International Journal of Mathematical Archive-9(4), 2018, 125-135
Available online through www.ijma.info ISSN 2229 – 5046

INTUITIONISTIC FUZZY COSETS ON HX RING

R. MUTHURAJ[1] AND M. S. MUTHURAMAN*[2]

[1]P. G. & Research Department of Mathematics,
H. H. The Rajahs' College, Pudukkotai, India.

[2]Department of Mathematics,
P. S. N. A. College of Engineering & Technology, Dindigul, India.

(Received On: 03-03-18; Revised & Accepted On: 11-04-18)

ABSTRACT

In this paper, we introduce the notion of intuitionistic fuzzy coset, intuitionistic pseudo fuzzy coset of a HX ring. We discussed some related properties of intuitionistic fuzzy coset and intuitionistic pseudo fuzzy coset of intuitionistic fuzzy HX subring

Keywords: *intuitionistic fuzzy set, fuzzy HX ring, intuitionistic fuzzy coset and intuitionistic pseudo fuzzy coset of intuitionistic fuzzy HX subring, homomorphism and anti homomorphism of an intuitionistic fuzzy coset and intuitionistic pseudo fuzzy coset of intuitionistic fuzzy HX subring, image and pre-image of an intuitionistic fuzzy coset and intuitionistic pseudo fuzzy coset of intuitionistic fuzzy HX subring.*

1. INTRODUCTION

In 1965, Zadeh [13] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu [7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2, 3] gave the structures of HX ring on a class of ring. R.Muthuraj *et.al* [10]. introduced the concept of fuzzy HX ring. In this paper we we introduce the notion of intuitionistic fuzzy coset, intuitionistic pseudo fuzzy coset of a HX ring. We discussed some related properties of intuitionistic fuzzy coset and intuitionistic pseudo fuzzy coset of intuitionistic fuzzy HX subring

2. PRELIMINARY

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R, +, ·) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition [6]: Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation '+' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. $A + B = \{a + b / a \in A \text{ and } b \in B\}$, which its null element is denoted by Q, and the negative element of A is denoted by – A.
ii. $AB = \{ab / a \in A \text{ and } b \in B\}$,
iii. $A (B + C) = AB + AC$ and $(B + C) A = BA + CA$.

3.1 Intuitionistic fuzzy coset of a HX ring

In this section we define the concept of an intuitionistic fuzzy coset of a HX subring of a HX ring and discuss some related results.

Corresponding Author: M. S. Muthuraman[2], [2]Department of Mathematics,*
P. S. N. A. College of Engineering & Technology, Dindigul, India.

3.1.1 Definition: Let R be a ring. Let H = $\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be an intuitionistic fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring .An intuitionistic fuzzy subset $\lambda^H = \{\langle A, \lambda^\mu(A), \lambda^\eta(A)\rangle / A\in\Re$ and $0\le \lambda^\mu(A) + \lambda^\eta(A) \le 1\}$ of $\Re$ is called an intuitionistic fuzzy HX subring on $\Re$ or an intuitionistic fuzzy sub ring induced by H and if the following conditions are satisfied. For all A, B $\in \Re$,

i. $\lambda^\mu(A - B) \ge \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,
ii. $\lambda^\mu(AB) \ge \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,
iii. $\lambda^\eta(A- B) \le \max\{\lambda^\eta(A), \lambda^\eta(B)\}$,
iv. $\lambda^\eta(AB) \le \max\{\lambda^\eta(A), \lambda^\eta(B)\}$,

where $\lambda^\mu(A) = \max\{\mu(x) /$ for all $x\in A \subseteq R\}$ and $\lambda^\eta(A) = \min\{\eta(x) /$ for all $x\in A \subseteq R\}$.

3.1.2 Definition: Let R be a ring. Let H =$\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be an intuitionistic fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. Let $\lambda^H = \{\langle A,\lambda^\mu(A), \lambda^\eta(A)\rangle / A\in\Re$ and $0 \le\lambda^\mu(A)+\lambda^\eta(A) \le 1\}$ be an intuitionistic fuzzy HX subring of a HX ring $\Re$ and $A\in \Re$. Then the intuitionistic fuzzy coset of an intuitionistic fuzzy HX subring λ^H of a HX ring $\Re$ determined by the element $A\in \Re$ is denoted as $(A + \lambda^H)$ and is defined by $(A + \lambda^H)(X) = \{\langle X-A, \lambda^\mu(X-A),\lambda^\eta(X-A)\rangle\}$ for every $X\in \Re$.

3.1.3 Remark

i. If A = Q, then the intuitionistic fuzzy coset $A+ \lambda_H = \lambda_H$.
ii. If λ_H is an intuitionistic fuzzy HX subring of a HX ring $\Re$, and A = Q then the intuitionistic fuzzy coset $(A + \lambda_H)$ is also an intuitionistic fuzzy HX subring of a HX ring $\Re$.

3.1.4 Theorem: Let R be a ring. Let H be an intuitionistic fuzzy set defined on a ring R. Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$. For any $A, B\in \Re$, $(A + \lambda^H) = (B + \lambda^H)$ iff $\lambda^\mu(B-A) = \lambda^\mu(A-B) = \lambda^\mu(Q)$ and $\lambda^\eta(B-A) = \lambda^\eta(A-B) = \lambda^\eta(Q)$.

