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ARITHMETIC LABELING OF $C_m \times P_n$ AND $P_m \times P_n$

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ABSTRACT

Acharya.B.D. and Hedge.S.M.was introduced the concept of arithmetic labeling and many research articles have published in this topic. In this paper, we have proved that the Cartesian product of $C_m \times P_n$ (where m is odd) and $P_m \times P_n$ are arithmetic graphs. Also we established a general formula to find the labeling the vertices of the graph G.

Key words: *Cartesian product, Labeling, Arithmetic Labeling.*

INTRODUCTION

In 1989 Acharya.B.D.andHedge.S.M. Introduced a new version of sequential graph known as arithmetic graph and is defined as follows: Let G is a graph with q edges a and d are the positive integers. A labeling f of G is said to be (a, d) – arithmetic if the vertices are labeled by distinct nonnegative integers and the edge labels are induced by $f(x) + f(y)$ for each xy are in the form of $a, a + d, a + 2d, \dots, a + (q - 1)d$. A graph is called arithmetic if it is an (a, d) - arithmetic for some a and d.

Definition 1.1: A **graph** G is an ordered pair $(V(G), E(G))$ consisting of a non empty set $V(G)$ of vertices and a set $E(G)$, disjoint from $V(G)$, of edges, together with an incidence function ψ_G that associates with each edge of G *is* an unordered pair of (not necessarily distinct) vertices of G.

Definition 1.2: Walk is an alternating sequence of vertices and edges starting and ending with vertices.
A walk in which all the vertices are distinct is called a **path.**
A closed path is called a **cycle**.

Definition 1.3: The **cartesian product** of the graphs G and H is denoted by $G \times H$ and defined the vertex set of $G \times H$ is the cartesian product $V(G) \times V(H)$ and any two vertices (u, u') and (v, v') are adjacent in $G \times H$ if and only if either $u = v$ and u' is adjacent with v' in H or $u' = v'$ and u is adjacent with v in G.

Definition 1.4: A **labeling** or valuation of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$.

Definition 1.5: A graph is said to be arithmeticif its vertices can be assigned distinct non negative integers in such a way that the value of the edges are obtained as sum of the values assigned in an arithmetic progression.

Lemma 2.1: Let G be the graph consisting of a cycle $C_{2t+1} = (u_1, u_2, \dots, u_{2t+1}, u_1)$ and a path$P_3 = (u_{2t-1}, w, u_{2t+1})$. Then G isan $(t + 2,1)$, $t \geq 1$arithmetic graph.

Lemma 2.2: The graph $P_m * P_n$ is an$(2y + 1,2)$- arithmetic graph for all m, n and for any non negative integer y. where $y = f(v_{1,1})$

Theorem 2.3: Let the graph G is the cartesian product of C_m and P_n. That is $G = C_m \times P_n$Where$m, n > 0$and m is odd is an $(a, 1)$ - arithmetic graph.

Proof: Let $G = C_m \times P_n$ where $m, n > 0$ and m is odd. Then the graph is given in (figure: 1) as below.

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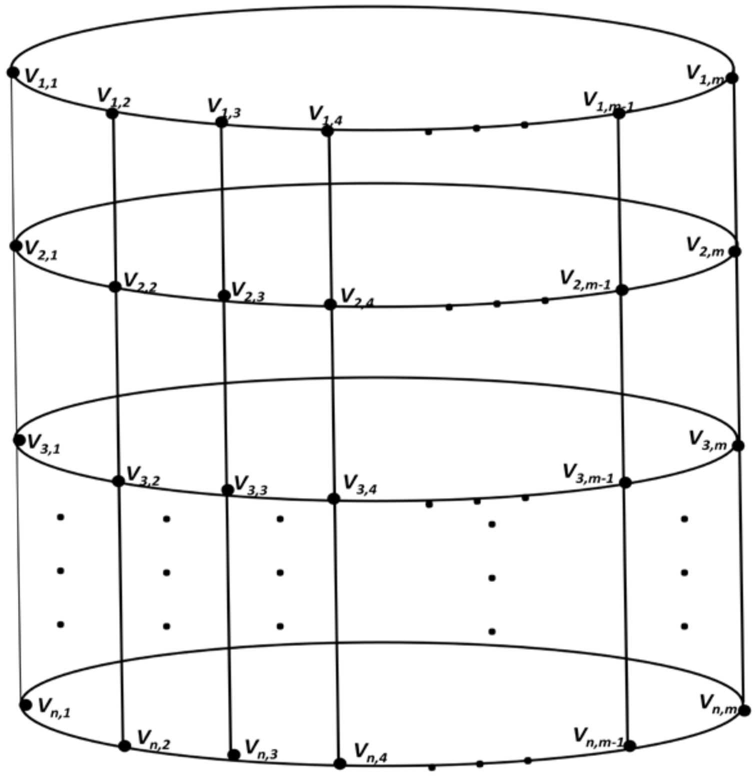

