International Journal of Mathematical Archive-2(9), 2011, Page: 1734-1741
Available online through www.ijma.info ISSN 2229 – 5046

πsg-CLOSED SETS IN BITOPOLOGICAL SPACES

K. Mohana* and I. Arockiarani

Department of Mathematics, Nirmala College for Women, Coimbatore-641 018, India

E-mail: mohanamaths@yahoo.co.in

(Received on: 05-08-11; Accepted on: 17-08-11)

ABSTRACT

In this paper the concept of πsg-closed sets in bitopological spaces is introduced. Properties of these sets are investigated and we introduce new bitopological spaces (τ_i, τ_j)-πsg $-T^2{}_{1/2}$ as applications. Further we discuss and study πsg-continuity in bitopological spaces.

Key Words: *(τ_i, τ_j)-πsg closed sets, (τ_i, τ_j)-πsg $-T^2{}_{1/2}$ spaces, (τ_i, τ_j)-πsg continuous maps.*

1. INTRODUCTION

A triple (X,τ_1,τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly[8] initiated the study of such spaces. In 1985, Fukutake [7] introduced the concepts of g-closed sets in bitopological spaces and after that [2, 3, 10, 11, 12, 13, 14] several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Dontchev. J, Noiri. T [6] introduced and studied the concepts of πg closed set in topological spaces.

The purpose of this paper is to introduce the concepts of πsg closed sets**,** πsg–$T^2{}_{1/2}$ spaces, πsg continuity in bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl(A) or cl(A),the interior of A is denoted by τ-int(A) or int (A) and the complement of A in X is denoted by A^c. When A is a subset of X, cl(A), int(A) denote the closure, the interior of A respectively. A subset A of a topological space X is called regular open if A=int(cl(A)).The finite union of regular open sets is said to be π- open. The complement of π- open is said to be π- closed.

Throughout this paper (X,τ_1,τ_2) and (Y,σ_1,σ_2) means a bitopological space on which no separation axioms are assumed unless explicitly mentioned and the integers $i,j\in\{1,2\}$. For a subset A of X τ_i-cl(A)(resp. τ_i-int(A))denote the closure (resp.interior)of A with respect to the topology τ_i .We denote the family of all π-open sets of X with respect to the topology τ_i by π-O, the family of all τ_j -closed sets is denoted by F_j .We denote the family of all (τ_i , τ_j)-g closed sets is denoted by the symbol $D(\tau_i , \tau_j)$.

we recall the following known definitions and results which are useful for our paper.

Definition: 2.1-A subset A of a bitopological space(X, τ_1 , τ_2) is called
(i). (τ_i , τ_j)-g closed **[7]** if τ_j –cl(A) $\subseteq$ U when ever A $\subseteq$ U and U$\in\tau_i$.
(ii). (τ_i, τ_j)-sg closed **[4]** if τ_j –scl(A) $\subseteq$ U when ever A $\subseteq$ U and U is semi open in (X, τ_i).

Definition: 2.2 [5] A space (X, τ_i) is called a sub maximal space if every dense subset of X is open in X.

Definition: 2.3 [6] A subset A of a space (X, τ_i) is said to be πg-closed if cl(A) $\subseteq$ U when ever A$\subseteq$ U and U is π-open in (X, τ_i).

Definition: 2.4[1] A subset A of a space (X, τ_i, τ_j) is said to be(τ_i ,τ_j)-πg-closed if τ_j -cl(A) $\subseteq$ U when ever A$\subseteq$ U and U is π-open in (X, τ_i).

***Corresponding author: K. Mohana*, *E-mail: mohanamaths@yahoo.co.in**

Definition: 2.5[9] A function f: $(X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called generalized -bi-continuous (briefly g-bi-continuous) if f is $D(\tau_i, \tau_j)$- σ_k -continuous if the inverse image of every σ_k–closed set is (τ_i, τ_j)- g -closed.

Definition: 2.6[9] A function f: $(X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called generalized –strongly-bi-continuous (briefly g-s-bi -continuous) if f is g-bi-continuous, $D(\tau_j, \tau_i)$- σ_k –continuous and $D(\tau_i, \tau_j)$- σ_k–continuous.

3. (τ_i, τ_j)- πsg CLOSED SETS

Definition: 3.1 Let i, j∈ {1, 2} be fixed integers. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (τ_i , τ_j)- πsg closed if τ_j -scl(A) ⊆ U whenever A ⊆ U and U is π -open in τ_i .