Proof: Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$. Let $(A + \lambda^H) = (B + \lambda^H)$, for any $A, B\in \Re$. Then, $(A + \lambda^\mu) = (B + \lambda^\mu)$ and $(A + \lambda^\eta) = (B + \lambda^\eta)$. For any $A, B\in \Re$,

i. $(A + \lambda^\mu)(B) = (B + \lambda^\mu)(B)$
$\lambda^\mu(B-A) = \lambda^\mu(B-B)$
$= \lambda^\mu(Q)$.
$\lambda^\mu(B-A) = \lambda^\mu(Q)$.
ii. $(A + \lambda^\mu)(A) = (B + \lambda^\mu)(A)$
$\lambda^\mu(A-A) = \lambda^\mu(A-B)$
$\lambda^\mu(Q) = \lambda^\mu(B-A)$.
iii. $(A + \lambda^\eta)(B) = (B + \lambda^\eta)(B)$
$\lambda^\eta(B-A) = \lambda^\eta(B-B)$
$= \lambda^\eta(Q)$.
$\lambda^\eta(B-A) = \lambda^\eta(Q)$.
iv. $(A + \lambda^\eta)(A) = (B + \lambda^\eta)(A)$
$\lambda^\eta(A-A) = \lambda^\eta(A-B)$
$\lambda^\eta(Q) = \lambda^\eta(B-A)$.

Hence, $\lambda^\mu(B-A) = \lambda^\mu(A-B) = \lambda^\mu(Q)$ and $\lambda^\eta(B-A) = \lambda^\eta(A-B) = \lambda^\eta(Q)$.

Conversely, let $\lambda^\mu(B-A) = \lambda^\mu(A-B) = \lambda^\mu(Q)$ and $\lambda^\eta(B-A) = \lambda^\eta(A-B) = \lambda^\eta(Q)$.

For any $X\in \Re$,
$$\begin{aligned}(A + \lambda^\mu)(X) &= \lambda^\mu(X-A)\\ &= \lambda^\mu(X-B+B-A)\\ &\ge \min\{\lambda^\mu(X-B), \lambda^\mu(B-A)\}\\ &= \min\{\lambda^\mu(X-B), \lambda^\mu(Q)\}\\ &= \lambda^\mu(X-B)\\ &= (B + \lambda^\mu)(X).\end{aligned}$$
$$(A + \lambda^\mu)(X) \ge (B + \lambda^\mu)(X).$$

Similarly, $(A + \lambda^\mu)(X) \le (B + \lambda^\mu)(X)$.

Hence, $(A + \lambda^\mu)(X) = (B + \lambda^\mu)(X)$.

Similarly, we have, $(A + \lambda^\eta)(X) = (B + \lambda^\eta)(X)$.

Hence, $(A + \lambda^H) = (B + \lambda^H)$.

3.1.5 Theorem: Let R be a ring. Let H be an intuitionistic fuzzy set defined on a ring R. Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$. For any A, B$\in \Re$, $(A + \lambda^H) = (B + \lambda^H)$, then $\lambda^\mu(A) = \lambda^\mu(B)$ and $\lambda^\eta(A) = \lambda^\eta(B)$.

Proof: Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$.

For any A, B$\in \Re$, $(A + \lambda^H) = (B + \lambda^H)$.

Now,
$$\begin{aligned}\lambda^\mu(A) &= \lambda^\mu((A-B)+B)\\ &\geq \min\{\lambda^\mu(A-B), \lambda^\mu(B)\}\\ &\geq \min\{\lambda^\mu(Q), \lambda^\mu(B)\}\\ &= \lambda^\mu(B).\\ \lambda^\mu(A) &\geq \lambda^\mu(B).\end{aligned}$$

Similarly, $\lambda^\mu(A) \leq \lambda^\mu(B)$.

Hence, $\lambda^\mu(A) = \lambda^\mu(B)$.

Now,
$$\begin{aligned}\lambda^\eta(A) &= \lambda^\eta((A-B)+B)\\ &\leq \max\{\lambda^\eta(A-B), \lambda^\eta(B)\}\\ &\leq \max\{\lambda^\eta(Q), \lambda^\mu(B)\}\\ &= \lambda^\eta(B).\\ \lambda^\eta(A) &\leq \lambda^\eta(B).\end{aligned}$$

Similarly, $\lambda^\eta(A) \geq \lambda^\eta(B)$.

Hence, $\lambda^\eta(A) = \lambda^\eta(B)$.

Hence, $\lambda^\mu(A) = \lambda^\mu(B)$ and $\lambda^\eta(A) = \lambda^\eta(B)$.

3.2 Homomorphism and anti homomorphism of an intuitionistic fuzzy coset of an intuitionistic fuzzy HX subring of a HX ring

In this section, we introduce the concept of an image, pre-image of intuitionistic fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring λ^H of a HX ring $\Re$ determined by the element A$\in \Re$.

3.2.1 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let $(A+\lambda^H)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring λ^H of a HX ring $\Re_1$ determined by the element A$\in \Re_1$.Then $f((A + \lambda^H))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^H)$ of a HX ring $\Re_2$ determined by the element $f(A)\in \Re_2$ and $f((A + \lambda^H)) = (f(A) + f(\lambda^H))$, if λ^H has a supremum property and λ^H is f-invariant.

Proof: Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings.