Figure: 1

Let V be the vertex set of G and is denoted by $V(G) = \{V_{ij}/1 \le i \le n; 1 \le j \le m\}$

Define $f: V(G) \to N$.Now we are giving the label to the vertices of G as below

$$f(v_{2r-1,2k-1}) = m((2r-1)-1) + k \text{ where} 1 \le k \le \frac{m+1}{2};$$
$$1 \le r \le \frac{n+1}{2} \ \forall \ odd\ n \text{ (Or) } 1 \le r \le \frac{n}{2} \ \forall \ even\ n \tag{1}$$

$$f(v_{2r-1,2k}) = m((2r-1)-1) + \left(\frac{m+1}{2}\right) + k \text{ where } 1 \le k \le \frac{m-1}{2}$$
$$1 \le r \le \frac{n+1}{2} \ \forall \ odd\ n \text{ (Or) } 1 \le r \le \frac{n}{2} \ \forall \ even\ n \tag{2}$$

$$f(v_{2r,2k-1}) = m(2r) - \left(\frac{m-1}{2}\right) + (k-1) \text{ where} 1 \le k \le \frac{m+1}{2};$$
$$1 \le r \le \frac{n-1}{2} \ \forall \ odd\ n \text{ (Or) } 1 \le r \le \frac{n}{2} \ \forall \ even \tag{3}$$

$$f(v_{2r,2k}) = m(2r-1) + k \text{ where } 1 \le k \le \frac{m-1}{2};$$
$$1 \le r \le \frac{n-1}{2} \ \forall \ odd\ n \text{ (Or) } 1 \le r \le \frac{n}{2} \ \forall \ even\ n \tag{4}$$

Clearly all the vertices of $G = C_m \times P_n$ which are labeled by distinct labels from 1 to mn. The edge labels induced by $f(uv) = f(u) + f(v)$ for each uv are

$$a, a+d, a+2d, \ldots, a+(q-1)d.$$

Therefore, the graph $G = C_m \times P_n$ is an$(a, 1)$ - Arithmetic graph.

Example 2.4: Consider the graph $G = C_7 \times P_5$

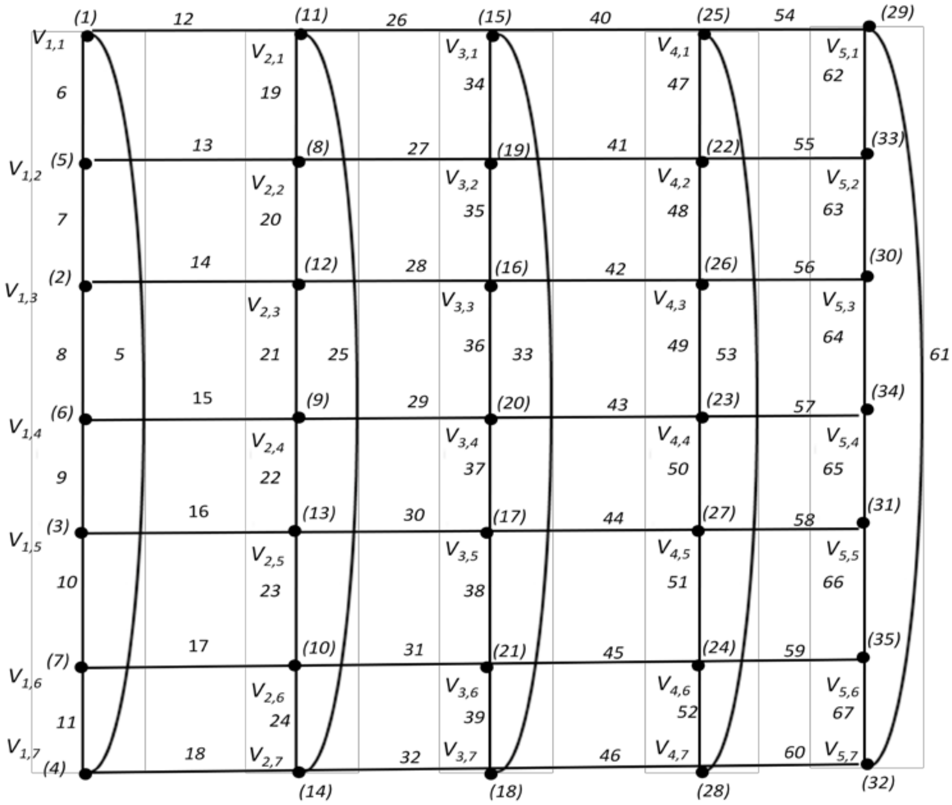

Figure: 2

Here, $m = 7; n = 5$ and $q = 63$

The vertex labels are given below.

