International Journal of Mathematical Archive-9(4), 2018, 77-83
Available online through www.ijma.info ISSN 2229 – 5046

SOME RESULTS ON THE FIXED POINT THEOREM ON CONE METRIC SPACE

JIGMI DORJEE BHUTIA*
Dept. of Mathematics, Cooch Behar College, West Bengal, India.

KALISHANKAR TIWARY
Dept. of Mathematics, Raiganj University, Raiganj, West Bengal, India.

(Received On: 26-02-18; Revised & Accepted On: 02-04-18)

ABSTRACT

In this paper we obtain some results on fixed point theorem in cone metric space which extends some known results that are already proved in [6,13].

***Keywords**: Cone metric space, coincidence point, fixed point, Weakly Compatible mappings.*

INTRODUCTION

Jungck [1] introduced the concept of commuting maps and proved the results that generalize the Banach contraction principle. Sessa [2] further gave the idea of weakly commuting maps thereby generalizing the commuting maps. Jungck [3] again extended this concept to compatible mappings. In 1998 [4] Jungck and Rhoades introduced the notion of weakly compatible maps. Compatible maps are weakly compatible but the converse is not true. It is during the year 2007 when Huang and Zhang [5] introduced the concept of cone metric space by replacing the range set of non negative real numbers of the metric *d* by the ordered Banach space. The existence of a common fixed point in cone metric space has been considered recently in [6-18] and references therein. The following theorem is the extension of the result in [6, 13]. Recently, a new generalization of contraction mappings on complete metric spaces is introduced by Beiranvand *et al.* [17] called T-contraction and T-contractive mappings which are depending on another function. On the other hand Morales and Rojas [7, 12] have extended the concept of T-contraction mappings to cone metric space where the cone is normal by provingfixed point theorems for T -Kannan, T-Zamfirescu, T-weakly contraction mappings. In contrary M. Filipovi´c et al. [6] extended this result without considering the cones to be normal and hence defined T-Hardy–Rogers contraction and obtained the fixed point and periodic point results on cone metric space. In the framework of T-Hardy-Rogers Contraction Mapping in a Cone Metric Space Sumitra [11] *et al.* and Soleimani have given a new directions thereby obtaining common fixed point for two mappings.

PRELIMINARIES

We recall some definitions and other results that will be needed in the sequel.

Definition 1.1: Let E be a real Banach space and $P \subset E$. Then P is said to be a cone if it satisfies the following condition:

i) P is a non-empty closed subset of E and $P \neq \{\theta\}$.

ii) If $x, y \in P,\ and\ a, b \in R,\ a \geq 0,\ b \geq 0,$ then $ax + by \in P$.

iii) If $x \in P\, and - x \in P$, then $x = \theta$.

Cone induces a Partial order relation

We can define a partial order relation on E with respect to the cone P in the following way $x \leq y$ if and only if $y - x \in P$. Also $x << y$ if and only if $y - x \in \operatorname{int} P$ and $x < y$ implies $x \leq y$ but $x \neq y$. If $\operatorname{int} P \neq \phi$ then the cone is a solid cone.

Corresponding Author: Jigmi Dorjee Bhutia*
Dept. of Mathematics, Cooch Behar College, West Bengal, India.

Definition 1.2: Let *X* be a set and $d : X \times X \to E$ satisfying

i) $d(x, y) \geq \theta, \forall x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$.

ii) $d(x, y) = d(y, x), \forall x, y \in X$.

iii) $d(x, y) \leq d(x, z) + d(z, y), \forall x, y, z \in X$.

Then *d* is called the cone metric and the pair (X, d) is called the cone metric space.

E.g. a) [5]: Let $E = R^2$ and $P = \{(x, y) \in R^2 : x \geq 0, y \geq 0\}$, $X = R$ and

$d(x, y) = (|x - y|, \alpha|x - y|), \forall x, y \in X, \alpha \geq 0.$ Then (X, d) is a cone metric space and P is a normal cone with normal constant 1.

There are two different kinds of cones: normal (with a normal constant K) and non-normal cones. Let E be a real Banach space, $P \subset E$ a cone and $\leq$ the partial ordering defined by P. Then P is said to be normal if there exist positive real number K such that for all $x, y \in P$

$\theta \leq x \leq y$ implies $\|x\| \leq K\|y\|$.

