Define $f:V(G)\rightarrow\{0,1\}$ that introduces three types of labelings as given below.Of which only type A is cordial.

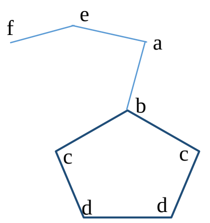

Fig 3.16 G = tail(C_5,P_4)

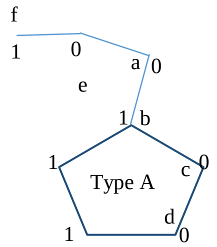

Fig 3.17 $v_f(0,1) =(4,4)$, $e_f(0,1)=(4,4)$

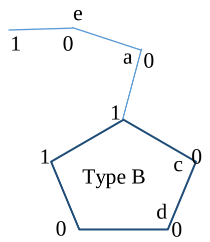

Fig 3.18 $v_f(0,1) =(5,3)$, $e_f(0,1)=(4,4)$

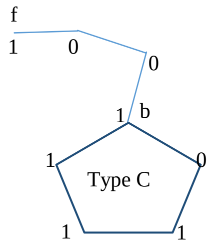

Fig 3.19 $v_f(0,1) =(4,4)$, $e_f(0,1)=(4,4)$

We obtain different structures on $G^{(k)}$ depending on any of the vertices ‘f’, ‘e’, ‘a’, ‘b’, ‘c’, ‘d’ (refer fig 4.5) of G used to fuse to obtain one point union. We can take one point union at).This will produce six different structures given by structure 1, structure 2, tructure3, structure 4, structure 5 and structure 6 respectively.

To obtain **structure 2** we fuse vertex ‘e’ from type A with vertex ‘e’ from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type B if $i \equiv 0 \pmod 2$, i = 1, 2,..k

To obtain **structure 3** we fuse vertex ‘a’ from type A with vertex ‘a’ from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type B if $i\equiv0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 5** we fuse vertex ‘c’ from type A with vertex ‘c’ from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type B if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above four structures the label numbers are: on vertices $v_f(0, 1) = (4 + 7x, 7 + 7x)$ and $e_f(0, 1) = (4x, 4x)$ when k is of type $2x + 1$, x = 0, 1, 2,…and $v_f(0, 1) = (7x+1, 7x)$ and $e_f(0, 1) = (8x, 8x)$ when k is of type 2x, x = 1, 2, , ,.

To obtain **structure 1** we fuse vertex ‘f’ from type A with vertex ‘f’ from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 6** we fuse vertex ‘d’ from type A with vertex ‘d’ from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above two structures the label distribution is given by : on vertices $v_f(0, 1) = (4 + 7x, 4 + 7x)$ and on edges $e_f(0,1) = (4x, 4x)$ when $k = 2x+1$, x= 0, 1, 2.. and when k is of type $k = 2x$, on vertices we have $v_f(0, 1) = (7x, 7x+1)$ and on edges $e_f(0, 1) = (8x, 8x)$. The graph is cordial.

Theorem 3.5: Let G = tail(C_5, p_2, p_3), a two tailed graph with p_2 and p_3 as tail, then all structures of $G^{(k)}$ are cordial.

Define $f:V(G)\rightarrow\{0,1\}$ that introduces three types of labelings as given below of which only type A is cordial.

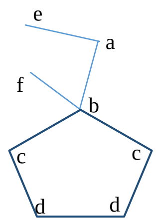

Fig 3.20 G = tail(C_5,P_2,P_3)

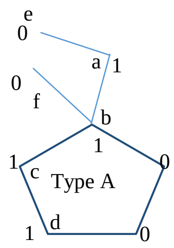

Fig 3.21 $v_f(0,1)$ =(4,4), $e_f(0,1)$=(4,4)

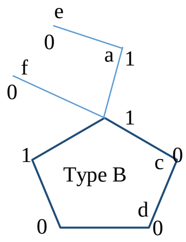

Fig 3.22 $v_f(0,1)$ =(5,3), $e_f(0,1)$=(4,4)

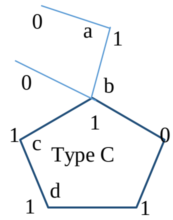

Fig 3.23 $v_f(0,1)$ =(3,5), $e_f(0,1)$=(4,4)

We obtain different structures on $G^{(k)}$ depending on any of the vertices 'e', 'a', 'b', 'f' 'c', 'd' (refer fig 4.5) of G used to fuse to obtain one point union.This will produce six different structures given by structure 1, structure 2, structure 3, structure 4, structure 5 and structure 6 respectively.