Equation (1) $\Rightarrow f(v_{2r-1,2k-1}) = m((2r-1)-1)+k$
where $1 \le k \le \frac{m+1}{2}$; $1 \le r \le \frac{n+1}{2}$ $\forall$ $odd\ n$

When $r = 1,2,3$ and $k = 1,2,3,4$ $\Rightarrow$ $f(v_{1,1}) = 1$; $f(v_{1,3}) = 2$;
$f(v_{1,5}) = 3$; $f(v_{1,7}) = 4$; $f(v_{3,1}) = 15$; $f(v_{3,3}) = 16$; $f(v_{3,5}) = 17$;
$f(v_{3,7}) = 18$; $f(v_{5,1}) = 29$; $f(v_{5,3}) = 30$; $f(v_{5,5}) = 31$; $f(v_{5,7}) = 32$.

Equation (2)$\Rightarrow f(v_{2r-1,2k}) = m((2r-1)-1)+\left(\frac{m+1}{2}\right)+k$
where $1 \le k \le \frac{m-1}{2}$; $1 \le r \le \frac{n+1}{2}$ $\forall$ $odd\ n$.

When $r = 1,2,3$and $k = 1,2,3$ $\Rightarrow$ $f(v_{1,2}) = 5$; $f(v_{1,4}) = 6$; $f(v_{1,6}) = 7$;
$f(v_{3,2}) = 19$; $f(v_{3,4}) = 20$; $f(v_{3,6}) = 21$; $f(v_{5,2}) = 33$;
$f(v_{5,4}) = 34$; $f(v_{5,6}) = 35$.

Equation (3) $\Rightarrow f(v_{2r,2k-1}) = m(2r)-\left(\frac{m-1}{2}\right)+(k-1)$ where $1 \le k \le \frac{m+1}{2}$; $1 \le r \le \frac{n-1}{2}$ $\forall$ $odd\ n$

When $r = 1,2$ and $k = 1,2,3,4$ $\Rightarrow f(v_{2,1}) = 11$; $f(v_{2,3}) = 12$;
$f(v_{2,5}) = 13$; $f(v_{2,7}) = 14$; $f(v_{4,1}) = 25$; $f(v_{4,3}) = 26$; $f(v_{4,5}) = 27$;
$f(v_{4,7}) = 28$.

Equation (4) $\Rightarrow f(v_{2r,2k}) = m(2r-1)+k$ where $1 \le k \le \frac{m-1}{2}; 1 \le r \le \frac{n-1}{2}$ $\forall$ $odd\ n$

When $r = 1,2$ and $k = 1,2,3$ $\Rightarrow$ $f(v_{2,2}) = 8$; $f(v_{2,4}) = 9$;
$f(v_{2,6}) = 10$; $f(v_{4,2}) = 22$; $f(v_{4,4}) = 23$; $f(v_{4,6}) = 24$.

In this graph, $a = 5$ and $d = 6 - 5 = 1$.

The edge labels are in the arithmetic progression

$$a = 5, a + d = 6, a + 2d = 7, a + 3d = 8, a + 4d = 9, a + 5d = 10,$$
$$a + 6d = 11, a + 7d = 12, a + 8d = 13, a + 9d = 14, a + 10d = 15,$$
$$a + 11d = 16, a + 12d = 17, a + 13d = 18, a + 14d = 19,$$
$$a + 15d = 20, a + 16d = 21, a + 17d = 22, a + 18d = 23,$$
$$a + 19d = 24, a + 20d = 25, a + 21d = 26, a + 22d = 27,$$
$$a + 23d = 28, a + 24d = 29, a + 25d = 30, a + 26d = 31,$$
$$a + 27d = 32, a + 28d = 33, a + 29d = 34, a + 30d = 35,$$
$$a + 31d = 36, a + 32d = 37, a + 33d = 38, a + 34d = 39,$$
$$a + 35d = 40, a + 36d = 41, a + 37d = 42, a + 38d = 43,$$
$$a + 39d = 44, a + 40d = 45, a + 41d = 46, a + 42d = 47,$$
$$a + 43d = 48, a + 44d = 49, a + 45d = 50, a + 46d = 51,$$
$$a + 47d = 52, a + 48d = 53, a + 49d = 54, a + 50d = 55,$$
$$a + 51d = 56, a + 52d = 57, a + 53d = 58, a + 54d = 59,$$
$$a + 55d = 60, a + 56d = 61, a + 57d = 62, a + 58d = 63,$$
$$a + 59d = 64, a + 60d = 65, a + 61d = 66,$$
$$a + (q-1)d = a + 62d = 67.$$

Then the graph $G = C_7 \times P_5$ is an $(5,1)$ - arithmetic graph.

Corollary 2.5: The graph G = $C_{2t+1} \times P_n$ is an (t+2(f ($v_{1,1}$)) ,1) – arithmetic graph.

Proof: Let the graph G is the cartesian product of C_{2t+1} and $P_n \; \forall\, t, n > 0$. Then the vertex labeling of G is given below.