Or, equivalently if $x_n \leq y_n \leq z_n, \forall n$ and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} z_n = x \Rightarrow \lim_{n\to\infty} y_n = x.$

The least of all such constant K is known as normal constant.

Definition 1.3 [5]: Let (X, d) be a cone metric space. Let x_n be a sequence in X and $x \in X$. If for every $c \in E$ with $c >> \theta$ there is N such that for all $n > N$, $d(x_n, x) << c$. Then x_n is said to be convergent to x and x is the limit of x_n. We denote this by

$\lim_{n\to\infty} x_n = x, \ or \ x_n \to x(n \to \infty)$.

Definition 1.4 [5]: Let (X, d) be a cone metric space. Let x_n be a sequence in X. If for any $c \in E$ with $c >> \theta$ there is N such that for all $n, m > N$, $d(x_n, x_m) << c$. Then x_n is said to be a Cauchy sequence in X.

Definition 1.5 [7]: Let (X, d) be a cone metric space, P a solid cone and $T : X \to X$. Then

i) T is said to be continuous if $\lim_{n\to\infty} x_n = x$ implies that $\lim_{n\to\infty} Tx_n = Tx$, for all x_n in T.

ii) T is said to be Subsequentially Convergent, if we have for every sequence $\{x_n\}$ that $\{Tx_n\}$ is convergent, implies $\{x_n\}$ has a convergent subsequence.

iii) T is said to be sequentially convergent if we have, for every sequence $\{x_n\}$, if $\{Tx_n\}$ is convergent, then $\{x_n\}$ also is convergent.

Definition 1.6 [8]: Let f and g be two self maps on X. If $fw = gw = v$ for some $w \in X$ then w is the coincidence point of f and g and v is the point of coincidence of f and g.

Definition 1.7[5]: Let (X, d) be a cone metric space. If every Cauchy sequence is convergent then X is said to be complete cone metric space

Let (X, d) be a cone metric space.

i) If $u \leq v$ and $v << w$ then $u << w$.

ii) If $\theta \leq u << c$ for each $c \in \text{int} P$, then $u = \theta$.

iii) If E is a real Banach space with cone P and if $a \leq a\lambda$ where $0 \leq \lambda < 1$ and $a \in P$, then $a = \theta$.

Definition 1.8 [4]: Let S and T be two self-mappings of a cone metric space (X, d). The pair (S, T) is said to be weakly compatible if $STu = TSu$ whenever $Su = Tu$ for some $u \in X$.

Remark 2.1 [15]: If $c \in \operatorname{int} P$, $\theta \le a_n$ and $a_n \to \theta$, then there exists n_0 such that for all $n > n_0$ we have $a_n << c$.

The following proposition is proved in [8].

Proposition 2.2 [8]: If *f* and *g* be two weakly compatible maps on *X*. If *f* and *g* have unique point of coincidence $fw = gw = v$ then *v* is the unique common fixed point of *f* and *g*.

M. Abbas, B.E. Rhoades [10] in corollary 2.4 have proved the following result

Theorem 2.3 [10]: Let (X, d) be a complete cone metric space with normal constant *K*. Suppose that the mapping $f : X \to X$ satisfies

$$d(fx, fy) \le a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, fy) + a_4 d(x, fy) + a_5 d(fx, y), \quad \forall x, y \in X.$$

where $a_i \ge 0$; $i = 1,2,3,4,5$ and $\sum_{i=1}^{5} a_i < 1$. Then f has a unique fixed point.

The G.Song *et al.* [13] have proved the following theorem.

Theorem 2.4[13]: Let (X, d) be cone metric space and $a_i \ge 0$; $i = 1,2,3,4,5$ be constants such that $\sum_{i=1}^{5} a_i < 1$.

Suppose that the mappings $f, g : X \to X$ satisfy the condition

$$d(fx, fy) \le a_1 d(gx, gy) + a_2 d(fx, gx) + a_3 d(gy, fy) + a_4 d(fx, gy) + a_5 d(gx, fy), \text{ for all } x, y \in X.$$

If the range of *f* is contained in the range of *g* and $g(X)$ is a complete subspace, then f and g have a unique point of coincidence. Moreover, if f and g are weakly compatible then they have a unique fixed point.