Let $(A+ \lambda^H)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring λ^H of a HX ring $\Re_1$ determined by the element A$\in \Re_1$.
Then, $f(\lambda^H) = \{\langle f(X), f(\lambda^\mu)(f(X)), f(\lambda^\eta)(f(X))\rangle / X\in\Re_1\}$.
There exist X, Y$\in \Re_1$ such that $f(X), f(Y) \in \Re_2$,

i. $$\begin{aligned}(f(\lambda^\mu))(f(X) - f(Y)) &= (f(\lambda^\mu))(f(X-Y)),\\ &= \lambda^\mu(X-Y)\\ &\geq \min\{\lambda^\mu(X), \lambda^\mu(Y)\}\\ &= \min\{(f(\lambda^\mu))(f(X)), (f(\lambda^\mu))(f(Y))\}\\ (f(\lambda^\mu))(f(X) - f(Y)) &\geq \min\{(f(\lambda^\mu))(f(X)), (f(\lambda^\mu))(f(Y))\}.\end{aligned}$$

ii. $$\begin{aligned}(f(\lambda^\mu))(f(X)f(Y)) &= (f(\lambda^\mu))(f(XY))\\ &= \lambda^\mu(XY)\\ &\geq \min\{\lambda^\mu(X), \lambda^\mu(Y)\}\\ &= \min\{(f(\lambda^\mu))(f(X)), (f(\lambda^\mu))(f(Y))\}\\ (f(\lambda^\mu))(f(X)f(Y)) &\geq \min\{(f(\lambda^\mu))(f(X)), (f(\lambda^\mu))(f(Y))\}\end{aligned}$$

iii. $(f(\lambda^{\eta}))(f(X) - f(Y)) = (f(\lambda^{\eta}))(f(X-Y))$,
$= \lambda^{\eta}(X-Y)$
$\leq \max\{\lambda^{\eta}(X), \lambda^{\eta}(Y)\}$
$= \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$
$(f(\lambda^{\eta}))(f(X) - f(Y)) \leq \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

iv. $(f(\lambda^{\eta}))(f(X)f(Y)) = (f(\lambda^{\eta}))(f(XY))$
$= \lambda^{\eta}(XY)$
$\leq \max\{\lambda^{\eta}(X), \lambda^{\eta}(Y)\}$
$= \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$
$(f(\lambda^{\eta}))(f(X)f(Y)) \leq \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

Hence, $f(\lambda^H)$ is an intuitionistic fuzzy HX subring of $\Re_2$. Then clearly $f((A + \lambda^H))$ is the intuitionistic fuzzy coset of an intuitionistic intuitionistic fuzzy HX ring $f(\lambda^H)$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$.

Let $A, X \in \Re_1$, then $f(A), f(X) \in \Re_2$.

Now, $(f(A) + f(\lambda^{\mu}))f(X) = (f(\lambda^{\mu}))(f(X) - f(A))$
$= (f(\lambda^{\mu}))(f(X - A))$
$= \lambda^{\mu}(X - A)$
$= (A + \lambda^{\mu})(X)$
$= f((A + \lambda^{\mu}))f(X)$.
$(f(A) + f(\lambda^{\mu}))f(X) = f((A + \lambda^{\mu}))f(X)$, for any $f(X) \in \Re_2$.

Hence, $f((A+\lambda^{\mu})) = (f(A) + f(\lambda^{\mu}))$.

Now, $(f(A) + f(\lambda^{\eta}))f(X) = (f(\lambda^{\eta}))(f(X) - f(A))$
$= (f(\lambda^{\eta}))(f(X - A))$
$= \lambda^{\eta}(X - A)$
$= (A + \lambda^{\eta})(X)$
$= f((A + \lambda^{\eta}))f(X)$.
$(f(A) + f(\lambda^{\eta}))f(X) = f((A + \lambda^{\eta}))f(X)$, for any $f(X) \in \Re_2$.

Hence, $f((A+\lambda^{\eta})) = (f(A) + f(\lambda^{\eta}))$.

Hence, $f((A + \lambda^H)) = (f(A) + f(\lambda^H))$.

3.2.2 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let $(B + \gamma^G)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\Re_2$ determined by the element $B \in \Re_2$.Then $f^{-1}((B + \gamma^G))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^G)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B+\gamma^G)) = (f^{-1}(B) + f^{-1}(\gamma^G))$.

Proof: Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings.

Let $(B + \gamma^G)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\Re_2$ determined by the element $B \in \Re_2$. Let $B = f(Y)$, $Y \in \Re_1$.
Then, $f^{-1}(\gamma^G) = \{ \langle X, f^{-1}(\gamma^{\alpha})(X), f^{-1}(\gamma^{\beta})(X) \rangle / X \in \Re_1 \}$.

For any $X, Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

i. $(f^{-1}(\gamma^{\alpha}))(X-Y) = \gamma^{\alpha}(f(X-Y))$
$= \gamma^{\alpha}(f(X) - f(Y))$
$\geq \min\{\gamma^{\alpha}(f(X)), \gamma^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$
$(f^{-1}(\gamma^{\alpha}))(X-Y) \geq \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$.

ii. $(f^{-1}(\gamma^{\alpha}))(XY) = \gamma^{\alpha}(f(XY))$
$= \gamma^{\alpha}(f(X)f(Y))$
$\geq \min\{\gamma^{\alpha}(f(X)), \gamma^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$
$(f^{-1}(\gamma^{\alpha}))(XY) \geq \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$

iii. $(f^{-1}(\gamma^{\beta}))(X-Y) = \gamma^{\beta}(f(X-Y))$
$= \gamma^{\beta}(f(X) - f(Y))$
$\leq \max\{\gamma^{\beta}(f(X)), \gamma^{\beta}(f(Y))\}$
$= \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$
$(f^{-1}(\gamma^{\beta}))(X-Y) \leq \max\{(f^{-1}(\eta^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}.$

iv. $(f^{-1}(\gamma^{\beta}))(XY) = \gamma^{\beta}(f(XY))$
$= \gamma^{\beta}(f(X)f(Y))$
$\leq \max\{\gamma^{\beta}(f(X)), \gamma^{\beta}(f(Y))\}$
$= \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$
$(f^{-1}(\gamma^{\beta}))(XY) \leq \max\{(f^{-1}(\eta^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}.$

Hence, $f^{-1}(\gamma^{G})$ is an intuitionistic fuzzy HX subring of $\mathfrak{R}_1$. Then, $f^{-1}((B + \gamma^{G}))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^{G})$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B) \in \mathfrak{R}_1$.