$f(v_{1,1})$ = Any one positive integer.

$$f(v_{2r-1,2k-1}) = m((2r-1)-1) + k + (f(v_{1,1}) - 1)$$
where $1 \le k \le \frac{m+1}{2}; 1 \le r \le \frac{n+1}{2} \;\forall\; odd\; n.$
(Or) $1 \le r \le \frac{n}{2} \;\forall\; even\; n$ (1)

$$f(v_{2r-1,2k}) = m((2r-1)-1) + \left(\frac{m+1}{2}\right) + k + (f(v_{1,1}) - 1)$$
where$1 \le k \le \frac{m-1}{2}; 1 \le r \le \frac{n+1}{2} \;\forall\; odd\; n.$
(Or) $1 \le r \le \frac{n}{2} \;\forall\; even\; n$ (2)

$$f(v_{2r,2k-1}) = m(2r) - \left(\frac{m-1}{2}\right) + (k-1) + (f(v_{1,1}) - 1)$$
where$1 \le k \le \frac{m+1}{2}; 1 \le r \le \frac{n-1}{2} \;\forall\; odd\; n.$
(or)$1 \le r \le \frac{n}{2} \;\forall\; even\; n$ (3)

$$f(v_{2r,2k}) = m(2r-1) + k + (f(v_{1,1}) - 1)$$
where $1 \le k \le \frac{m-1}{2}; 1 \le r \le \frac{n-1}{2} \;\forall\; odd\; n$
(Or) $1 \le r \le \frac{n}{2} \;\forall\; even\; n$ (4)

Clearly all the vertices which are denoted by distinct labels from $f(v_{1,1})$ to $(mn + f(v_{1,1}) - 1)$.

The edge labels induced by $f(uv) = f(u) + f(v)$ for each uv are $a, a+d, a+2d, \dots, a+(q-1)d$.

Then the graph $G = C_{2t+1} \times P_n$ is an $(t + 2(f(v_{1,1})), 1)$ - arithmetic graph.

Example 2.6: Consider the graph $G = C_5 \times P_4$.

Here, $m = 5; n = 4$ and $q = 35$

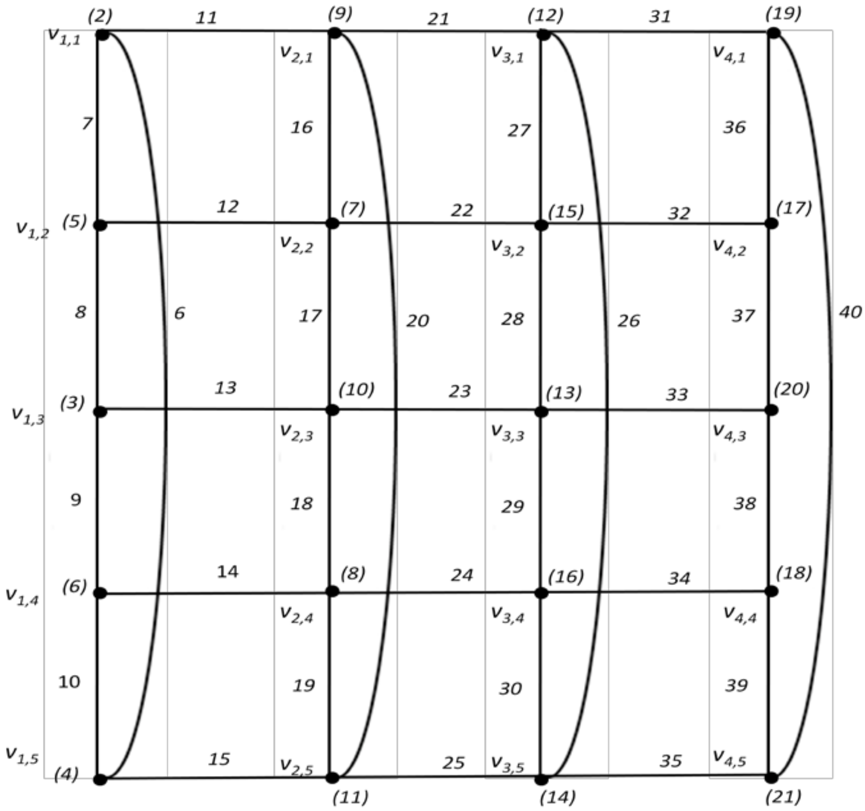

Figure: 3

The vertex labels are given below:

Let $f(v_{1,1}) = 2$

Equation (1) $\Rightarrow f(v_{2r-1,2k-1}) = m((2r-1)-1) + k + (f(v_{1,1}) - 1)$
where$1 \le k \le \frac{m+1}{2}; 1 \le r \le \frac{n}{2}\ \forall\ even\ n$

When $r = 1{,}2$and$k = 1{,}2{,}3 \Rightarrow f(v_{1,3}) = 3$; $f(v_{1,5}) = 4$;
$f(v_{3,1}) = 12$; $f(v_{3,3}) = 13$; $f(v_{3,5}) = 14$.