Remark: 3.2 By setting $\tau_1 = \tau_2$ in definition 3.1, a (τ_i ,τ_j)-πsg closed is a πsg closed.

Theorem: 3.3

1. Every τ_j -closed set is (τ_i , τ_j)-πsg closed .
2. Every τ_j -semi closed set is (τ_i , τ_j)-πsg closed **.**
3. Every (τ_i ,τ_j)-πg closed set is (τ_i ,τ_j)- πsg closed set.
4. Every (τ_i , τ_j)-sg closed set is (τ_i , τ_j)- πsg closed set.
5. Every (τ_i , τ_j)- g closed set is (τ_i , τ_j)- πsg closed set.

The converses of the above are not true may be seen by the following examples.

Example:3.4 Let X = {a, b, c}, τ_1 = {ϕ, X,{a},{b},{a, b}},τ_2 = {ϕ,X,{a},{a, b},{a,c}}. Then the subset {a, c} is (τ_1, τ_2)- πsg closed set, but not τ_2 -closed.

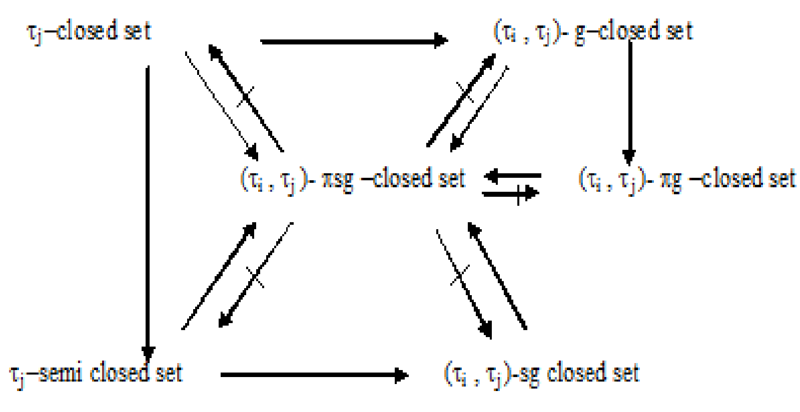
Example: 3.5 Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} ,τ_2 = {ϕ, X,{a},{b},{a, b}}. Then the subset {a, b} is (τ_1, τ_2)- πsg closed sets ,but not τ_2 - semi closed sets.

Example :3.6 Let X = {a, b, c}, τ_1 = {ϕ, X,{a},{b},{a, b}} ,τ_2 = {ϕ, X,{a},{a, b}}. Then the subsets {a}, {b}are (τ_1,τ_2)-πsg closed set, but not (τ_1,τ_2)-πg closed set.

Example :3.7 Let X = {a, b, c}, τ_1 = {ϕ, X,{a},{a, b}} ,τ_2 = {ϕ, X,{a},{b},{a, b}}. Then the subset {a, b } is (τ_1 ,τ_2)-πsg closed set , but not (τ_1 , τ_2)- sg closed set.

Example: 3.8 Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {c}, {a, c}} ,τ_2 = {ϕ, X,{a},{b, c}}. Here {c} is (τ_1 , τ_2)- πsg closed set, but not (τ_1 , τ_2)- g closed set.

From the above results and examples, we have the following implications



Theorem: 3.9 If A is an (τ_i , τ_j)- πsg closed set of X such that A⊆B⊆τ_j–scl(A),then B is also an (τ_i ,τ_j)- πsg closed set of (X,τ_1,τ_2).

Proof: Let B ⊆ U,where U is π-open in τ_i ,then A ⊆ B implies A ⊆ U. Since A is an (τ_i ,τ_j)-πsg closed set=> τ_j -scl (A) ⊆ U, given B⊆ τ_j -scl (A) => τ_j -scl(B)⊆τ_j -scl(scl(A))⊆τ_j-scl(A)⊆ U

=> τ_j -scl (B) ⊆ G. Therefore B is (τ_i , τ_j)- πsg closed set.

Theorem: 3.10 If A is (τ_i, τ_j)- πsg closed set then τ_j-scl(A) – A contains no non-empty τ_i-π-closed set.

Proof: Let A be an (τ_i, τ_j)- πsg closed set and F be a τ_i-π closed set such that $F \subseteq \tau_j$-scl(A) – A .Then $F \subseteq X – A$, since A is (τ_i, τ_j)- πsg closed set and X –F is τ_i -π-open.Therefore τ_j-scl(A) $\subseteq$ X – F (i.e)., F$\subseteq$ X - τ_j-scl(A). Hence F$\subseteq$ τ_j-scl(A) $\cap$ (X - τ_j-scl(A) = ϕ

The converse of the above theorem is not true as seen from the following example.

