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APPLICATION OF DIFFERENTIAL EQUATION IN ECONOMICAL SOLOW GROWTH MODEL

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ABSTRACT

The Solow growth model is an economic model. This model is the so-called Solow-swan model name after Robert (Bob) Solow and Trevor swan, or the simple Solow model. Bob Solow developed many implications and applications of his model and was awarded the Nobel prize in economics for his contributions. Solow model is a simple and abstract representation of complex economy. Solow uses differential equation for solving model. To represent the process of growth macroeconomic equilibrium. We have to consider households and individuals and individuals with different tastes, abilities, incomes and roles in society, various sectors, and multiple social interactions. The Solow model cuts through there complications by constructing simple one best economy. With small reference of particular individual decisions. Solow model is closed relation with economic growth model. The Solow model makes relatively few speculation about household. The Solow growth model which models the growth of the ratio of capital to labour under the assumption. The model predicts that is the long term, capital will grow exponentially along with the labour. If the capital is very low, It will rapidly increase till it becomes equitiesly proportional to the labour. It remarkably represents that capital remains proportional to the labour.

1. INTRODUCTION

The Growth Model is a statistical technique used in the structural equation modeling framework to estimate growth trajectory. It is a longitudinal scrutiny method to calculate growth over a previous of time. It is particularly called an economics.

Solow growth models ensures steady growth in the long run period without any difficulty. Prof. Solow counterfeit that Harrod Domar's model was depends up on some non realistic assumptions like fixed factor proportions, constant capitals output ratio.

In Solow growth model these assumptions has dropped while formulating its models of long - run growth. It shows that by the introduces of the factors influencing economic growth, the instability of Harrod Domar's model can be reduced to some limit. The Solow growth model represent that if technical co-efficient of production one assumed to be variable , the capital labour ratio may adjust itself to stability ratio with respect to time.[1] [2] [3] [4] [5]

However in Harrod Domar's model of steady growth, the economic system attains a knife-edge balance of stability is growth in the long-run period. This balance is recognition for a result of pulls and counter pulls exerted by natural growth rate which depends on the increase in labour force in the absence of technical changes and necessitate natural growth rate. Which necessitate on the saving and investment norm of household and firms.

The important specification for the Solow model is the exchangeability between capital and labour Prof. Solow indicate in his model that, this fundamental opposition of assured and natural rates turns out in the end to flow from the necessary assumption that production takes place under conditions of fixed proportions. The knife edge balance established under Harrodin steady growth path can be shattered by a slight change in key parameters. [1] [2] [9]

2. SOLOW'S ECONOMIC GROWTH MODEL

We assume a model from macro economics. Let K be the capital, the labour L and the production output of an economy is P. We can constructs a dynamic economy equation so k(t) , L(t) and P(t) are all functions of time but we can conceal the parameter.

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In primary Economic It seems that a universal Hypothesis is that p can be expressed as function F, K and L.

$$P = f(K, L) \tag{1}$$

It postulate that '*f* ' has, using economics terminology, constant returns to scale. Mathematically, it means that taking product, of K and L by the some results in P should be multiplied by same term. Such that for some constant.

$$f(CK,CL) = bf(C,L) \tag{2}$$

Now For making some more hypothesis. It postulates that the constant quantum (proportion) of Y is invested in capital. It states that the rate of change of capital K is proportional to production P.[6] [7] [8] [10]

$$\frac{dK}{dt} \alpha P$$

$$\therefore \frac{dK}{dt} = aP \tag{3}$$

where a > 0, is the proportionality constant

Also, the labour force is increasing with respect to time *t*.

$$\frac{dl}{dt} = \alpha L \tag{4}$$

where $\alpha > 0$ is the per capita growth rate. This is a first order first degree differential equation for L which we can solve to find.

$$L = Loe^{\alpha t}$$

Now by substituting equation (1) in equation (3) then we get

$$\frac{dK}{dt} = af(K, L) \tag{5}$$

Since L(t) is a known function, the only unknown function is K(t). This is a first order and first degree differential equation for K(t).[13] [14] [17]

$$L(t) = L_0 e^{\alpha t},$$

So the right hand side of the differential equation depends on t explicitly. We shall quiet try to analyse this equation. But it shall be nice if we shall find an autonomous first order differential equation.