Equation (2) $\Rightarrow f(v_{2r-1,2k}) = m((2r-1)-1) + \left(\frac{m+1}{2}\right) + k + (f(v_{1,1}) - 1)$
where $1 \le k \le \frac{m-1}{2}; 1 \le r \le \frac{n}{2}\ \forall\ even\ n.$

When $r = 1{,}2$ and$k = 1{,}2 \Rightarrow f(v_{1,2}) = 5$; $f(v_{1,4}) = 6$; $f(v_{3,2}) = 15$;
$f(v_{3,4}) = 16$.

Equation (3) $\Rightarrow f(v_{2r,2k-1}) = m(2r) - \left(\frac{m-1}{2}\right) + (k-1) + (f(v_{1,1}) - 1)$
where $1 \le k \le \frac{m+1}{2}; 1 \le r \le \frac{n}{2}\ \forall\ even\ n$

When $r = 1{,}2\ and\ k = 1{,}2{,}3 \Rightarrow f(v_{2,1}) = 9$; $f(v_{2,3}) = 10$; $f(v_{2,5}) = 11$;
$f(v_{4,1}) = 19$; $f(V_{4,3}) = 20$; $f(V_{4,5}) = 21$.

Equation (4) $\Rightarrow f(v_{2r,2k}) = m(2r-1) + k + (f(v_{1,1}) - 1)$
where $1 \le k \le \frac{m-1}{2}; 1 \le r \le \frac{n}{2}\ \forall\ even\ n.$
When $r = 1{,}2\ and\ k = 1{,}2 \Rightarrow f(v_{2,2}) = 7$; $f(v_{2,4}) = 8$; $f(v_{4,2}) = 17$;
$f(v_{4,4}) = 18$

In this graph $a = 6$ and $d = 7 - 6 = 1$

The edge labels are in the arithmetic progression

$$a = 6, a + d = 7, a + 2d = 8, a + 3d = 9, a + 4d = 10, a + 5d = 11,$$
$$a + 6d = 12, a + 7d = 13, a + 8d = 14, a + 9d = 15, a + 10d = 16,$$
$$a + 11d = 17, a + 12d = 18, a + 13d = 19, a + 14d = 20,$$
$$a + 15d = 21, a + 16d = 22, a + 17d = 23, a + 18d = 24,$$
$$a + 19d = 25, a + 20d = 26, a + 21d = 27, a + 22d = 28,$$
$$a + 23d = 29, a + 24d = 30, a + 25d = 31, a + 26d = 32,$$
$$a + 27d = 33, a + 28d = 34, a + 29d = 35, a + 30d = 36,$$
$$a + 31d = 37, a + 32d = 38, a + 33d = 39,$$
$$a + (q - 1)d = a + 34d = 40.$$

Here t = 2, (t + 2 (f ($v_{1,1}$)), 1) = (6,1).

Therefore, the graph $G = C_5 \times P_4$ is an (6,1)- arithmetic graph.

Theorem 2.7: Let the graph G is the cartesian product of $P_m$andP_n. That is $G = P_m \times P_n$ where $m, n > 0$ is an $(a, 2)$-arithmetic graph.

Proof: Let $G = P_m \times P_n$ where $m, n > 0$. Then the graph is given in the (figure: 4) as below.