MAIN RESULT

Theorem 3.1: Let (X, d) be complete cone metric space. Let $T : X \to X$ be a continuous and one-one function and $f, g : X \to X$ be two functions satisfying

$$d(Tfx, Tfy) \le a_1 d(Tgx, Tgy) + a_2 d(Tfx, Tgx) + a_3 d(Tgy, Tfy) + a_4 d(Tfx, Tgy) + a_5 d(Tgx, Tfy) \tag{1}$$

for all $x, y \in X$ Where $a_i \ge 0$; $i = 1,2,3,4,5$ and $\sum_{i=1}^{5} a_i < 1$.

Suppose if the range of *f* is contained in the range of *g* and $g(X)$ is a complete subspace. Then

i) There exist $u \in X$ such that $\lim_{n\to\infty} Tf(x_n) = u = \lim_{n\to\infty} Tg(x_{n+1})$.
ii) If T is subsequentially convergent then $f(x_n) = g(x_{n+1})$ have convergent subsequence.
iii) f and g have a unique point of coincidence.
iv) Further if f and g are weakly compatible then they have a unique common fixed point.

Proof: Let $x_0 \in X$ be an arbitrary point of X. Since $f(X) \subset g(X)$ then there exist $x_1 \in X$ such that $f(x_0) = g(x_1)$ and so on. Therefore, we have $f(x_n) = g(x_{n+1})$ where $n = 0,1,2,3,\ldots\ldots\ldots$
Then

$$\begin{aligned} d(Tfx_n, Tfx_{n+1}) &\le a_1 d(Tgx_n, Tgx_{n+1}) + a_2 d(Tfx_n, Tgx_n) + a_3 d(Tgx_{n+1}, Tfx_{n+1}) + a_4 d(Tfx_n, Tgx_{n+1}) \\ &\quad + a_5 d(Tgx_n, Tfx_{n+1}) \\ &\le a_1 d(Tfx_{n-1}, Tfx_n) + a_2 d(Tfx_n, Tfx_{n-1}) + a_3 d(Tfx_n, Tfx_{n+1}) + a_4 d(Tfx_n, Tfx_n) + a_5 d(Tfx_{n-1}, Tfx_{n+1}) \\ &\le a_1 d(Tfx_{n-1}, Tfx_n) + a_2 d(Tfx_n, Tfx_{n-1}) + a_3 d(Tfx_n, Tfx_{n+1}) + a_5 \{d(Tfx_{n-1}, Tfx_n) + d(Tfx_n, Tfx_{n+1})\} \\ &\le (a_1 + a_2 + a_5) d(Tfx_{n-1}, Tfx_n) + (a_3 + a_5) d(Tfx_n, Tfx_{n+1})\} \end{aligned} \tag{2}$$

and

$$d(Tfx_{n+1},Tfx_n) \le a_1 d(Tgx_n,Tgx_{n+1}) + a_2 d(Tfx_{n+1},Tgx_{n+1}) + a_3 d(Tgx_n,Tfx_n) + a_4 d(Tfx_{n+1},Tgx_n) + a_5 d(Tgx_{n+1},Tfx_n)$$

$$\le a_1 d(Tfx_{n-1},Tfx_n) + a_2 d(Tfx_{n+1},Tfx_n) + a_3 d(Tfx_{n-1},Tfx_n) + a_4 d(Tfx_{n+1},Tfx_{n-1}) + a_5 d(Tfx_n,Tfx_n)$$

$$\le a_1 d(Tfx_{n-1},Tfx_n) + a_2 d(Tfx_{n+1},Tfx_n) + a_3 d(Tfx_{n-1},Tfx_n) + a_4 \{d(Tfx_{n+1},Tfx_n) + d(Tfx_{n-1},Tfx_n)\}.$$

$$\le (a_1 + a_3 + a_4) d(Tfx_{n-1},Tfx_n) + (a_2 + a_4) d(Tfx_{n+1},Tfx_n) \qquad (3)$$

Adding (2) and (3), we get,

$$2d(Tfx_{n+1},Tfx_n) \le (2a_1 + a_2 + a_3 + a_4 + a_5) d(Tfx_{n-1},Tfx_n) + (a_2 + a_3 + a_4 + a_5) d(Tfx_{n+1},Tfx_n).$$

$$d(Tfx_{n+1},Tfx_n) \le \frac{(2a_1 + a_2 + a_3 + a_4 + a_5)}{(2 - a_2 - a_3 - a_4 - a_5)} d(Tfx_{n-1},Tfx_n).$$

$$d(Tfx_{n+1},Tfx_n) \le b d(Tfx_{n-1},Tfx_n) \le \dots\dots\dots \le b^n d(Tfx_0,Tfx_1). \qquad (4)$$

where, $b = \frac{(2a_1 + a_2 + a_3 + a_4 + a_5)}{(2 - a_2 - a_3 - a_4 - a_5)} < 1.$