It forms out we can obtain an autonomous equation for the ratio K/L instead of K

First, because *f* has constant output to scale we can write.

$$f(K, L) = f\left(L\frac{K}{L}, L\right) = Lf\left(\frac{K}{L}, 1\right) \tag{6}$$

Then, later dividing by L equation (5) because

$$\frac{1}{L}\frac{dK}{dt} = a \ f\left(\frac{K}{L}, 1\right) \tag{7}$$

Now we assume the derivative of $\dfrac{K}{L}$ given by the quotient rule and using equation (4) we get

$$\frac{d\left(\frac{K}{L}\right)}{dt} = \frac{L\frac{dK}{dt} - K\frac{dL}{dt}}{L^2}$$

$$= \frac{L\frac{dK}{dt}}{L^2} - \frac{K\frac{dL}{dt}}{L^2}$$

$$= \frac{1}{L}\frac{dK}{dt} - \frac{K}{L^2}\frac{dL}{dt}$$

$$= \frac{1}{L}\frac{dK}{dt} - \alpha\frac{K}{L} \tag{8}$$

$$\left(\because \frac{dL}{dt} = \alpha L \quad \text{which is equation } (4)\right)$$

Now subtracting $\alpha\frac{K}{L}$ from both side of equation (7) [8] [9] [10] [11]

$\therefore$ Equation (7) becomes.

$$\frac{1}{L}\frac{dK}{dt} - \alpha\frac{K}{L} = a \;\; f\left(\frac{K}{L},1\right) - \alpha\frac{K}{L}$$

From equation (8) we can get

$$\frac{d\left(\frac{K}{L}\right)}{dt} = a \;\; f\left(\frac{K}{L},1\right) - \alpha\frac{K}{L} \tag{9}$$

Now we have a differential equation in which the unknown function is available i.e. $\frac{K}{L}$.

Now, we define $\rho = \frac{K}{L}$ (10)

and $g(\rho) = f(\rho,1)$ (11)

Then above equation (9) becomes

$$\frac{d\rho}{dt} = ag(\rho) - \propto \rho \tag{12}$$

This is the Solow's Growth model. Above model, models the growth of the ratio of capital to labour under the hypothesis mentioned systematically.

3. SUMMARY OF MODEL

Hypothesis

1. $p = f(K,L)$ where $f(K,L)$ is a function with constant returns to scale.
2. $\frac{dK}{dt} = ap$ is the function of fraction of the production output is invested in capital.
3. $\frac{dL}{dt} = \alpha L$; labour increase according to equation.
4. $\rho = \frac{K}{L}$, It is the ratio of capital to labour.
5. $g(\rho) = f(\rho,1)$

   Model $\frac{d\rho}{dt} = a\,g(\rho) - \alpha\rho$

We now consider for example of the production function [3] [4] [5]

$$f(K,L) = K^{\frac{1}{5}}L^{\frac{4}{3}} \tag{13}$$

The $g(\rho) = f(\rho,1) = K^{\frac{1}{5}}$ (14)

and the differential equation for ρ is $\frac{d\rho}{dt} = a\rho^{\frac{1}{5}} - \alpha\rho$ (15)

Now, the graph shows $\frac{d\rho}{dt}$ verses ρ

by solving $a\rho^{\frac{1}{5}} - \alpha\ \rho = 0$

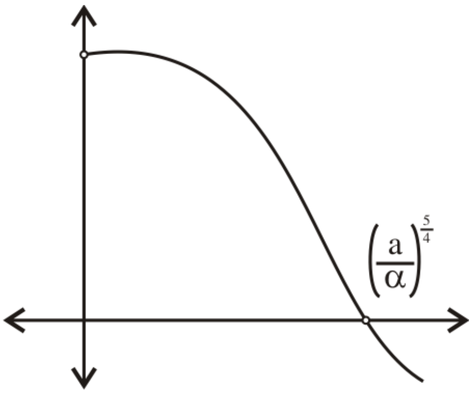
$$a\rho^{\frac{1}{5}} = \alpha\rho$$

$$\left(\frac{a}{\alpha}\right) = \rho\ \rho^{-\frac{1}{5}}$$

$$\left(\frac{a}{\alpha}\right) = \rho^{\frac{4}{5}}$$

$$\therefore\ \rho = \left(\frac{a}{\alpha}\right)^{\frac{5}{4}}$$

Solow graph model rate



We got the equilibrium solutions for $\rho = 0$ or $\rho = \left(\frac{S}{\lambda}\right)^{\frac{5}{4}}$

Changing α or a will change the scale and the collection values of the non-zero equilibrium.