Let $X \in \mathfrak{R}_1$ and $B \in \mathfrak{R}_2$.

Now, $(f^{-1}(B) + f^{-1}(\gamma^{\alpha}))(X) = (f^{-1}(f(Y) + f^{-1}(\gamma^{\alpha}))(X)$
$= (Y + f^{-1}(\gamma^{\alpha}))(X)$
$= (f^{-1}(\gamma^{\alpha}))(X- Y)$
$= (\gamma^{\alpha}(f(X- Y))$
$= (\gamma^{\alpha}(f(X) - f(Y))$
$= (f(Y) + \gamma^{\alpha})f(X)$
$= (B + \gamma^{\alpha})f(X)$
$= f^{-1}((B + \gamma^{\alpha}))(X).$
$(f^{-1}(B) + f^{-1}(\gamma^{\alpha}))(X) = f^{-1}((B + \gamma^{\alpha}))(X).$

Hence, $f^{-1}((B + \gamma^{\alpha})) = (f^{-1}(B) + f^{-1}(\gamma^{\alpha})).$

Now, $(f^{-1}(B) + f^{-1}(\gamma^{\beta}))(X) = (f^{-1}(f(Y) + f^{-1}(\gamma^{\beta}))(X)$
$= (Y + f^{-1}(\gamma^{\beta}))(X)$
$= (f^{-1}(\gamma^{\beta}))(X- Y)$
$= (\gamma^{\beta}(f(X- Y))$
$= (\gamma^{\beta}(f(X) - f(Y))$
$= (f(Y) + \gamma^{\beta})f(X)$
$= (B + \gamma^{\beta})f(X)$
$= f^{-1}((B + \gamma^{\beta}))(X).$
$(f^{-1}(B) + f^{-1}(\gamma^{\beta}))(X) = f^{-1}((B + \gamma^{\beta}))(X).$

Hence, $f^{-1}((B + \gamma^{\beta})) = (f^{-1}(B) + f^{-1}(\gamma^{\beta})).$

Hence, $f^{-1}((B+\gamma^{G})) = (f^{-1}(B) + f^{-1}(\gamma^{G})).$

3.2.3 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism onto HX rings. Let $(A+\lambda^{H})$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring λ^{H} of a HX ring $\mathfrak{R}_1$ determined by the element $A \in \mathfrak{R}_1$.Then $f((A + \lambda^{H}))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^{H})$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A) \in \mathfrak{R}_2$ and $f((A + \lambda^{H})) = (f(A) + f(\lambda^{H}))$, if λ^{H} has a supremum property and λ^{H} is f-invariant.

Proof: Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism onto HX rings.

Let $(A+ \lambda^{H})$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring λ^{H} of a HX ring $\mathfrak{R}_1$ determined by the element $A \in \mathfrak{R}_1$.
Then, $f(\lambda^{H}) = \{ \langle f(X), f(\lambda^{\mu})(f(X)), f(\lambda^{\eta})(f(X))\rangle / X \in \mathfrak{R}_1 \}$.
There exist $X, Y \in \mathfrak{R}_1$ such that $f(X), f(Y) \in \mathfrak{R}_2$,

i. $(f(\lambda^{\mu}))(f(X) - f(Y)) = (f(\lambda^{\mu}))(f(X-Y)),$
$= \lambda^{\mu}(X-Y)$
$= \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$
$= \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$
$(f(\lambda^{\mu}))(f(X) - f(Y)) \geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$

ii. $(f(\lambda^{\mu}))(f(X)f(Y)) = (f(\lambda^{\mu}))(f(YX))$
$= \lambda^{\mu}(YX)$
$\geq \min\{\lambda^{\mu}(Y), \lambda^{\mu}(X)\}$
$= \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$
$= \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$
$(f(\lambda^{\mu}))(f(X) - f(Y)) \geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$

iii. $(f(\lambda^{\eta}))(f(X) - f(Y)) = (f(\lambda^{\eta}))(f(X-Y))$,
$= \lambda^{\eta}(X-Y)$
$\leq \max\{\lambda^{\eta}(X), \lambda^{\eta}(Y)\}$
$= \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$
$(f(\lambda^{\eta}))(f(X) - f(Y)) \leq \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

iv. $(f(\lambda^{\eta}))(f(X)f(Y)) = (f(\lambda^{\eta}))(f(YX))$
$= \lambda^{\eta}(YX)$
$\leq \max\{\lambda^{\eta}(Y), \lambda^{\eta}(X)\}$
$= \max\{\lambda^{\eta}(X), \lambda^{\eta}(Y)\}$
$= \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$
$(f(\lambda^{\eta}))(f(X)f(Y)) \leq \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

Hence, $f(\lambda^H)$ is an intuitionistic fuzzy HX subring of $\mathfrak{R}_2$. Then $f((A + \lambda^H))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^H)$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A)\in \mathfrak{R}_2$.

Let $A, X \in \mathfrak{R}_1$, then $f(A), f(X)\in \mathfrak{R}_2$.