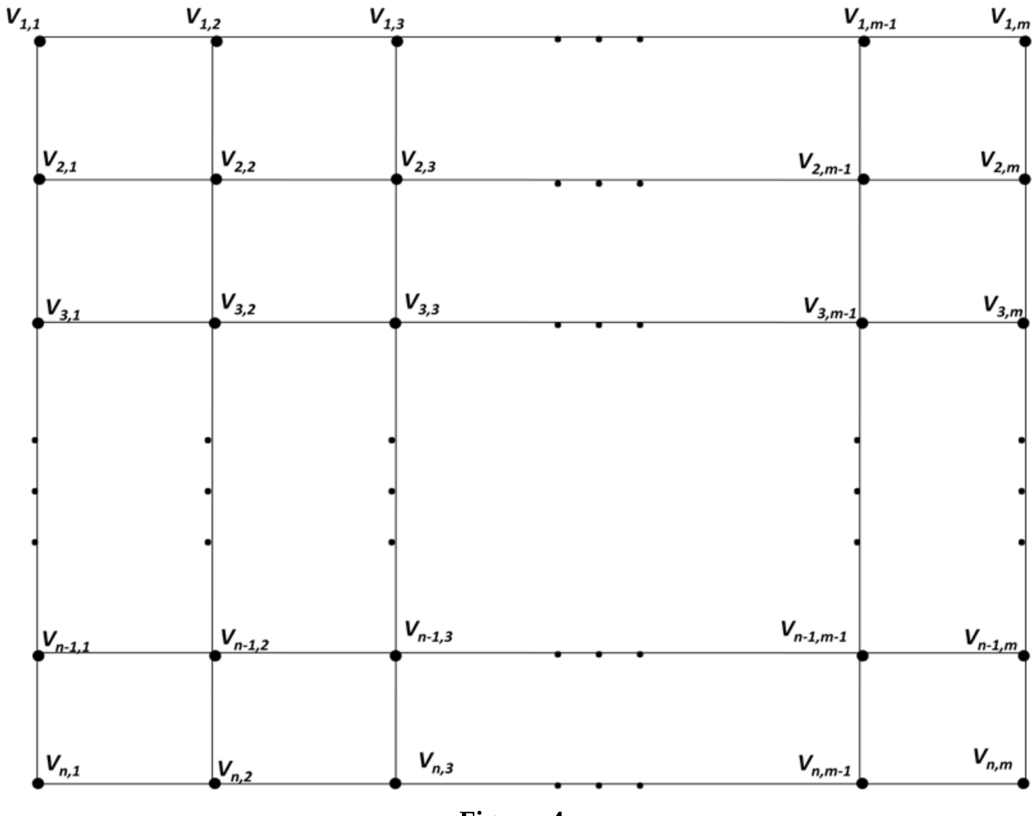

Figure: 4

Let V be the vertex set of G and is denoted by$V(G) = \{V_{ij}/1 \leq i \leq n; 1 \leq j \leq m\}$

Define $f: V(G) \rightarrow N$.

Now we are giving the label to the vertices of G as below:

$$f(v_{ij}) = (2m - 1)(i - 1) + j$$
$$\text{where } 1 \leq i \leq n; 1 \leq j \leq m.$$

Clearly all the vertices of $G = P_m \times P_n$ are labeled by distinct labels from 1 to $q + 1$.

The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows

$$f(v_{i,k}v_{i,k+1}) = (2k + 1) + 2(2m - 1)(i - 1)$$
$$\text{where } 1 \leq i \leq n; 1 \leq k \leq m - 1.$$

$$f(v_{r,j}v_{r+1,j}) = 2m + (2j-1) + 2(2m-1)(r-1)$$
$$\text{where } 1 \le r \le n-1; 1 \le j \le m.$$

Clearly the edges are labeled as $f(E(G)) = \left\{ \begin{matrix} a, a+d, a+2d, \dots, \\ a+(q-1)d \end{matrix} \right\}$.

Therefore, f is an arithmetic labeling.

Hence the graph $G = P_m \times P_n$ is an $(a, 2)$- arithmetic graph.

Example 2.8: Consider the graph $G = P_4 \times P_4$

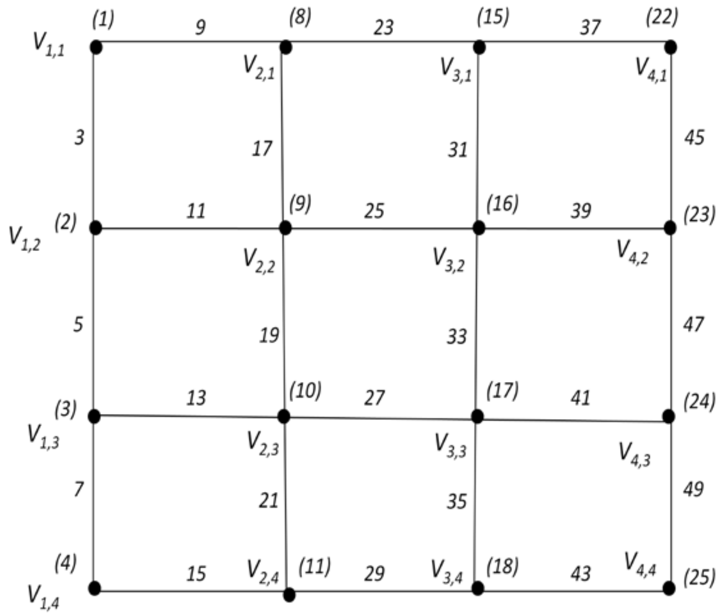

Figure: 5

Here, $m = 4; n = 4$ and $q = 24$

The vertex labels are given below.
$f(v_{ij}) = (2m-1)(i-1) + j$ Where $1 \le i \le n; 1 \le j \le m$.

When $i = 1,2,3,4$ and $j = 1,2,3,4 \Rightarrow f(v_{1,1}) = 1;\ f(v_{1,2}) = 2;\ f(v_{1,3}) = 3;$
$f(v_{1,4}) = 4;\ f(v_{2,1}) = 8;\ f(v_{2,2}) = 9;\ f(v_{2,3}) = 10;\ f(v_{2,4}) = 11;$
$f(v_{3,1}) = 15;\ f(v_{3,2}) = 16;\ f(v_{3,3}) = 17;\ f(v_{3,4}) = 18;$
$f(v_{4,1}) = 22;\ f(v_{4,2}) = 23;\ f(v_{4,3}) = 24;\ f(v_{4,4}) = 25.$

The edge labels are given below.
$$f(v_{i,k}v_{i,k+1}) = (2k+1) + 2(2m-1)(i-1)$$
$$\text{where } 1 \le i \le n; 1 \le k \le m-1$$

