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# THE SQUARE ROOT TRANSFORMATION

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## INTRODUCTION

Taking $p = 2$, $Z_1 = X_1^{1/2}$, $Z_2 = X_2^{1/2}$ in the equation (1.1), we obtain the model

$$Y = \beta_0 + \beta_1 X_1^{1/2} + \beta_2 X_2^{1/2} + \varepsilon . \tag{1.2}$$

Thus there are many possible transformations, and models can be postulated which contain few or many such terms. Several different transformations may occur in the same model. The choice of transformation would often be made on the basis of previous knowledge of the variables under study. The purpose of making transformations of this type is to be able to use a regression model of simple form in the transformed variables, rather than a more complicated one in the original variables.

## NONLINEAR MODELS THAT ARE INTRINSICALLY LINEAR

We can divide nonlinear models (*i.e.* nonlinear in the parameters to be estimated) into two types. One is called intrinsically linear and the other is called intrinsically nonlinear models. If a model is intrinsically linear, it can be expressed, by suitable transformation of the variables, in the standard linear model form of equation (1.1). If a nonlinear model cannot be expressed in this form then it is intrinsically nonlinear. Some examples are given below.

### 1. THE MULTIPLICATIVE MODEL

$$Y = \alpha X_1^{\beta} X_2^{\gamma} X_3^{\delta} \varepsilon \tag{1.2}$$

where $\alpha, \beta, \gamma$ and $\delta$ are unknown parameters, $\varepsilon$ is the multiplicative random error which has a continuous distribution with mean 1 and some finite variance. Taking logarithms to the base $e$ in equation (3·25) converts the model into the linear form

$$\log_e Y = \log_e \alpha + \beta \log_e X_1 + \gamma \log_e X_2 + \delta \log_e X_3 + \log_e \varepsilon . \tag{1.3}$$

The transformed model (1.3) is now in the form of equation (1.1) and so can be handled by the standard linear regression procedures.

An alternative which is often considered applicable in this case is

$$Y = \alpha X_1^{\beta} X_2^{\gamma} X_3^{\delta} + \varepsilon \tag{1.4}$$

This model is intrinsically nonlinear.

### 2. THE EXPONENTIAL MODEL

$$Y = e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2} \varepsilon \tag{1.5}$$

Taking natural logarithms of both sides

$$\log_e Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \log_e \varepsilon \tag{1.6}$$

which is of the form of equation (1.1).

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**3. A RECIPROCAL MODEL**

$$Y = \frac{1}{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon} \tag{1.7}$$

Taking reciprocals on both sides

$$\frac{1}{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \tag{1.8}$$

**4. A MORE COMPLICATED EXPONENTIAL MODEL**

$$Y = \frac{1}{1 + e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon}} \tag{1.9}$$

Taking reciprocals, subtracting 1, and then taking natural logarithm of both sides

$$\log_e \left( \frac{1}{Y} - 1 \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon . \tag{1.10}$$

This is an example of an iterated transformation on the dependent variable in order to reduce a complicated nonlinear model to a linear model.

In all cases, where the models are transformed as in these examples, the least squares analysis is applied to the transformed model of equation (1.1), and so the estimated coefficients are “least squares estimates” only as far as transformed model is concerned.

Two other examples of models which are intrinsically nonlinear are:

$$Y = \beta_0 + \beta_1 e^{-\beta_2 X} + \varepsilon$$

and $$Y = \beta_0 + \beta_1 X + \beta_2 \left(\beta_3\right)^X + \varepsilon$$

From the above discussion we have seen that the use of transformations sometimes help to achieve the linearity of the regression function. The transformations are also used to stabilize the error variance, that is, to make error variance constant to all the observations. The constancy of error variance is one of the standard assumptions of least square theory. It is often referred to as the assumption of homoscedasticity. When the error variance is not constant, over all the observations, the error is said to be heteroscedastic. Homoscedasticity and heteroscedasticity are described below with figures.

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