Now, $(f(A) + f(\lambda^{\mu}))f(X) = (f(\lambda^{\mu}))(f(X) - f(A))$
$= (f(\lambda^{\mu}))(f(X - A))$
$= \lambda^{\mu}(X - A)$
$= (A + \lambda^{\mu})(X)$
$= f((A + \lambda^{\mu}))f(X)$.
$(f(A) + f(\lambda^{\mu}))f(X) = f((A + \lambda^{\mu}))f(X)$, for any $f(X)\in \mathfrak{R}_2$.

Hence, $f((A+\lambda^{\mu})) = (f(A) + f(\lambda^{\mu}))$.

Now, $(f(A) + f(\lambda^{\eta}))f(X) = (f(\lambda^{\eta}))(f(X) - f(A))$
$= (f(\lambda^{\eta}))(f(X - A))$
$= \lambda^{\eta}(X - A)$
$= (A + \lambda^{\eta})(X)$
$= f((A + \lambda^{\eta}))f(X)$.
$(f(A) + f(\lambda^{\eta}))f(X) = f((A + \lambda^{\eta}))f(X)$, for any $f(X)\in \mathfrak{R}_2$.

Hence, $f((A+\lambda^{\eta})) = (f(A) + f(\lambda^{\eta}))$.

Hence, $f((A + \lambda^H)) = (f(A) + f(\lambda^H))$.

3.2.4 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let $(B + \gamma^G)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$.Then $f^{-1}((B + \gamma^G))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^G)$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B)\in\mathfrak{R}_1$ and $f^{-1}((B+\gamma^G)) = (f^{-1}(B) + f^{-1}(\gamma^G))$.

Proof: Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism on HX rings.

Let $(B + \gamma^G)$ be the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$. Let $B = f(Y)$, $Y\in \mathfrak{R}_1$. Then, $f^{-1}(\gamma^G) = \{\langle X, f^{-1}(\gamma^{\alpha})(X), f^{-1}(\gamma^{\beta})(X)\rangle / X\in\mathfrak{R}_1\}$.

For any $X,Y \in \mathfrak{R}_1$, $f(X), f(Y) \in \mathfrak{R}_2$,

i. $(f^{-1}(\gamma^{\alpha}))(X-Y) = \gamma^{\alpha}(f(X-Y))$
$= \gamma^{\alpha}(f(X) - f(Y))$
$\geq \min\{\gamma^{\alpha}(f(X)), \gamma^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$
$(f^{-1}(\gamma^{\alpha}))(X-Y) \geq \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$.

ii. $(f^{-1}(\gamma^{\alpha}))(XY) = \gamma^{\alpha}(f(XY))$
$= \gamma^{\alpha}(f(Y)\, f(X))$
$\geq \min\{\gamma^{\alpha}(f(Y)), \gamma^{\alpha}(f(X))\}$
$= \min\{\gamma^{\alpha}(f(X)), \gamma^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$
$(f^{-1}(\gamma^{\alpha}))(XY) \geq \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$.

iii. $(f^{-1}(\gamma^{\beta}))(X-Y) = \gamma^{\beta}(f(X-Y))$
$= \gamma^{\beta}(f(X) - f(Y))$
$\leq \max\{\gamma^{\beta}(f(X)), \gamma^{\beta}(f(Y))\}$
$= \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$
$(f^{-1}(\gamma^{\beta}))(X-Y) \leq \max\{(f^{-1}(\eta^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$.

iv. $(f^{-1}(\gamma^{\beta}))(XY) = \gamma^{\beta}(f(XY))$
$= \gamma^{\beta}(f(Y)\, f(X))$
$\leq \max\{\gamma^{\beta}(f(Y)), \gamma^{\beta}(f(X))\}$
$= \max\{\gamma^{\beta}(f(X)), \gamma^{\beta}(f(Y))\}$
$= \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$
$(f^{-1}(\gamma^{\beta}))(XY) \leq \max\{(f^{-1}(\eta^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}$.

Hence, $f^{-1}(\gamma^{G})$ is an intuitionistic fuzzy HX subring of $\Re_1$.

Then, $f^{-1}((B + \gamma^{G}))$ is the intuitionistic fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^{G})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$.

Let $X \in \Re_1$ and $B \in \Re_2$.

Now,
$$\begin{aligned}(f^{-1}(B) + f^{-1}(\gamma^{\alpha}))(X) &= (f^{-1}(f(Y) + f^{-1}(\gamma^{\alpha}))(X)\\ &= (Y + f^{-1}(\gamma^{\alpha}))(X)\\ &= (f^{-1}(\gamma^{\alpha}))(X - Y)\\ &= (\gamma^{\alpha}(f(X - Y))\\ &= (\gamma^{\alpha}(f(X) - f(Y))\\ &= (f(Y) + \gamma^{\alpha})f(X)\\ &= (B + \gamma^{\alpha})f(X)\\ &= f^{-1}((B + \gamma^{\alpha}))(X).\end{aligned}$$
$$(f^{-1}(B) + f^{-1}(\gamma^{\alpha}))(X) = f^{-1}((B + \gamma^{\alpha}))(X).$$

Hence, $f^{-1}((B + \gamma^{\alpha})) = (f^{-1}(B) + f^{-1}(\gamma^{\alpha}))$.