To obtain **structure 1** we fuse vertex 'e' from type A with vertex 'e' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type B if $i \equiv 0 \pmod 2$, i = 1, 2,..k

To obtain **structure 4** we fuse vertex 'f' from type A with vertex 'f' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type B if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above two structures the label numbers are: on vertices $v_f(0, 1) = (4 + 7x, 7 + 7x)$ and $e_f(0, 1) = (4x, 4x)$ when k is of type 2x+1, x= 0, 1, 2,…and $v_f(0, 1) = (7x+1, 7x)$ and $e_f(0, 1) = (8x, 8x)$ when k is of type 2x, x= 1, 2, ,,.

To obtain **structure 2** we fuse vertex 'a' from type A with vertex 'a' from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 3** we fuse vertex 'b' from type A with vertex 'b' from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 5** we fuse vertex 'c' from type A with vertex 'c' from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

To obtain **structure 6** we fuse vertex 'd' from type A with vertex 'd' from type C. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1 \pmod 2$ and type C if $i \equiv 0 \pmod 2$, i = 1, 2,..k.

For above two structures the label distribution is given by : on vertices $v_f(0, 1) = (4 + 7x, 4 + 7x)$ and on edges $e_f(0, 1) = (4x, 4x)$ when k = 2x+1, x = 0, 1, 2.. and when k is of type k = 2x, on vertices we have $v_f(0, 1) = (7x, 7x+1)$ and on edges $e_f(0, 1) = (8x, 8x)$. The graph is cordial.

Theorem 3. 6: Let G = tail(C_5, $3p_2$), a three tailed graph with each tail a p_2 copy, then all structures of $G^{(k)}$ are cordial.

Define f:V(G)$\rightarrow${0,1} that introduces two types of labelings as given below Of which only type A is cordial.

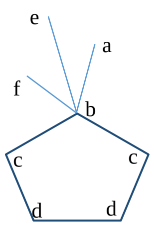

Fig 3.24 G = tail(C_5,3P_3)

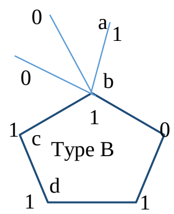

Fig 3.25 $v_f(0,1)$ =(3,5), $e_f(0,1)$=(4,4)

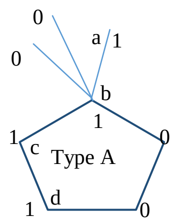

Fig 3.26 $v_f(0,1)$ =(4,4), $e_f(0,1)$=(4,4)

We obtain different structures on $G^{(k)}$ depending on any of the vertices 'a', 'b', 'c', 'd' (refer fig 4.5) of G used to fuse to obtain one point union.This will produce four different structures given by structure 1, structure 2, structure 3, structure 4 respectively.

To obtain **structure 1** we fuse vertex 'a' from type A with vertex 'a' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1$ (mod 2) and type B if $i \equiv 0$ (mod 2), i = 1, 2,..k

To obtain **structure 2** we fuse vertex 'b' from type A with vertex 'b' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1$ (mod 2) and type B if $i \equiv 0$ (mod 2), i = 1, 2,..k.

To obtain **structure 3** we fuse vertex 'c' from type A with vertex 'c' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1$ (mod 2) and type B if $i \equiv 0$ (mod 2), i = 1, 2,..k.

To obtain **structure 4** we fuse vertex 'd' from type A with vertex 'd' from type B. In $G^{(k)}$ the i^{th} copy is type A if $i \equiv 1$ (mod 2) and type B if $i \equiv 0$ (mod 2), i = 1, 2,..k.

For above structures the label numbers are: on vertices $v_f(0,1) = (4+7x,4+7x)$ and $e_f(0,1) = (4x,4x)$ when k is of type 2x+1, x= 0, 1, 2,…and $v_f(0, 1) = (7x, 7x+1)$ and $e_f(0, 1) = (8x, 8x)$ when k is of type 2x, x= 1, 2, ,,. Thus the graph is cordial.

CONCLUSIONS

We have defined tail graph tail(G, P_k), multiple tailgraph and have obtained their cordial labeling. We have taken $G = C_5$ and k = 2, 3, 4. Further we have obtained all possible structures of $G^{(k)}$ by changing common point of union and shown that all these structures are cordial.

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