The graph of $\frac{dR}{dE}$ verses will always have the same qualitative shape as the graph shown above.

If $\rho > 0$ is small $\frac{d\rho}{dt} > 0$, so ρ will increase, the equilibrium $\rho = 0$.

Which is unstable.

The graph of ρ (t) have an inflection point when ρ reaches $\left(\frac{a}{5^{\alpha}}\right)^{\frac{5}{4}}$

The right hand side of equation (15) has maximum value f then ρ converge asymptotically to the positive equilibrium. The equilibrium $\rho\left(\frac{a}{\alpha}\right)^{\frac{5}{4}}$ is asymptotically stable particular solution that starts close to the equilibrium will concentrate to the equilibrium will concentrate to the equilibrium as $t \to \infty$.

Moreover total solutions will $\rho(0) > 0$ will concentrate asymptotically to this equilibrium.

We can express this means interms of capital K and labour L.

Since, $\rho(t) = \dfrac{K(t)}{L(t)}$, and $L(t) = L_O e^{\propto t}$

particularly $\rho(t)$ converges to an asymptotically stable equilibrium K, then K(t) must treat asymptotically like $\rho, L(t)$.

This means that in the long term K(t) must grow exponentially with the same exponent as L(t).

The present model suggests that in the long term, capital will grow exponentially along with the labour.

In same case if capital is livelihood law, capital rapidly increase till it becomes about proportional to the labour and the capital settle into long term behavior in which capital remains proportional to the labour.[1] [5] [6] [7] [14]

The Solow Growth Model is discrete Time:

I next represent the dynamics of economic growth in the discrete-time Solow model.

Fundamental Law of Motion of the Solow Model:

Remember that K capital depreciates exponentially at the rate λ, so that the law of motion of the capital stock is given by

$$K(t+1) = (1-\lambda)K(t) + I(t) \tag{16}$$

Where I(*t*) is investment at time *t*.

From national income accounting for a closed economy, C(t) is the consumption.

The total Amount of Final production in the economic Growth must be invested or consumed. Thus,

$$p(t) = C(t) + I(t) \tag{17}$$

From equation (1), (16) and (17) any feasible dynamic distribution is satisfied by given economy.

$$K(t+1) \le f\left(K(t), L(t)\right) + (1-\lambda)K(t) - C(t) \tag{18}$$

For t = 0, 1, 2, 3, …….. The question is to obtained the equilibrium dynamic allocation among the set of feasible dynamic distribution. Have the behavior rule that households save a constant friction of their income simplifies the construction of stability considerably are implication of given hypothesis is that any welfare comparisons based on the Solow growth model have be taken with small amount. [7] [8] [13]

Since we do not know that the need of the households are

The economy doesn't have any government grant. Since economy is closed. In this economy total investment is equal to savings.

$$S(t) = I(t) = p(t) - C(t)$$

The hypothesis that households save a constant friction S(0, 1) of their profit can be represented as.

$$S(t) = sp(t) \tag{19}$$

It represents that the remaining 1-S fraction of their income and Hence

$$C(t) = (1-S)p(t) \tag{20}$$

The equation (19) represents that the supply of capital for time t + 1 resulting from households behaviour can be represents that $K(t+1) = (1-\lambda)K(t) + S(t) = (1-\lambda)K(t) + sp(t)$.