Now,
$$\begin{aligned}(f^{-1}(B) + f^{-1}(\gamma^{\beta}))(X) &= (f^{-1}(f(Y) + f^{-1}(\gamma^{\beta}))(X)\\ &= (Y + f^{-1}(\gamma^{\beta}))(X)\\ &= (f^{-1}(\gamma^{\beta}))(X - Y)\\ &= (\gamma^{\beta}(f(X - Y))\\ &= (\gamma^{\beta}(f(X) - f(Y))\\ &= (f(Y) + \gamma^{\beta})f(X)\\ &= (B + \gamma^{\beta})f(X)\\ &= f^{-1}((B + \gamma^{\beta}))(X).\end{aligned}$$
$$(f^{-1}(B) + f^{-1}(\gamma^{\beta}))(X) = f^{-1}((B + \gamma^{\beta}))(X).$$

Hence, $f^{-1}((B + \gamma^{\beta})) = (f^{-1}(B) + f^{-1}(\gamma^{\beta}))$.

Hence, $f^{-1}((B+\gamma^{G})) = (f^{-1}(B) + f^{-1}(\gamma^{G}))$.

3.4 Intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring of a HX ring

In this section, we introduce the notion of intuitionistic pseudo fuzzy cosets of a intuitionistic intuitionistic fuzzy HX ring and discuss its properties.

3.4.1 Definition: Let H be an intuitionistic fuzzy set defined on R. Let $\Re \subset 2^{R} - \{\phi\}$ be a HX ring. Let λ^{H} be an intuitionistic fuzzy HX subring of a HX ring $\Re$ and $A \in \Re$. Then the intuitionistic pseudo fuzzy coset of a intuitionistic fuzzy HX ring λ^{H} of a HX ring $\Re$ determined by the element $A \in \Re$ is denoted as $(A + \lambda^{H})^{P}$ and is defined by $(A + \lambda^{\mu})^{P}(X) = p(A)\, \lambda^{\mu}(X)$ and $(A + \lambda^{\eta})^{P}(X) = p(A)\, \lambda^{\eta}(X)$ for every $X \in \Re$ and for some $p \in P$, where $P = \{p(X) / p(X) \in [0,1] \text{ for all } X \in \Re\}$.

3.4.2 Theorem: Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$, then the intuitionistic pseudo fuzzy coset $(A + \lambda^H)^P$ is an intuitionistic fuzzy HX subring λ^H of a HX ring $\Re$ determined by the element $A \in \Re$.

Proof: Let λ^H be an intuitionistic fuzzy HX subring of a HX ring $\Re$.

For every X, Y, $A \in \Re$ we have,

i. $(A + \lambda^\mu)^P (X-Y) = p(A)\, \lambda^\mu (X-Y)$
$\geq p(A) \min \{ \lambda^\mu (X), \lambda^\mu (Y)\}$
$= \min \{p(A)\lambda^\mu (X), p(A)\, \lambda^\mu (Y)\}$
$= \min \{(A + \lambda^\mu)^P (X), (A + \lambda^\mu)^P (Y)\}$

Therefore, $(A + \lambda^\mu)^P(X-Y) \geq \min \{(A + \lambda^\mu)^P (X), (A + \lambda^\mu)^P (Y)\}$

ii. $(A + \lambda^\mu)^P (XY) = p(A)\, \lambda^\mu(XY)$
$\geq p(A) \min\{ \lambda^\mu(X), \lambda^\mu (Y)\}$
$= \min \{p(A)\lambda^\mu (X), p(A)\, \lambda^\mu (Y)\}$
$= \min \{(A + \lambda^\mu)^P (X), (A + \lambda^\mu)^P(Y)\}$

Therefore, $(A + \lambda^\mu)^P (XY) \geq \min \{(A + \lambda^\mu)^P (X), (A + \lambda^\mu)^P (Y)\}$

iii. $(A + \lambda^\mu)^P (X-Y) = p(A)\, \lambda^\eta (X-Y)$
$\leq p(A) \max \{ \lambda^\eta(X), \lambda^\eta (Y)\}$
$= \max \{p(A)\lambda^\eta (X), p(A)\, \lambda^\eta (Y)\}$
$= \max \{(A + \lambda^\eta)^P (X), (A + \lambda^\eta)^P (Y)\}$

Therefore, $(A + \lambda^\eta)^P(X-Y) \leq \max \{(A + \lambda^\eta)^P (X), (A + \lambda^\eta)^P (Y)\}$

iv. $(A + \lambda^\eta)^P (XY) = p(A)\, \lambda^\eta(XY)$
$\leq p(A) \max\{ \lambda^\eta(X), \lambda^\eta (Y)\}$
$= \max \{p(A)\lambda^\eta (X), p(A)\, \lambda^\eta (Y)\}$
$= \max \{(A + \lambda^\eta)^P (X), (A + \lambda^\eta)^P(Y)\}$

Therefore, $(A + \lambda^\eta)^P (XY) \leq \max \{(A + \lambda^\eta)^P (X), (A + \lambda^\eta)^P (Y)\}$

Hence, the intuitionistic pseudo fuzzy coset $(A + \lambda^\mu)^P$ determined by the element $A \in \Re$ is an intuitionistic fuzzy HX subring of a HX ring $\Re$.

3.5 Homomorphism and anti homomorphism of a intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX subring of a HX ring

In this section, we introduce the concept of an image, pre-image of intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of intuitionistic pseudo fuzzy coset of a intuitionistic fuzzy HX ring λ^H of a HX ring $\Re$ determined by the element $A \in \Re$.

3.5.1 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let $(A+\lambda^H)^P$ be the intuitionistic pseudo fuzzy coset of a intuitionistic fuzzy HX ring λ^H of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.Then $f((A + \lambda^H)^P)$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^H)$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and
$f((A + \lambda^H)^P) = (f(A) + f(\lambda^H))^p$, if λ^μ has a supremum property and λ^μ is f-invariant.