Let $m > n$,

$$d(Tfx_n,Tfx_m) \le (b^n + b^{n+1} + b^{n+2} + \cdots\cdots + b^{m-1}) d(Tfx_0,Tfx_1).$$

$$\le \frac{b^n}{1-b} d(Tfx_0,Tfx_1) \to \theta, n \to \infty.$$

So from Remark 2.1, for given $c \in \operatorname{int} P$ with $\theta << c$, such that we have $\frac{b^n}{1-b} d(Tfx_0,Tfx_1) << c$.

Hence $d(Tfx_n,Tfx_m) \le \frac{b^n}{1-b} d(Tfx_0,Tfx_1) << c$ for each $c \in \operatorname{int} P$, which implies that $d(Tfx_n,Tfx_m) << c$, for each $c \in \operatorname{int} P$.

Consequently, $\{Tf(x_n)\}$ $[resp\{Tg(x_{n+1})\}]$ is a Cauchy sequence.

Since (X,d) is a complete cone metric space the above sequence is convergent in *X*.

That is $\{Tf(x_n)\}$ $[resp\{Tg(x_{n+1})\}]$ is convergent.

Now since T is sub sequentially convergent so $f(x_n) = g(x_{n+1})$ have a convergent subsequence.

Let $f(x_{n_k}) = g(x_{n_k+1})$ be the convergent subsequence such that

$\lim_{k\to\infty} g(x_{n_k+1}) = v$. $\left[resp \lim_{k\to\infty} f(x_{n_k}) = v\right]$.

Since T is continuous, we get, $\lim_{k\to\infty} Tg(x_{n_k+1}) = Tv.$ (5)

Again as $g(X)$ is complete, so there exist $w \in X$ such that $gw = v$.

Now,

$$d(Tfw,Tgw) = d(Tfw,Tv) \le d(Tfw,Tfx_{n_i+1}) + d(Tfx_{n_i+1},Tv).$$

$$\le a_1 d(Tgw,Tgx_{n_i+1}) + a_2 d(Tfw,Tgw) + a_3 d(Tgx_{n_i+1},Tfx_{n_i+1}) + a_4 d(Tfw,Tgx_{n_i+1}) + a_5 d(Tgw,Tfx_{n_i+1}) + d(Tfx_{n_i+1},Tv).$$

From definition 1.2 (iii),
$$a_4 d(Tfw, Tgx_{n_i+1}) \le a_4\{d(Tfw, Tv) + d(Tv, Tgx_{n_i+1})\}.$$

Using definition 1.2 (iii) and (4), we get,
$$a_3 d(Tgx_{n_i+1}, Tfx_{n_i+1}) \le a_3\{d(Tgx_{n_i+1}, Tfx_{n_i}) + d(Tfx_{n_i}, Tfx_{n_i+1})\}. \le a_3 d(Tgx_{n_i+1}, Tfx_{n_i}) + a_3 b^{n_i} d(Tfx_0, Tfx_1).$$

Then from the above relation we get
$$\le a_1 d(Tgw, Tgx_{n_i+1}) + a_2 d(Tfw, Tgw) + a_3 d(Tgx_{n_i+1}, Tfx_{n_i}) + a_3 b^{n_i} d(Tfx_0, Tfx_1) +$$
$$a_4\{d(Tfw, Tv) + d(Tv, Tgx_{n_i+1})\} + a_5 d(Tgw, Tfx_{n_i+1}) + d(Tfx_{n_i+1}, Tv).$$

