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LAPLACE EXPANSION IN RHOTRIX

VIJAYABARATHI.S*

Assistant professor of Mathematics,
Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya, Enathur, Kancheepuram, India.

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ABSTRACT

Laplace expansion of rhotrix matrices has been defined and the results $\Delta R_1 = \Delta R_3 = \Delta C_1 = \Delta C_3$ in the third order and in the fifth order $\Delta R_1 = \Delta C_1$ & $\Delta R_5 = \Delta C_5$ have been proved.

Keywords: *Laplace expansion, cofactors, determinants, NE-elements.*

1. PRELIMINARY

Definition [2]1.1:

Rhotrix is defined as $R = \left\{ \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle : a,b,c,d,e \in R \right\}$ R is of dimension 3.This is a rhomboidal arrangement.

Entry c in R is the heart of R and denoted by h(R).This is analogous to concepts in Matrix Theory.

Multiplication of Rhotrix matrices [2]1.2:

$$A \circ B = \left\langle \begin{matrix} & a & \\ b & h(A) & d \\ & e & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & f & \\ g & h(B) & i \\ & j & \end{matrix} \right\rangle = \left\langle \begin{matrix} & af+dg & \\ bf+eg & h(A)h(B) & ai+dj \\ & bi+ej & \end{matrix} \right\rangle$$

Cardinality is $\frac{1}{2}\left(n^2+1\right)$

Right triangular Rhotrix1.3:

All the entries to the left of the main diagonal in R are zero.

Left triangular Rhotrix1.4:
All the entries to the right of the main diagonal in R are zero.

Upper triangular Rhotrix1.5:
All the entries to the below the horizontal diagonal in R are zero.

Lower triangular Rhotrix1.6:
All the entries to the above the horizontal diagonal in R are zero

Corresponding Author: Vijayabarathi.S*, Assistant professor of Mathematics,
Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya, Enathur, Kancheepuram, India.

2. LAPLACE EXPANSION

Determinants of the Rhotrix 2.1: $|A| = \left| \left\langle \begin{matrix} & a & \\ c & h(A) & d \\ & b & \end{matrix} \right\rangle \right|$ is called the determinant of the rhotrix of the third order. The diagonals from top to bottom which contains the element $a.h(A), b$ called the leading or principal diagonal.

Cofactors: 2.2: The sign of an element in the i^{th} row and j^{th} column is $(-1)^{i+j}$. But in Rhotrix if i+j is odd, null entries will be there.

$$A = \left\langle \begin{matrix} & a & \\ c & h(A) & d \\ & b & \end{matrix} \right\rangle$$

Co-factors of $a = h(A) \times b$
Co-factors of $d = h(A) \times c$
Co-factors of $c = h(A) \times d$
Co-factors of $b = h(A) \times a$
Co-factors of $h(A) = (a \times b) - (c \times d)$

If the order of rhotrix increases then convert this into coupled (ordinary) matrix and find the del value of that matrix.

$$\left\langle \begin{matrix} & a & \\ d & b & e \\ & c & \end{matrix} \right\rangle$$

The elements a and e are called as North –East elements (NE-elements)

Similarly the elements e and c are called as East-South elements (ES-elements).

The elements c and d are called as South-west elements (NE-elements) and the elements d and a are called as West-North elements (WN-elements).

Laplace Expansion 2.3:
A determinant can be expanded in terms of North –East elements (row) as follows:
Multiply each element of the NE in terms of which we intend expanding the del, by its cofactor then add up all these terms.

Expanding NE-Row

$$\Delta = a[h(A) \times b] + d[h(A) \times c]$$

Expanding by NW-column

$$\Delta = a[h(A) \times b] + c[h(A) \times d]$$

Thus Δ is the sum of the products of the elements of any NE-row(or column) by their corresponding cofactors

Let the rhotrix be $\left\langle \begin{matrix} & a & \\ d & b & e \\ & c & \end{matrix} \right\rangle$

$$\Delta_{NE} = \Delta_{R_1} = abc - deb .$$

$$\Delta_{NW} = \Delta_{C_1} = abc - dbe$$

$$\Delta_{WS} = \Delta_{R_3} = -deb + abc$$
$$\Delta_{ES} = \Delta_{C_3} = abc - deb$$
$$\Delta_{R_1} = \Delta_{C_1} = \Delta_{R_3} = \Delta_{C_3}$$

Let the rhotrix be

$$\left\langle \begin{array}{ccccc} & & a & & \\ & m & b & n & \\ f & g & c & h & l \\ & p & d & q & \\ & & e & & \end{array} \right\rangle$$

$$\Delta_{NE} = \Delta_{R_1} = abcde - abqpd - ahcge + ahqgp - nbmde + nbqfd$$
$$+nhmge - nhqfg + lbmpd - lbcfd - lhmgp + lhcfg$$

$$\Delta_{NW} = \Delta_{C_1} = abcde - abqpd - ahcge + ahqgp - nbmde + nbqfd$$
$$+nhmge - nhqfg + lbmpd - lbcfd - lhmgp + lhcfg$$

$$\Rightarrow \Delta_{R_1} = \Delta_{C_1}$$

$$\Delta_{WS} = \Delta_{R_5} = fnbdq - nhgqf - flbcd + flhgc - pabdq +$$
$$+phgqa + plbmd - plhmg + eabcd - heagc + enhmg$$

$$\Delta_{SE} = \Delta_{C_5} = fnbdq - nhgqf - flbcd + flhgc - pabdq +$$
$$+phgqa + plbmd - plhmg + eabcd - heagc + enhmg$$

$$\Rightarrow \Delta_{R_5} = \Delta_{C_5}$$

These results have been supported by numerical judgment.

- The Laplace Expansion of the rhotrix $A = \left\langle \begin{array}{ccc} & 5 & \\ 3 & 4 & 8 \\ & 6 & \end{array} \right\rangle$

  Cofactors of 5=24
  Cofactors of 8=12
  Cofactors of 3=32
  Cofactors of 6=20
  Cofactors of 4=6
  NE-elements (R_1)
  $\Delta R_1 = 5(24) + 8(12) = 216$
  NW- elements(C_1)
  $\Delta C_1 = 5(24) + 3(32) = 216$
  WS-elements (R_3)
  $\Delta R_3 = 3(32) + 6(20) = 216$
  ES-elements (C_3)
  $\Delta C_3 = 8(12) + 6(20) = 216$
  $\therefore \Delta R_1 = \Delta R_3 = \Delta C_1 = \Delta C_3$

Note 2.4: In the third order Rhotrix, the above problems leads to the conclusion that $\Delta R_1 = \Delta R_3 = \Delta C_1 = \Delta C_3$.

The Laplace expansion of $\left\langle \begin{array}{ccccc} & & 2 & & \\ & 3 & 6 & 4 & \\ 1 & 4 & 5 & 25 & 6 \\ & 7 & 17 & 3 & \\ & & 8 & & \end{array} \right\rangle$

Expansion of NE- elements (R_1)

$$\Delta R_1 = 2(38) + 4(-42) + 6(32) = 100$$

Expansion of NW- elements (C_1)

$$\Delta C_1 = 2(38) + 3(20) + 1(-36) = 100$$

Expansion of ES- elements (C_5)

$$\Delta C_5 = 6(32) + 3(-20) + 8(-4) = 100$$

Expansion of WS- elements (R_5)

$$\Delta R_5 = 1(-36) + 7(24) + 8(-4) = 100$$

$$\therefore \Delta R_1 = \Delta C_1 \ \&\ \Delta R_5 = \Delta C_5$$

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