Proof: Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let $(A+ \lambda^H)^P$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring λ^H of a HX ring $\Re_1$ determined by the element $A \in \Re_1$. By Theorem 3.2.1, $f(\lambda^H)$ is an intuitionistic fuzzy HX subring of a HX ring $\Re_2$. Then $f((A + \lambda^H)^P)$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^H)$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$.

Let $A, X \in \Re_1$, then $f(A), f(X) \in \Re_2$.

Now, $(f(A) + f(\lambda^\mu))^p f(X) = p(f(A))(f(\lambda^\mu)(f(X)))$
$= p(A)\, \lambda^\mu (X)$
$= (A + \lambda^\mu)^P(X)$
$= f((A + \lambda^\mu)^P)f(X).$
$(f(A) + f(\lambda^\mu))^p f(X) = f((A + \lambda^\mu)^P)f(X)$, for any $f(X) \in \Re_2$.

Hence, $f((A+\lambda^{\mu})^{P}) = (f(A) + f(\lambda^{\mu}))^{p}$.

Now,
$$\begin{aligned}(f(A) + f(\lambda^{\eta}))^{p} f(X) &= p(f(A))(f(\lambda^{\eta})(f(X)))\\ &= p(A)\,\lambda^{\eta}(X)\\ &= (A + \lambda^{\eta})^{P}(X)\\ &= f((A + \lambda^{\eta})^{P})f(X).\\ (f(A) + f(\lambda^{\eta}))^{p} f(X) &= f((A + \lambda^{\eta})^{P})f(X), \text{ for any } f(X)\in \Re_2.\end{aligned}$$

Hence, $f((A+\lambda^{\eta})^{P}) = (f(A) + f(\lambda^{\eta}))^{p}$.

Hence, $f((A + \lambda^{H})^{P}) = (f(A) + f(\lambda^{H}))^{p}$.

3.5.2 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let $(B + \gamma^{G})^{P}$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring γ^{G} of a HX ring $\Re_2$ determined by the element $B\in \Re_2$. Then $f^{-1}((B + \gamma^{G})^{P})$ is the intuitionistic pseudo fuzzy coset of an intuitionistic f uzzy HX ring $f^{-1}(\gamma^{G})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$ and $f^{-1}((B+\gamma^{G})^{P}) = (f^{-1}(B) + f^{-1}(\gamma^{G}))^{p}$.

Proof: Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings.

Let $(B + \gamma^{G})^{P}$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring γ^{G} of a HX ring $\Re_2$ determined by the element $B\in \Re_2$.

By Theorem 3.2.2, $f^{-1}(\gamma^{G})$ is an intuitionistic fuzzy HX subring of a HX ring $\Re_1$.Then, $f^{-1}((B + \gamma^{G})^{P})$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^{G})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$.

Let $X \in \Re_1$ and $B\in \Re_2$.

Now,
$$\begin{aligned}(f^{-1}(B) + f^{-1}(\gamma^{\alpha}))^{p}(X) &= p(f^{-1}(B))(f^{-1}(\gamma^{\alpha}))(X)\\ &= p(B)(\gamma^{\alpha}(f(X))\\ &= (B + \gamma^{\alpha})^{P}f(X)\\ &= f^{-1}((B + \gamma^{\alpha})^{P})(X).\\ (f^{-1}(B) + f^{-1}(\gamma^{\alpha}))^{p}(X) &= f^{-1}((B + \gamma^{\alpha})^{P})(X).\end{aligned}$$

Hence, $f^{-1}((B + \gamma^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\gamma^{\alpha}))^{p}$.

Now,
$$\begin{aligned}(f^{-1}(B) + f^{-1}(\gamma^{\beta}))^{p}(X) &= p(f^{-1}(B))(f^{-1}(\gamma^{\beta}))(X)\\ &= p(B)(\gamma^{\beta}(f(X))\\ &= (B + \gamma^{\beta})^{P}f(X)\\ &= f^{-1}((B + \gamma^{\beta})^{P})(X).\\ (f^{-1}(B) + f^{-1}(\gamma^{\beta}))^{p}(X) &= f^{-1}((B + \gamma^{\beta})^{P})(X).\end{aligned}$$

Hence, $f^{-1}((B + \gamma^{\beta})^{P}) = (f^{-1}(B) + f^{-1}(\gamma^{\beta}))^{p}$.

Hencee, $f^{-1}((B+\gamma^{G})^{P}) = (f^{-1}(B) + f^{-1}(\gamma^{G}))^{p}$.

3.5.3 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A+\lambda^{H})^{P}$ be the intuitionistic pseudo fuzzy coset of a intuitionistic fuzzy HX ring λ^{H} of a HX ring $\Re_1$ determined by the element $A\in \Re_1$.Then $f((A + \lambda^{H})^{P})$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^{H})$ of a HX ring $\Re_2$ determined by the element $f(A)\in\Re_2$ and $f((A + \lambda^{H})^{P}) = (f(A) + f(\lambda^{H}))^{p}$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings.

Let $(A+ \lambda^{H})^{P}$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring λ^{H} of a HX ring $\Re_1$ determined by the element $A\in \Re_1$.

By Theorem 3.2.3, $f(\lambda^{H})$ is an intuitionistic fuzzy HX subring of a HX ring $\Re_2$. Then $f((A + \lambda^{H})^{P})$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f(\lambda^{H})$ of a HX ring $\Re_2$ determined by the element $f(A)\in \Re_2$.

Let A, X $\in \Re_1$, then f(A), f(X)$\in \Re_2$.

