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A STUDY ON FUZZY γ^* BOUNDARY SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce the concept of fuzzy γ^* boundary sets in fuzzy topological space.Some characterizations are discussed, examples are given and properties are established.

***Key words**: Fuzzy γ -semi open, fuzzy γ- semi closed, fuzzy γ -semi interior, fuzzy γ –semi closure, fuzzy γ^*-semi open and fuzzy γ^*-semi closed, γ boundary sets.*

I.INTRODUCTION

The concept of fuzzy sets operations were first introduced by L.A.Zadeh [2] in his paper. After that chang [1] defined and studied the notion of fuzzy topological space. Pu and Liu [6] defined the notion of fuzzy boundary in fuzzy topological spaces in 1980. Following this, Ahmad and Athar [5] studied their properties. They are also defined the concept of fuzzy semi boundary and the concept of fuzzy semi boundary and discussed their properties. In 2011, swidi and oon [9] introduce fuzzy closed set and discussed their properties. Usha *et al.* [8] defined the concept of fuzzy -semi closed sets in fuzzy topological spaces.

II. PRELIMINARIES

Through this paper (X,τ) and (Y,σ) denote fuzzy topological spaces. For a fuzzy set A in a fuzzy topological space $X\ or\ Y$. $cl(A), int(A), A^c$ denote the closure, interior, complement of A respectively. By 0_X and 1_X we mean the constant fuzzy sets taking on the values 0 and 1 respectively.

Definition 2.1 (Fuzzy Sets): A fuzzy set X is a function with domain X and values in I. That is an element of I^X. let $A \in I^X$. The subset of X in which A assume non-zero values is known as the support of A for every $x \in X$, $A(x)$ is called the grade of membership of x in A. And X is called carrier of the fuzzy set A. If A takes only 0 and 1, then A is a crisp set in X.

OPERATIONS OF THE FUZZY SETS

Definition 2.2: Let $A, B \in I^X$ we define the following fuzzy sets:
1. (Union) $A \vee B \in I^X$ by $(A \vee B)(x) = \max\{A(x), B(x)\}$ for every $x \in X$.
2. (Intersection) $A \wedge B \in I^X$ by $(A \wedge B)(x) = \min\{A(x), B(x)\}$ for every $x \in X$.
3. (Complement) $A^c \in I^X$ by $A^c(x) = 1 - A(x), \forall\, x \in X$.
4. Let $f : X \to Y$, $A \in I^X$ and $B \in I^Y$ then $f(A)$ is a fuzzy set in Y defined by
$$f(A)(y) = \begin{cases} \sup\{A(x): x \in f^{-1}(y)\} & if\ f^{-1}(y) \neq 0 \\ 0 & ;\ if\ f^{-1}(y) = 0 \end{cases}$$

Definition 2.3: A family $\tau \subseteq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms.
1. $\bar{0}, \bar{1} \in \tau$
2. $\forall\, A, B \in \tau \implies A \wedge B \in \tau$
3. $\forall \left(A_j\right)_{j\in J} \in \tau \implies \vee_{j\in J}\ A_j \in \tau$

The pair (X,τ) is called a fuzzy topological space (or) fts for short. The elements of τ are called fuzzy open sets.

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Example 2.4: Let $X = \{a, b, c\}$ and the topology is

$$\tau = \{0, 1, \{a_{.1}, b_{.2}, c_{.3}\}, \{a_{.5}, b_{.1}, c_{.4}\}, \{a_{.1}, b_{.1}, c_{.3}\}, \{a_{.5}, b_{.2}, c_{.4}\}\}$$

(i) $0 \in \tau$ and $1 \in \tau$.
(ii) $\{a_{.1}, b_{.2}, c_{.3}\} \vee \{a_{.1}, b_{.1}, c_{.3}\} = \{a_{.1}, b_{.2}, c_{.3}\} \in \tau$
(iii) $\{a_{.5}, b_{.1}, c_{.4}\} \wedge \{a_{.1}, b_{.1}, c_{.3}\} = \{a_{.1}, b_{.1}, c_{.3}\} \in \tau$
$\therefore$ τ is a fuzzy topology and the pair (x, τ) is called fuzzy topological space.