Example: 3.11 Let X = {a, b, c}, τ_1 = {ϕ, X,{a},{b},{a, b}} ,τ_2 = {ϕ,X,{a},{a, b},{a, c}}. If A = {c} then τ_2-scl(A) – A = X – {c}={a, b} does not contain any non-empty τ_1- π-closed set. But A = {a, b} is not (τ_i, τ_j)-πsg closed set .

Corollary: 3.12Let A be (τ_i, τ_j)-πsg closed set in X then A is τ_j-closed iff τ_j-scl(A) – A is τ_i π-closed.

Proof: Necessity: Let A be τ_j-closed then τ_j-cl(A) = A=> τ_j- scl(A) – A =ϕ. by theorem **3.10**.,Which is τ_i- π-closed.

Sufficiency: If τ_j- scl(A) – A is τ_i π-closed. by theorem **3.10**.,τ_j- scl(A) – A = ϕ,since A is (τ_i, τ_j)- πsg closed set. Therefore A is τ_j –closed set.

Theorem: 3.13 If D [E] $\subset$ D_S [E] for each subset E of a Bitopological space X then the union of two (τ_i, τ_j)- πsg closed set is (τ_i, τ_j)- πsg closed set.

Proof: Let A,B be (τ_i, τ_j)- πsg-closed subsets of X and let U be a π-open set in τ_i such that A$\cup$B $\subset$ U. Given A and B are (τ_i, τ_j)- πsg closed sets then τ_j-scl(A) $\subseteq$U , τ_j-scl(B) $\subseteq$U, since D[A] $\subset$ D_S[A] and D[B] $\subset$ D_S[B]
τ_j-cl(A) = τ_j-scl(A) and τ_j-cl(B) = τ_j-scl(B)

Therefore τ_j-cl(A$\cup$B) = τ_j-cl(A)$\cup$ τ_j-cl(B)

=> τ_j-scl(A$\cup$B) = τ_j-scl(A)$\cup\tau_j$-scl(B) $\subseteq$ U => τ_j- scl(A$\cup$B) $\subseteq$ U

=> A$\cup$B is (τ_i, τ_j)- πsg closed set.

Proposition: 3.14 If A,B $\in$ (τ_i, τ_j)- πsg closed set in X and (X, τ_j) is sub maximal, then A$\cup$B$\in$ (τ_i, τ_j)- πsg closed set

Proof: Suppose A$\cup$B $\subseteq$ G ,where G is π-open, since A, B$\in$ (τ_i, τ_j)- πsg closed set,τ_j-scl(A) $\subseteq$G , τ_j-scl(B) $\subseteq$G. Hence τ_j-scl(A) $\cup$ τ_j-scl(B) $\subseteq$G. As (X, τ_j) is sub maximal, finite union of semi closed set is semi closed.

So, it follows that τ_j-scl (A$\cup$B) $\subseteq$G. =>A$\cup$B$\in$ (τ_i, τ_j)- πsg-closed set.

Definition: 3.15 A subset A $\subseteq$ X is called (τ_i, τ_j)- πsg open set iff its complement is (τ_i, τ_j)- πsg closed set.

Theorem : 3.16 A subset A of a Bitopological space X is (τ_i, τ_j)- πsg open set iff F $\subseteq$ τ_j-Sint A whenever F is π-closed set in τ_i and F $\subseteq$ A.

Proof: Necessity: Let A be (τ_i, τ_j)- πsg open set. Let F be π-closed set in τ_i and F $\subseteq$ A. Then X-A $\subseteq$ X-F ,where X – F is π-open in τ_i .(τ_i, τ_j)- πsg closeness of X – A => X –A is (τ_i, τ_j)- πsg closed=> τ_j–scl(X-A) $\subseteq$ X-F=> X - τ_j-Sint A $\subseteq$ X-F.

Hence F $\subseteq$ τ_j-Sint A.

Sufficiency: Suppose F is π-closed set in τ_i and F$\subseteq$ A => F $\subseteq$ τ_j-Sint A. Let X-A $\subseteq$ U, where U is π-open in τ_i. Then X-U $\subseteq$ A, where X – U is π-closed in τ_i. By hypothesis, X-U $\subseteq$ τ_j-Sint A=> X- τ_j-Sint A $\subseteq$ U => τ_j- scl(X-A) $\subseteq$ U.
=> X-A is (τ_i, τ_j)- πsg closed set. Therefore A is (τ_i, τ_j)- πsg open set.