Now, $$(f(A) + f(\lambda^\mu))^p f(X) = p(f(A))(f(\lambda^\mu)(f(X)))$$
$$= p(A)\, \lambda^\mu (X)$$
$$= (A + \lambda^\mu)^P(X)$$
$$= f((A + \lambda^\mu)^P)f(X).$$
$$(f(A) + f(\lambda^\mu))^p f(X) = f((A + \lambda^\mu)^P)f(X), \text{ for any } f(X)\in \Re_2.$$

Hence, $f((A+\lambda^\mu)^P) = (f(A) + f(\lambda^\mu))^p$.

Now, $$(f(A) + f(\lambda^\eta))^p f(X) = p(f(A))(f(\lambda^\eta)(f(X)))$$
$$= p(A)\, \lambda^\eta (X)$$
$$= (A + \lambda^\eta)^P(X)$$
$$= f((A + \lambda^\eta)^P)f(X).$$
$$(f(A) + f(\lambda^\eta))^p f(X) = f((A + \lambda^\eta)^P)f(X), \text{ for any } f(X)\in \Re_2.$$

Hence, $f((A+\lambda^\eta)^P) = (f(A) + f(\lambda^\eta))^p$.

Hence, $f((A + \lambda^H)^P) = (f(A) + f(\lambda^H))^p$.

3.5.4 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings. Let $(B + \gamma^G)^P$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\Re_2$ determined by the element B$\in \Re_2$.Then $f^{-1}((B + \gamma^G)^P)$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^G)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$ and
$f^{-1}((B+\gamma^G)^P) = (f^{-1}(B) + f^{-1}(\gamma^G))^p$.

Proof: Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings.

Let $(B + \gamma^G)^P$ be the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring γ^G of a HX ring $\Re_2$ determined by the element B$\in \Re_2$.

By Theorem 3.2.4,$f^{-1}(\gamma^G)$ is an intuitionistic fuzzy HX subring of a HX ring $\Re_1$.Then, $f^{-1}((B + \gamma^G)^P)$ is the intuitionistic pseudo fuzzy coset of an intuitionistic fuzzy HX ring $f^{-1}(\gamma^G)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$.

Let X $\in \Re_1$ and B$\in \Re_2$.

Now, $$(f^{-1}(B) + f^{-1}(\gamma^\alpha))^p (X) = p(f^{-1}(B))(f^{-1}(\gamma^\alpha))(X)$$
$$= p(B)(\gamma^\alpha(f(X))$$
$$= (B + \gamma^\alpha)^P f(X)$$
$$= f^{-1}((B + \gamma^\alpha)^P)(X).$$
$$(f^{-1}(B) + f^{-1}(\gamma^\alpha))^p (X) = f^{-1}((B + \gamma^\alpha)^P)(X).$$

Hence, $f^{-1}((B + \gamma^\alpha)^P) = (f^{-1}(B) + f^{-1}(\gamma^\alpha))^p$.

Now, $$(f^{-1}(B) + f^{-1}(\gamma^\beta))^p (X) = p(f^{-1}(B))(f^{-1}(\gamma^\beta))(X)$$
$$= p(B)(\gamma^\beta(f(X))$$
$$= (B + \gamma^\beta)^P f(X)$$
$$= f^{-1}((B + \gamma^\beta)^P)(X).$$
$$(f^{-1}(B) + f^{-1}(\gamma^\beta))^p (X) = f^{-1}((B + \gamma^\beta)^P)(X).$$

Hence, $f^{-1}((B + \gamma^\beta)^P) = (f^{-1}(B) + f^{-1}(\gamma^\beta))^p$.

Hence, $f^{-1}((B+\gamma^G)^P) = (f^{-1}(B) + f^{-1}(\gamma^G))^p$

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to all our friends for their help to make this paper a successful one.

REFERENCES

1. Atanassov.K.T, intuitionistic fuzzy sets,Fuzzy Sets and System 20 (1986), pp 87-96.
2. Bing-xueYao and Yubin-Zhong, Upgrade of algebraic structure of ring, Fuzzy information and Engineering (2009)2:219-228.
3. Bing-xueYao and Yubin-Zhong, The construction of power ring, Fuzzy information and Engineering (ICFIE), ASC 40, pp.181-187, 2007.
4. Dheena.P and Mohanraaj.G, On intuitionistic fuzzy k ideals of semi rings, International Journal of computational cognition, Volume 9, No.2, 45-50, June 2011.
5. Li Hong Xing, HX group, BUSEFAL, 33(4), 31-37, October 1987.
6. Li Hong Xing, HX ring, BUSEFAL, 34(1) 3-8, January 1988.
7. Liu.W.J., Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems,8,133 - 139.
8. Muthuraj.R, Manikandan K.H., Sithar Selvam.P.M., Intuitionistic Q-fuzzy normal HX group, International Journal of Mathematical Archive, Vol.2, no10., pp 1828-1832.
9. Muthuraj.R and Manikandan K.H, Pseudo Fuzzy Cosets of a HX Group, Applied Mathematical Science, Hikari Ltd,2013,Volume 7,no.86, pp.4259-4271.
10. Muthuraj.R, Manikandan K.H., Intuitionistic Q-fuzzy HX subgroup and $< \alpha, \beta >$ level sub HX groups, CIIT International DOI – FS 122011010.
11. Rosenfeld. A., Fuzzy groups,J.Math.Anal., 35(1971),512-517.
12. Sharma.P.K, On Intuitionistic Anti-Fuzzy Ideal and Quotient Ring, IJAMP, 2012, Volume 4, 2, pp 109-119.
13. Zadeh.L.A., Fuzzy sets, Information and control, 8, 338-353.

Source of support: Nil, Conflict of interest: None Declared.