$$(1 - a_2 - a_4) d(Tfw, Tgw) \le a_1 d(Tgw, Tgx_{n_i+1}) + a_3 d(Tgx_{n_i+1}, Tfx_{n_i})$$
$$+ a_3 b^{n_i} d(Tfx_0, Tfx_1) + a_4 d(Tv, Tgx_{n_i+1}) + (a_5 + 1) d(Tfx_{n_i+1}, Tv).$$
$$d(Tfw, Tgw) \le \frac{a_1}{(1 - a_2 - a_4)} d(Tgw, Tgx_{n_i+1}) + \frac{a_3}{(1 - a_2 - a_4)} d(Tgx_{n_i+1}, Tfx_{n_i})$$
$$+ \frac{a_3}{(1 - a_2 - a_4)} b^{n_i} d(Tfx_0, Tfx_1) + \frac{a_4}{(1 - a_2 - a_4)} d(Tv, Tgx_{n_i+1}) + \frac{(a_5 + 1)}{(1 - a_2 - a_4)} d(Tfx_{n_i+1}, Tv).$$
$$d(Tfw, Tgw) \le A_1 d(Tv, Tgx_{n_i+1}) + A_2 d(Tgx_{n_i+1}, Tfx_{n_i}) + A_2 b^{n_i} d(Tfx_0, Tfx_1) + A_3 d(Tv, Tgx_{n_i+1})$$
$$+ A_4 d(Tfx_{n_i+1}, Tv).$$
where,
$$A_1 = \frac{a_1}{(1 - a_2 - a_4)}, A_2 = \frac{a_3}{(1 - a_2 - a_4)}, A_3 = \frac{a_4}{(1 - a_2 - a_4)}, A_4 = \frac{(a_5 + 1)}{(1 - a_2 - a_4)}.$$

Let $c \in \operatorname{int} P$ with $\theta << c$ be given. Then from (5) and using definition 1.3 we can choose a natural number *N* such that for all $i \ge N$, we have,
$$d(Tv, Tgx_{n_i+1}) << \frac{c}{5A_1},\ d(Tgx_{n_i+1}, Tfx_{n_i}) << \frac{c}{5A_2},\ d(Tv, Tgx_{n_i+1}) << \frac{c}{5A_3},\ d(Tfx_{n_i+1}, Tv) << \frac{c}{5A_4},$$

$A_2 b^{n_i} d(Tfx_0, Tfx) \to \theta\, an\, i \to \infty$, so from remark 2.1 we have, $A_2 b^{n_i} d(Tfx_0, Tfx) << \dfrac{c}{5A_2}$.

This implies $d(Tfw, Tgw) << \frac{c}{5} + \frac{c}{5} + \frac{c}{5} + \frac{c}{5} + \frac{c}{5} + \frac{c}{5} = c.$

Thus we get,
$d(Tfw, Tgw) << c$ for all $c \in \operatorname{int} P$ which implies that $d(Tfw, Tgw) = \theta.$

So it follows that $Tfw = Tgw.$ Now since *T* is one – one $fw = gw$. Hence we have $fw = gw = v$.

So, w is the coincidence point and v is the point of coincidence.

We now prove that the point of coincidence is unique.

If not let there be another point of coincidence q then there is $p \in X$ such that $fp = gp = q$.

Then we have
$$d(Tfp, Tfw) \le a_1 d(Tgp, Tgw) + a_2 d(Tgp, Tfp) + a_3 d(Tgw, Tfw) + a_4 d(Tgp, Tfw)$$
$$+ a_5 d(Tfp, Tgw).$$
$$d(Tfp, Tfw) \le a_1 d(Tfp, Tfw) + a_2 d(Tfp, Tfp) + a_3 d(Tfw, Tfw) + a_4 d(Tfp, Tfw) + a_5 d(Tfp, Tfw).$$

$$d(Tfp, Tfw) \le (a_1 + a_4 + a_5) d(Tfp, Tfw).$$

Since, $\sum_{i=1}^{5} a_i < 1$ we get $d(Tfp, Tfw) = \theta$ therefore, $Tfp = Tfw$ since T is one-one $fp = fw$. Therefore $q = v$.

Since f and g is weakly compatible mappings the unique common fixed point exist from the Proposition [2.2].

The following corollary is the main result Theorem 2.1 in [13]

Corollary 3.2: Let (X, d) be a cone metric space. Let $f, g : X \to X$ be two functions satisfying

$$d(fx, fy) \le a_1 d(gx, gy) + a_2 d(fx, gx) + a_3 d(gy, fy) + a_4 d(fx, gy) + a_5 d(gx, fy)$$

where $a_i \ge 0;\ i = 1,2,3,4,5$ and $\sum_{i=1}^{5} a_i < 1$.

Suppose if the range of f is contained in the range of g and $g(X)$ is a complete subspace. Then f and g have a unique point of coincidence. Further if f and g are weakly compatible then they have a unique common fixed point.

Proof: If we take $Tx = x$ in Theorem 3.1 then we get the result.