When $i = 1,2,3,4\ and\ k = 1,2,3 \Rightarrow f(v_{1,1}v_{1,2}) = 3; f(v_{1,2}v_{1,3}) = 5; f(v_{1,3}v_{1,4}) = 7;$
$f(v_{2,1}v_{2,2}) = 17; f(v_{2,2}v_{2,3}) = 19; f(v_{2,3}v_{2,4}) = 21;$
$f(v_{3,1}v_{3,2}) = 31; f(v_{3,2}v_{3,3}) = 33; f(v_{3,3}v_{3,4}) = 35;$
$f(v_{4,1}v_{4,2}) = 45; f(v_{4,2}v_{4,3}) = 47; f(v_{4,3}v_{4,4}) = 49.$
$$f(v_{r,j}v_{r+1,j}) = 2m + (2j-1) + 2(2m-1)(r-1)$$
$$\text{where } 1 \le r \le n-1; 1 \le j \le m$$

When $r = 1,2,3$ andj= 1,2,3,4 $\Rightarrow f(v_{1,1}v_{2,1}) = 9; f(v_{2,1}v_{3,1}) = 23;$
$f(v_{3,1}v_{4,1}) = 37; f(v_{1,2}v_{2,2}) = 11; f(v_{2,2}v_{3,2}) = 25;$
$f(v_{3,2}v_{4,2}) = 39; (v_{1,3}v_{2,3}) = 13, f(v_{2,3}v_{3,3}) = 27;$
$f(v_{3,3}v_{4,3}) = 41; f(v_{1,4}v_{2,4}) = 15; f(v_{2,4}v_{3,4}) = 29;$
$f(v_{3,4}v_{4,4}) = 43$

In this graph $a = 3$ and $d = 5 - 3 = 2$

The edge labels are in the arithmetic progression

$$a = 3, a + d = 5, a + 2d = 7, a + 3d = 9, a + 4d = 11,$$
$$a + 5d = 13, a + 6d = 15, a + 7d = 17, a + 8d = 19, a + 9d = 21,$$
$$a + 10d = 23, a + 11d = 25, a + 12d = 27, a + 13d = 29,$$
$$a + 14d = 31, a + 15d = 33, a + 16d = 35, a + 17d = 37,$$
$$a + 18d = 39, a + 19d = 41, a + 20d = 43, a + 21d = 45,$$
$$a + 22d = 47, a + (q - 1)d = a + 23d = 49.$$

Therefore, the graph $G = P_4 \times P_4$ is an (3,2)- arithmetic graph.

Corollary 2.9: The graph G = $P_m \times P_n$ is an (2y+1,2) - arithmetic graph.Where y =$f(v_{1,1})$.

Proof: Let the graph G is the cartesian product of $P_m$andP_n $\forall\ m, n > 0$. Then the vertex labeling of G is given below.

$f(v_{1,1})$ = any one positive integer

$$f(v_{ij}) = (2m - 1)(i - 1) + j + (f(v_{1,1}) - 1)$$
where $1 \le i \le n; 1 \le j \le m$

Clearly all the vertices which are denoted by distinct labels from

$f(v_{1,1})$ to $q + f(v_{1,1})$.

The edge labeling of G is given as below:

$$f(v_{i,k}v_{i,k+1}) = (2k + 1) + 2(2m - 1)(i - 1) + 2(f(v_{1,1}) - 1)$$
where $1 \le i \le n; 1 \le k \le m - 1$

$$f(v_{r,j}v_{r+1,j}) = 2m + (2j - 1) + 2(2m - 1)(r - 1) + 2(f(v_{1,1}) - 1)$$
where $1 \le r \le n - 1; 1 \le j \le m$

The induced edge labels are $f(E(G)) = \{a, a + d, a + 2d, a + 3d, \dots, a + (q - 1)d\}$. Then f is an arithmetic labeling.

Hence the graph $G = P_m \times P_n$is an $(2y + 1,2\ )$- arithmetic graph. Where $y = f(v_{1,1})$.

Example 2.10: Consider the graph $G = P_5 \times P_4$

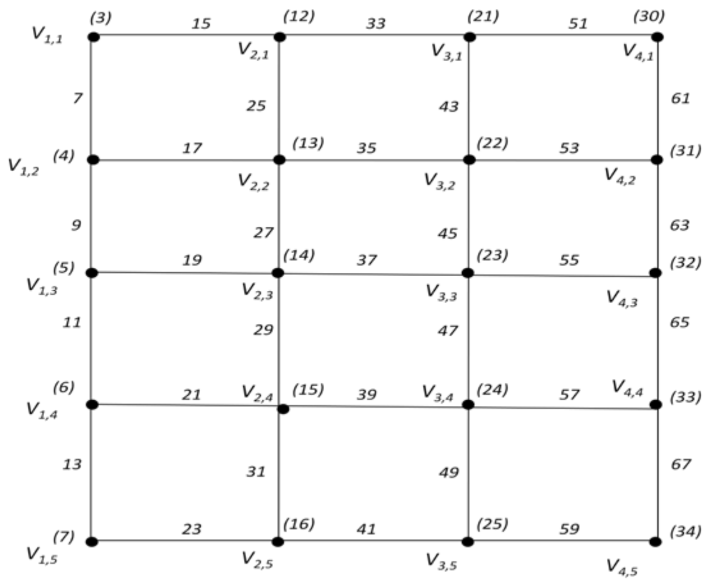

Figure: 6

Here, $m = 5; n = 4$ and $q = 31$

The vertex labels are given below

Let $f(v_{1,1}) = 3$

$$f(v_{ij}) = (2m - 1)(i - 1) + j + (f(v_{1,1}) - 1)$$
where $1 \le i \le n; 1 \le j \le m.$