Definition 2.5: The closure $\bar{A}$ and the interior A° of a fuzzy set A of X are defined as $\bar{A} = \inf\{K : A \le k,\ k^c \in \tau\}$ and $A^\circ = \sup\{O : O \le A, O \in \tau\}$.

Definition 2.6: A fuzzy set A of (X, τ) is called

1. Fuzzy semi open (in short Fs open) if $A \le cl\left(Int\,(A)\right)$ and a fuzzy semi closed (in short Fs – closed) if $Int\left(cl\,(A)\right) \le A$.
2. Fuzzy preopen (in short Fp-open if $A \le Int\left(cl(A)\right)$ and a fuzzy pre closed (in short Fp-closed) if $cl\left(Int(A)\right) \le A$.
3. Fuzzy strongly semi open (in short Fss –open if $A \le int\left(cl(Int\ A)\right)$ and a fuzzy strongly semi closed (in short Fss-closed) $cl\ (Int\left(cl(A)\right) \le A$.
4. Fuzzy γ-open if $A \le \left(int\left(cl(A)\right)\right) \vee \left(cl(int\ A)\right)$ and fuzzy γ-closed if $cl\left(int\ (A)\right) \wedge int\left(cl(A)\right) \le A$.
5. Fuzzy γ^*- semi open if $int(A) \le cl\ (\gamma - int\ (A))$ and fuzzy γ^*- semi closed if $cl(A) \ge int\left(\gamma - cl(A)\right)$.

Definition 2.7: Let (X, τ) be a fuzzy topological space, then for a fuzzy subset A of X, the fuzzy γ^*- interior of A (briefly γ^*-$int(A)$) is union of all fuzzy γ^*-semi open sets of X contained in A. (i.e) $\gamma^* int(A) = \vee\{B : B \le A, B$ is fuzzy γ^*-semi open in $X\}$.

Definition 2.8: Let (X, τ) be a fuzzy topological space then for a fuzzy subset A of X, the fuzzy γ^*-closure of A (briefly $\gamma^* - cl(A)$) is the intersection of all fuzzy γ^*-semi closed sets of X contained in A. (i.e)γ^*- $cl(A) = \wedge\{B : B \ge A, B$ is fuzzy γ^*-semi closed set in $X\}$.

Lemma 2.9:
i) γ^*-cl(A) is fuzzy γ^* closed set
ii) γ^*int(A) is fuzzy γ^*open set
iii) $\gamma^* cl(A) \le \gamma^* cl(B)$ if A ≤ B
iv) $\gamma^* int(\gamma^* int\ A) = \gamma^*(A)$
v) $\gamma^* cl(\gamma^* clA) = \gamma^* cl(A)$
vi) $\gamma^* int(A \wedge \mathrm{B}) = \gamma^*\mathrm{int(A)} \wedge \gamma^*\mathrm{int(B)}$
vii) $\gamma^* int(A \vee \mathrm{B}) = \gamma^*\mathrm{int(A)} \vee \gamma^*\mathrm{int(B)}$
viii) $\gamma^* int(A \wedge \mathrm{B}) = \gamma^* cl(A) \wedge \gamma^* cl(B)$
ix) $\gamma^* int(A \vee \mathrm{B}) = \gamma^*\mathrm{cl(A)} \vee \gamma^*\mathrm{cl(B)}$

III. γ^*BOUNDARY SET

Definition 3.1: Let A be a fuzzy set in an fuzzy topological space (X, τ), then the fuzzy γ^*-boundary of A is defined as γ^*- Bd-(A) = γ^*cl(A) $\wedge$ γ^*cl(A^c)

Remark: In Fuzzy topology, we have A $\vee$ γ^*Bd(A) ≤ γ^*cl(A)

Example 3.2: This example not hold above remark
Let X = {a, b}, τ = {0,1,{a.$_3$,b.$_4$}},Then (x, τ) is a fuzzy topological space the collection of closed sets of τ is τ^c = {0,1,{a.$_7$,b.$_6$}}. Let taking A = {a.$_6$, b.$_4$}
Then γ^*cl (A) = {a.$_7$, b.$_4$} and γ^*-Bd (A) = {a.$_5$, b.$_4$}
ie) A γ^*Bd (A) = { a.$_6$, b.$_4$ } {a.$_5$,b.$_4$}={a.$_6$,b.$_4$}
ie) A γ^*Bd(A) ≠ γ^*-cl(A).