From equation (1) and (16) the fundamental law of motion of the Solow growth model is:

$$K(t+1) = sf\left(K(t), L(t)\right) + (1-\lambda)Kt \tag{21}$$

This is a nonlinear difference equation. The equilibrium of the Solow growth model is described by (21) combine with the law of motion for $L(t)$ and $A(t)$.[3] [10] [11] [14]

The Solow Growth Model is continuous Time

From Difference to Differential Equations:

Recall that the time periods t = 0, 1, 2, ….... can refer to days, weeks, months or years. In some sense the parameter time is not considerable this arbitralness suggests that perhaps it may be more convenient to look at dynamics by making the time unit as small as possible, this is by going to continuous time. Where in Recent macro-economics consider discrete-time models. But particularly many growth models are defined in continuous time. In continuous time growth models have some merits. In same discrete time models pathological outcome vanishes when it is in continuous time. Finally i-conclude that continuous time Growth models have large Elasticity in the scrutiny of dynamics and allow explicit form solutions in a wider set of circumstances. These consequences motivate the detailed study of both the discrete and continuous time proposition of Growth models. [12] [15] [16]

Now we consider a difference equation

$$x(t+1) - x(t) = h(x(t)) \tag{22}$$

Above equation between t and t + 1 represents that the adequate growth in *x* is given by *h*(*x*(*t*)).

Assume that time is more surely divisible than that captured by our discrete indices, t = 0, 1, 2, ……..

In the limit, finally time is finally divisible as we consider. So that $t \in R_+$.

Above equation (22) gives us information about the variable *x* changes between two discrete points in time t and t + 1. Between the time periods t and t + 1 they are not too far apart, the following approximation is commensurable.

$$x(t+\rho t) - x(t) \simeq \rho t \cdot h(x(t)) \tag{23}$$

For some $\rho t \in [0,1]$. When $\rho t = 0$ equation (23) becomes an identity. When $\rho t = 1$, above equation becomes in the form (22) and in between (0,1) it is linear approximation. This approximation is comparatively accurate if the distance between t and t + 1 is small.[3] [7] [15]

Hence $h(x) \simeq h(x(t))$.

For all $x \in \left[x(t), x(t+1)\right]$.

however there is a conclusion that this approximation in fact can be abominable if the function g is highly nonlinear in this case its behaviour changes expressively between *x*(*t*) and *x*(*t* + 1).

We can divide both side of equation (23) by ρt and taking limit as $\delta t \to 0$.

$$\therefore \lim_{\delta t \to 0} \frac{x(t+\delta t) - x(t)}{\delta t} = x(t) = h(x(t)) \tag{24}$$

where $x(t) = \frac{d}{dt} x(t)$,

where time t is discrete.

Now equation (24) is the differential equation which have a same dynamics is differential equation (23). In the above model the gap between *t* and *t* + 1 is small increments.

4. CONCLUSION

The Solow model is remarkable in its simplicity. In the present era every once appreciate it for much of an intellectual development it has. Before the advent of the Solow growth model, Generally economic growth is Generally calculated by the model developed by the economists Ray Harrod and Evsay Domar the well known name Harrod-Domar model prominence potential dysfunctional angle of economic growth. For example, How economic growth can go hand in hand with increasing unemployment.

The Solow model practically states why the Harrod-Domar model was not an attractive place to start. At the center of the Solow growth model, recognizing it from the Harrod-Domar model. It is the neoclassical aggregate production function. This function enables the Solow model to make contact with microeconomics. Also it serves as a bridge between the model and the data.

The most important feature of the Solow model, is that it is a simple and abstract representation of complex economy. The Solow growth model is to simple and too abstract. Solow growth model do justice in growth of macroeconomic equilibrium and it consider various sectors such as households and individuals with different capital abilities incomes, and roles in society, various fields, and mix social interactions. The Solow model cuts through these complications by constructing simple are good economy will small reference to personal decisions. Therefore the Solow model should be through of as a initial point and a springboard for riche models. Economic growth and development are dynamic process and thus necessitate dynamic models regarding its simplicity the Solow Growth Model is dynamic general equilibrium model. The Solow model has many important key features of dynamic general equilibrium models emphasized.

The Solow model can be formulated in either discrete or continuous time. The Solow model in discrete time version is conceptually simpler and more commonly used in macroeconomic applications. Many Solow models are formulated in continuous time.

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