When $i = 1,2,3,4$and $j = 1,2,3,4,5 \Rightarrow f(v_{1,2}) = 4; f(v_{1,3}) = 5; f(v_{1,4}) = 6;$
$f(v_{1,5}) = 7; f(v_{2,1}) = 12; f(v_{2,2}) = 13; f(v_{2,3}) = 14;$
$f(v_{2,4}) = 15; f(v_{2,5}) = 16; f(v_{3,1}) = 21; f(v_{3,2}) = 22;$
$f(v_{3,3}) = 23; f(v_{3,4}) = 24; f(v_{3,5}) = 25; f(v_{4,1}) = 30;$
$f(v_{4,2}) = 31; f(v_{4,3}) = 32; f(v_{4,4}) = 33; f(v_{4,5}) = 34.$

The edge labels are given below.

$$f(v_{i,k}v_{i,k+1}) = (2k+1) + 2(2m-1)(i-1) + 2(f(v_{1,1}) - 1)$$
$$\text{where } 1 \le i \le n; 1 \le k \le m-1$$

When $i = 1,2,3,4$ and $k = 1,2,3,4 \Rightarrow f(v_{1,1}v_{1,2}) = 7; f(v_{1,2}v_{1,3}) = 9;$
$f(v_{1,3}v_{1,4}) = 11; f(v_{1,4}v_{1,5}) = 13; f(v_{2,1}v_{2,2}) = 25; f(v_{2,2}v_{2,3}) = 27;$
$f(v_{2,3}v_{2,4}) = 29; f(v_{2,4}v_{2,5}) = 31; f(v_{3,1}v_{3,2}) = 43; f(v_{3,2}v_{3,3}) = 45;$
$f(v_{3,3}v_{3,4}) = 47; f(v_{3,4}v_{3,5}) = 49; f(v_{4,1}v_{4,2}) = 61; f(v_{4,2}v_{4,3}) = 63;$
$f(v_{4,3}v_{4,4}) = 65; f(v_{4,4}v_{4,5}) = 67.$

$$f(v_{r,j}v_{r+1,j}) = 2m + (2j-1) + 2(2m-1)(r-1) + 2(f(1,1) - 1)$$

Where $1 \le r \le n-1; 1 \le j \le m$

When $r = 1,2,3$and $j = 1,2,3,4,5 \Rightarrow f(v_{1,1}v_{2,1}) = 15; f(v_{2,1}v_{3,1}) = 33;$
$f(v_{3,1}v_{4,1}) = 51; f(v_{1,2}v_{2,2}) = 17; f(v_{2,2}v_{3,2}) = 35; f(v_{3,2}v_{4,2}) = 53;$
$f(v_{1,3}v_{2,3}) = 19; f(v_{2,3}v_{3,3}) = 37; f(v_{3,3}v_{4,3}) = 55; f(v_{1,4}v_{2,4}) = 21;$
$f(v_{2,4}v_{3,4}) = 39; f(v_{3,4}v_{4,4}) = 57; f(v_{1,5}v_{2,5}) = 23; f(v_{2,5}v_{3,5}) = 41;$
$f(v_{3,5}v_{4,5}) = 59.$

In this graph $a = 7$ *and* $d = 9 - 7 = 2$.

The edge labels are in the arithmetic progression

$$a = 7, a+d = 9, a+2d = 11, a+3d = 13, a+4d = 15, a+5d = 17,$$
$$a+6d = 19, a+7d = 21, a+8d = 23, a+9d = 25, a+10d = 27,$$
$$a+11d = 29, a+12d = 31, a+13d = 33, a+14d = 35,$$
$$a+15d = 37, a+16d = 39, a+17d = 41, a+18d = 43, a+19d = 45,$$
$$a+20d = 47, a+21d = 49, a+22d = 51, a+23d = 53, a+24d = 55,$$
$$a+25d = 57, a+26d = 59, a+27d = 61, a+28d = 63, a+29d = 65,$$
$$a+(q-1)d = a+30d = 67.$$

Here $y = 3, (2y+1,2) = (7,2)$

Therefore, the graph $G = C_5 \times P_4$ is an (7,2) - arithmetic graph.

CONCLUSION

In this paper we have studied about arithmetic labeling of $C_m \times P_n$ $\forall$ odd m and $P_m \times P_n$. We assure that it will help to the new researches in this area.

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