Proposition 3.3: Let A be a Fuzzy set in fuzzy topological space (X, τ), then following are hold
(i) γ^*-Bd(A) $=$ γ^*- Bd (A^c)
(ii) If A is fuzzy γ^*-closed set then γ^*-Bd (A) ≤ A
(iii) If A is fuzzy γ^*-open, then γ^*-Bd(A) ≤ A^c

Proof:
(i) By definition(3.1) γ^*-Bd(A) $=$ γ^*-cl(A) γ^*- cl (A^c) (1)
and γ^*- Bd(A^c) $=$ γ^*-cl(A^c) γ^*-cl (A) (2)

From (1) and (2) We have γ^*-Bd(A) $= \gamma^*$- Bd (A^c) Hence by (i)
(ii) Let A be γ^*-closed By Lemma (2.9) γ^*cl(A) = A
γ^*- Bd(A) $\le$ γ^*-cl(A) γ^*- cl (A^c) $\le \gamma^*$-cl(A) $\le$ A
$\therefore \gamma^*$-Bd(A) $\le$ A Hence (ii)

(iii) Again we using lemma(2.9) γ^*-int(A) = A ,Its follow that
γ^*-Bd(A) $\le$ γ^*-cl (A^c) $\le [\gamma^* - int(A)]^c \le A^c$. Hence by γ^*- Bd(A) $\le A^c$

Proposition 3.4: Let A be a fuzzy set in fuzzy topological space then following are hold.
(i) $[\gamma^* Bd(A)]^c$ = γ^*-int(A) γ^*int (A^c)
(ii) γ^*Bd(A) $\le$ Bd(A)
(iii) γ^*cl(γ^*- Bd(A)) $\le$ Bd(A)

Proof:
(i) By definition 3.1$[\gamma^* Bd(A)]^c$ = $[\gamma^*$cl(A) γ^* cl(A^c) $]^c$ = $[\gamma^* cl(A)]^c[\gamma^* cl(A^c)]^c$ = γ^*int(A^c)$\vee\gamma^*$int(A).
(ii) γ^*-cl(A) $\le$ cl(A) and γ^*-cl(A^c) $\le$ cl(A^c) we have
γ^*Bd(A) = γ^*cl(A)^γ^*cl$(A)^c$ $\le$ cl(A)cl(A^c) = Bd(A)
(iii) γ^*cl(γ^*Bd(A) = γ^*cl(γ^*cl(A) γ^*cl(A^c) $\le \gamma^*$ cl(γ^* cl(A) γ^*cl($\gamma^* cl(A^c)$)) = γ^*cl(A) γ^*cl(A^c) = γ^*Bd(A) $\le$ Bd(A)

Theorem 3.5: Let A and B be a fuzzy set's in a fuzzy topological space (X,τ) then γ^*Bd(A B) $\le \gamma^*$Bd(A) γ^*Bd(B).

Proof: By known Lemma 2.9 implies γ^*cl((AB) = γ^*cl(A) γ^*cl(B) and γ^*cl((AB)) $\le \gamma^*$cl(A) γ^*cl(B).It follows that γ^*Bd(AB) = γ^*cl(AB) γ^*cl$(AB)^c$ = γ^*cl(AB)γ^*cl(A^cB^c) $\le$ (γ^*cl(A) γ^*cl(B)) (γ^*cl(A^C) γ^*cl(B^C)) $\le$ (γ^*cl(A) γ^*cl(A^C)) $\vee$(γ^*cl(B) γ^*cl(B^C)) $\le \gamma^*$Bd(A) γ^*Bd(B).

