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HIGHER ORDER FIBONACCI SERIES

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ABSTRACT

The Fibonacci numbers were first introduced in 1202 by Leonardo of Pisa, better known today as Fibonacci, in his book Liber abaci. The Fibonacci number sequence appeared in the solution to the following problem:

"A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?"

Higher order Fibonacci series is an extension of existing Fibonacci series which shows interesting pattern of the 'golden ratio' of the higher order Fibonacci series which has been shown here.

CONTENT

The very well-known Fibonacci series gives us a pattern like 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,….

The sequence can be formulated as

$F(0) = 0$
$F(1) = 1$
$F(n) = F(n-1) + F(n-2)$ where $n > 1$

So n^{th} order number in Fibonacci sequence is sum of the previous 2 numbers.

Now the ratio of any 2 consecutive Fibonacci numbers always approaches φ which is called *Golden* or divine ratio as seen by Johannes Kepler.

$$\varphi \approx 1.618 = \frac{(1+\sqrt{5})}{2}$$

Now if we consider the Fibonacci series as last 2 numbers sum and then if we take last 3 numbers, last 4 numbers, last 5 numbers and so on then we will have the series like shown below.

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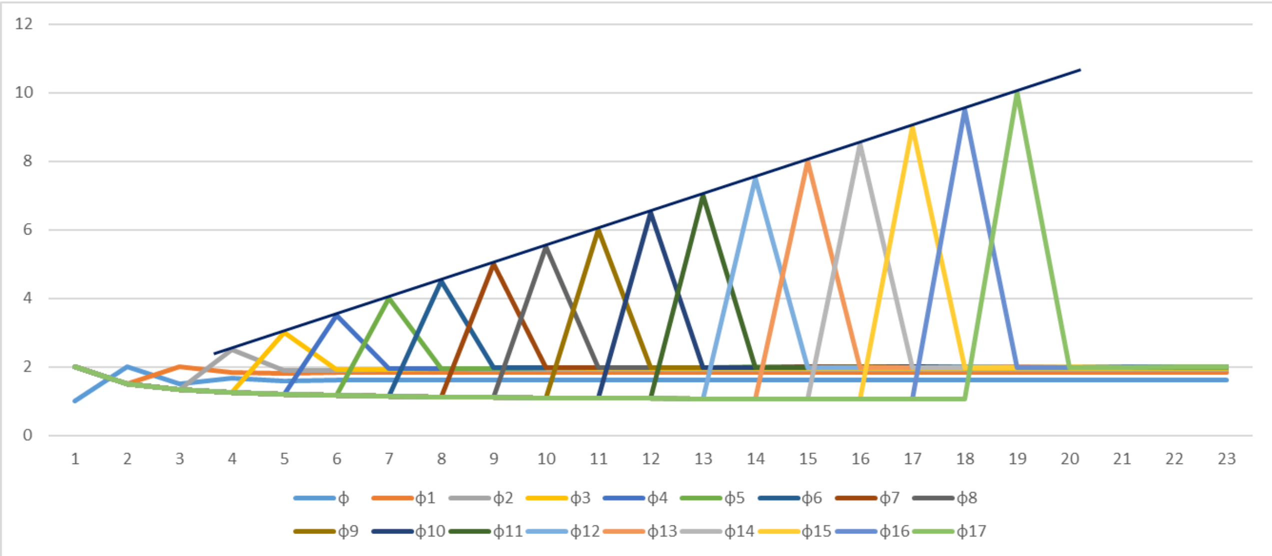
conventional Fibonacci series(2 number sum)	3 number sum series	4 number sum series	5 number sum series	6 number sum series	7 number sum series	8 number sum series	9 number sum series	10 number sum series	11 number sum series	12 number sum series
1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	2	2	2	2	2
2	3	3	3	3	3	3	3	3	3	3
3	6	4	4	4	4	4	4	4	4	4
5	11	10	5	5	5	5	5	5	5	5
8	20	19	15	6	6	6	6	6	6	6
13	37	36	29	21	7	7	7	7	7	7
21	68	69	56	41	28	8	8	8	8	8
34	125	134	109	80	55	36	9	9	9	9
55	230	258	214	157	108	71	45	10	10	10
89	423	497	423	310	213	140	89	55	11	11
144	778	958	831	615	422	277	176	109	66	12
233	1431	1847	1633	1224	839	550	349	216	131	78
377	2632	3560	3210	2427	1672	1095	694	429	260	155
610	4841	6862	6311	4813	3337	2184	1383	854	517	308
987	8904	13227	12408	9546	6646	4361	2760	1703	1030	613
1597	16377	25496	24393	18935	13237	8714	5513	3400	2055	1222
2584	30122	49145	47955	37560	26366	17392	11018	6793	4104	2439
4181	55403	94730	94277	74505	52519	34713	22027	13578	8201	4872
6765	101902	182598	185344	147786	104616	69286	44009	27147	16394	9737
10946	187427	351969	364377	293145	208393	138295	87929	54284	32779	19466
17711	344732	678442	716346	581477	415114	276040	175682	108513	65548	38923
28657	634061	1307739	1408299	1153408	826891	550985	351015	216917	131085	77836
46368	1166220	2520748	2768643	2287881	1647136	1099786	701336	433618	262104	155661