The following corollary is obtained which is the main result of Hardy and Rogers [14] in the setup of cone metric space as proved by M. Abbas, B.E. Rhoades [10] for normal cones.

Corollary 3.3: Let (X, d) be a complete cone metric space. Let $f : X \to X$ be two functions satisfying

$$d(fx, fy) \le a_1 d(x, y) + a_2 d(fx, x) + a_3 d(y, fy) + a_4 d(fx, y) + a_5 d(x, fy)$$

where $a_i \ge 0;\ i = 1,2,3,4,5$ and $\sum_{i=1}^{5} a_i < 1$.

Then f have a unique fixed point.

Proof: If we take g and T both as identity map in Theorem 3.1 then we get the result.

REFERENCES

1. G. Jungck, Commuting mappings and fixed points, Amer. Math. Monthly, 73 (1976), 261–263.
2. S. Sessa, On a weak commutativity condition of mappings in fixed point considerations, Publ. Inst. Math. (Beograd) (N.S.), 32 (1982), 149–153.
3. G. Jungck, Compatible mappings and common fixed p oints, International Journal of Mathematics and Mathematical Sciences, 9 (1986), 771–779.
4. G. Jungck and B.E. Rhoades, Fixed Points for set valued functions without continuity, Indian J. Pure Appl. Math., 29 (1998), 227–238.
5. L. G. Huang and X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, Journal of Mathematical Analysis and Applications, 332(2), (2007), 1468–1476.
6. M. Filipovi´c, L. Paunovi´c, S. Radenovi´c, M. Rajovi´c, Remarks on "Cone metric spaces and fixed point theorems of T-Kannan and T-Chatterjea contractive mappings", Math. Comput. Modelling. 54 (2011), pp. 1467–1472.
7. J.R. Morales, E. Rojas, Cone metric spaces and fixed point theorems of T-Kannan contractive mappings, Int. J. Math. Anal. 4 (4) (2010), pp. 175–184.
8. M. Abbas, G. Jungck, Common fixed point results for non commuting mappings without continuity in cone metric space, J. Math. Anal. Appl. 341 (2008) 416–420.
9. Sh. Rezapour and R. Hamlbarani, Some notes on the paper "Cone metric spaces and fixed point theorems of contractive mappings", J. Math. Anal. Appl., 345, (2), (2008), 719–724.
10. M. Abbas, B.E. Rhoades, Fixed and periodic point results in cone metric spaces, Appl. Math. Lett, 22 (2009), 511-515.
11. Sumitra, V.R. Uthariaraj, and R. Hemavathy, Common Fixed Point Theorem For T- Hardy-Rogers Contraction Mapping in a Cone Metric Space, International Mathematical Forum5(2010), 1495-1506.
12. J. Morales, E. Rojas, T-Zamfirescu and T-weak Contraction Mappings on Cone Metric Spaces, arxiv: 0909.1255v1.math. FA (2009).

13. G. Song, X. Sun, Y. Zhao, and G. Wang, New common fixed point theorems for maps on cone metric spaces, Appl. Math. Lett. 23 (2010), pp. 1033–1037.
14. G.E. Hardy, T.D. Rogers, A generalization of a fixed point theorem of Reich, Canada Math. Bull. 1 (6) (1973), pp. 201–206.
15. G. Jungck, S. Radenović, S. Radojević, and V. Rakočević, Common fixed point theorems for weakly compatible pairs on cone metric spaces, Fixed Point Theory Appl., vol. 2009, ArticleID 643840.
16. S. Janković, Z. Kadelburg, S. Radenović, and B.E. Rhoades, Assad-fixed-type point theorems for a pair of nonself mappings on cone metric spaces, Fixed Point Theory Appl., vol.2009, Article ID 761086, 16pages, doi:10.1155/2009/761086.
17. Gh. Soleimani Rad., New fixed and periodic point results on cone metric spaces, Journal of Linear and Topological Algebra, Vol. 01, No. 01, Summer 2012, 33- 40.
18. J. Morales and E. Rojas, Cone metric spaces and fixed point theorems of T−contractive mappings, Revista Notas de Matemática Vol.4(2),No. 269, 2008, pp. 66-78 http://www.saber.ula.ve/notasdematematica/.
19. Beiranvand A., Moradi S., Omid M. and Pazandeh H., Two fixed point theorem for special mappings, arxiv: 0903.1504 v1 [math.FA].

Source of support: Nil, Conflict of interest: None Declared.