Example 3.6: The reverse of in equality in above theorem is in general not true as shown by following example. Let X = {a, b} and τ= {0,1,{a.3,b.4}} Then (X,τ) is a fuzzy topological space.

The closed set is τ^c = {0,1,{a.7,b.6}} Let A = {a.3,b.6} and B = {a.4,b.5} then calculation give γ^*-Bd(A)={a.4,b.5} and γ^*-Bd(B)={a.6,b.5}. Now, AB = {a.4,a.6}, γ^*Bd(AB) = {a.5,b.5}

It follows that γ^*-Bd(A) $\vee$ γ^*Bd(B) ={a.6,b.5} $\not\le${a.5,b.5} = γ^*Bd(A$\vee$B) ie) γ^*Bd(A)$\wedge$ γ^*Bd(B) $\not\le$ γ^*Bd(A $\wedge$ B).

Theorem 3.7: For any fuzzy sets A and B in an fuzzy topological space (X, τ) one has γ^*Bd(A^B) $\le$ (γ^*Bd(A) γ^*cl(B)) (γ^*Bd(B) γ^*cl(A))

Proof: By known Lemma 2.9 γ^*-cl(AB) = γ^*cl(A) γ^*cl(B) and
γ^*-cl(A^B) = γ^*cl(A) γ^*cl(B) γ^*-Bd(A^B) = γ^*cl(A^B) γ^*cl($(AB)^C$
= γ^*cl(A^B) γ^*cl($A^C \vee B^C$) $\le$ (γ^*cl(A) γ^*cl(B))^(γ^*cl(A^C) γ^*cl(B^C)
= (γ^*cl(A) $\gamma^* c$l(B)^ γ^*cl(A^C))$\vee$ (γ^*cl(A) $\gamma^* c$l(B)) ^ γ^*cl(B^C)
= (γ^*Bd(A) $\gamma^* c$l(B))$\vee$(γ^*Bd(B) (γ^*cl(A)).
It is complete the proof

Theorem 3.8: For any fuzzy sets A in an fuzzy topological space (X, τ) are has
(i) γ^*Bd ($\gamma^* B$d(A)) $\le$ γ^*Bd(A)
(ii) γ^*Bd($\gamma^* Bd$(γ^*Bd(A)) $\le \gamma^*$Bd(γ^*Bd(A))

Proof:
(i) By Lemma (2.9) and definition 3.1 we have γ^*Bd ($\gamma^* B$d(A)) = γ^*cl(γ^*Bd(A))$\wedge$ γ^*cl (γ^*Bd$(A)^C$) $\le$ γ^*cl(γ^*Bd(A)) = γ^*cl(γ^*cl(A))^(γ^*cl(A^C) = γ^*cl(γ^*cl(A))^ γ^*cl (γ^*cl(A^C)) = γ^*cl(A) γ^*cl(A^C) = γ^*Bd(A) Hence by (i).
(ii) γ^*Bd(γ^*Bd ($\gamma^* B$d(A))) = $\gamma^* cl$(γ^*Bd (γ^*Bd(A))^ γ^*cl(γ^*Bd (γ^*Bd$(A))^C$) = γ^*Bd(γ^*Bd(A)^ γ^*cl(γ^*Bd γ^*Bd(A^C)) = γ^*Bd(γ^*Bd(A)) Hence by (ii).

CONCLUSION

Fuzzy γ- closed set and fuzzy γ- open set are the major role in fuzzy topology, Since its inception several weak forms of fuzzy γ-closed sets and fuzzy γ- open sets have been introduced in general fuzzy topology. The present paper is investigated in the new weak forms fuzzy γ^*- Boundary set in fuzzy topological spaces. Hence the propositions and theorems are justify the results. We hope that the findings in this paper will help researcher enhance and promote the further study on general fuzzy topology to carry out a general framework for their applications in practical life. This paper, not only can form the theoretical basis for further applications of fuzzy topology, but also lead to the development of information systems.

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