***Only 12 number series are shown here, however 'n' number of such series can be made.*

Now if we calculate ratio of any 2 numbers(like golden ratio φ for Fibonacci series) for these '*n*' number sum series and I demark it as φ^1, φ^2, φ^3 and so on then we will see φ^n will tend to reach '2' but will never cross it.(Except one spur)

ϕ	ϕ1	ϕ2	ϕ3	ϕ4	ϕ5	ϕ6	ϕ7	ϕ8	ϕ9	ϕ10	ϕ11	ϕ12	ϕ13	ϕ14	ϕ15	ϕ16	ϕ17
1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
1.5	2	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333	1.333333
1.666666667	1.833333	2.5	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
1.6	1.818182	1.9	3	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
1.625	1.85	1.894737	1.933333	3.5	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667	1.166667
1.615384615	1.837838	1.916667	1.931034	1.952381	4	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857	1.142857
1.619047619	1.838235	1.942029	1.946429	1.95122	1.964286	4.5	1.125	1.125	1.125	1.125	1.125	1.125	1.125	1.125	1.125	1.125	1.125
1.617647059	1.84	1.925373	1.963303	1.9625	1.963636	1.972222	5	1.111111	1.111111	1.111111	1.111111	1.111111	1.111111	1.111111	1.111111	1.111111	1.111111
1.618181818	1.83913	1.926357	1.976636	1.974522	1.972222	1.971831	1.977778	5.5	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
1.617977528	1.839243	1.927565	1.964539	1.983871	1.981221	1.978571	1.977528	1.981818	6	1.090909	1.090909	1.090909	1.090909	1.090909	1.090909	1.090909	1.090909
1.618055556	1.839332	1.927975	1.965102	1.990244	1.988152	1.98556	1.982955	1.981651	1.984848	6.5	1.083333	1.083333	1.083333	1.083333	1.083333	1.083333	1.083333
1.618025751	1.839273	1.92745	1.965707	1.982843	1.992849	1.990909	1.988539	1.986111	1.984733	1.987179	7	1.076923	1.076923	1.076923	1.076923	1.076923	1.076923
1.618037135	1.839286	1.927528	1.966044	1.983107	1.995813	1.994521	1.992795	1.990676	1.988462	1.987097	1.989011	7.5	1.071429	1.071429	1.071429	1.071429	1.071429
1.618032787	1.839289	1.927572	1.966091	1.983378	1.991609	1.996795	1.995662	1.994145	1.992263	1.99026	1.98895	1.990476	8	1.066667	1.066667	1.066667	1.066667
1.618034448	1.839286	1.927572	1.965909	1.983553	1.991724	1.998166	1.997464	1.996477	1.995146	1.993475	1.991667	1.990431	1.991667	8.5	1.0625	1.0625	1.0625
1.618033813	1.839287	1.927557	1.965933	1.983628	1.991841	1.995869	1.998549	1.997941	1.99708	1.995908	1.994421	1.992788	1.991632	1.992647	9	1.058824	1.058824
1.618034056	1.839287	1.927561	1.965947	1.983626	1.991921	1.995918	1.999183	1.998822	1.998294	1.99754	1.996503	1.995175	1.993697	1.99262	1.993464	9.5	1.055556
1.618033963	1.839287	1.927563	1.965951	1.983572	1.991965	1.995967	1.997957	1.999337	1.999025	1.998563	1.997898	1.996977	1.995785	1.994444	1.993443	1.994152	10
1.618033999	1.839287	1.927562	1.96595	1.983578	1.99198	1.996002	1.997978	1.999632	1.999451	1.999178	1.998773	1.998183	1.99736	1.996286	1.995066	1.994135	1.994737
1.618033985	1.839287	1.927562	1.965947	1.983582	1.991977	1.996023	1.997998	1.998987	1.999695	1.999538	1.999298	1.998939	1.998414	1.997674	1.996702	1.995588	1.994723
1.61803399	1.839287	1.927562	1.965948	1.983583	1.991961	1.996033	1.998013	1.998996	1.999832	1.999743	1.999605	1.999394	1.999074	1.998603	1.997936	1.997052	1.996032
1.618033988	1.839287	1.927562	1.965948	1.983583	1.991963	1.996036	1.998023	1.999004	1.999497	1.999859	1.999781	1.999659	1.999471	1.999185	1.99876	1.998155	1.997349

The plot of φ^1, φ^2, φ^3… will give a seesaw plot like shown below and the line connecting the tips of seesaw will always be a straight line.



REFERENCE

Fibonacci series

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