Theorem: 3.17 If τ_j-sint A$\subseteq$B $\subseteq$A and A is (τ_i, τ_j)-πsg open set then B is (τ_i, τ_j)- πsg open set.

Proof: τ_j-Sint A $\subseteq$ B $\subseteq$ A =>X-A$\subseteq$X- B $\subseteq$ X-τ_j-Sint A (i. e)., X-A$\subseteq$X- B $\subseteq$ τ_j-scl(X-A),since X –A is (τ_i, τ_j)- πsg closed set. by theorem :3.9., X-B is (τ_i, τ_j)- πsg closed set => B is (τ_i, τ_j)- πsg open set.

Remark: 3.18 For any A $\subseteq$ X, τ_j-Sint(τ_j-scl (A)-A) = ϕ.

Theorem: 3.19 If A$\subseteq$X is (τ_i, τ_j) - πsg closed set, then τ_j-scl(A)–A is (τ_i, τ_j)-πsg open set.

Proof: Let A be (τ_i, τ_j)- πsg closed. Let F be π-closed set in τ_i such that F $\subseteq$ τ_j-scl(A) – A .Then by theorem 3.10., F=ϕ.So

$F \subseteq \tau_j$-Sint(τ_j-scl(A) – A).

=> τ_j-scl(A) – A is (τ_i, τ_j)- πsg closed set.

The reverse implication does not hold.

Example:3.20 Let X = {a, b, c}, τ_1 ={ϕ, X,{a},{b},{a, b}},τ_2 = {ϕ, X,{a},{a, b}}. Let A = {a, b}, which is (τ_1, τ_2)-πsg open sets, but not (τ_1, τ_2)-πsg closed sets.

4. (τ_i, τ_j)- πsg –T $^2 1_{/2}$ space

Definition: 4.1 A Bitopological space (X,τ_1,τ_2) is called (τ_i, τ_j)- πsg –T $^2 1_{/2}$ if every (τ_i, τ_j)- πsg closed set is τ_j -semi closed.

Definition: 4.2 For a subset A of (X, τ_1, τ_2),we define the (τ_i, τ_j)- πsg closure of A and (τ_i, τ_j)- πsg interior of A as follows:
(τ_i, τ_j)- πsg -cl(A) = ∩{ F: F is (τ_i, τ_j)- πsg closed in X,A⊂ F}
(τ_i, τ_j)- πsg -int(A) = ∪{ F: F is (τ_i, τ_j)- πsg open in X, F⊂ A}

Theorem:4.3 Let A be a subset of (X, τ_i, τ_j) and x ∈ X. Then
(a). X- ((τ_i, τ_j)- πsg -cl(A)) = (τ_i, τ_j)- πsg -int(X-A).
(b). X- ((τ_i, τ_j)- πsg -int(A)) = (τ_i, τ_j)- πsg -cl(X-A).

Proof: (a) We have (τ_i,τ_j)- πsg -cl(A) = ∩{ F: F is (τ_i,τ_j)- πsg -closed in X, A⊂ F}Taking complement on both the sides, we have X- ((τ_i, τ_j)- πsg -cl(A)) = ∪{ F^c: F^c is (τ_i, τ_j)- πsg -open in X, F^c⊂ A^c} = ∪{ U: U is (τ_i, τ_j)- πsg -open in X,U⊂ A^c}.Where U = F^c is (τ_i, τ_j)- πsg -open = (τ_i, τ_j)- πsg –int(A^C) = (τ_i, τ_j)- πsg –int(X-A).

(b) (τ_i, τ_j)- πsg -int(A) = ∪{ F: F is (τ_i, τ_j)- πsg -open in X,F⊂ A} Taking complement on both the sides, we have X- ((τ_i, τ_j)- πsg -int(A)) =∩{ F^c: F^c is (τ_i, τ_j)-πsg closed in X, $A^c \subset F^c$}= ∩{ U: U is (τ_i, τ_j)- πsg – closed in X,U⊃ A^c},where U = F^c is (τ_i, τ_j)-πsg closed = (τ_i, τ_j)- πsg cl(A^C) = (τ_i, τ_j)- πsg cl(X-A).

Lemma:4.4 Let A be a subset of (X, τ_i, τ_j) and x∈X.Then x∈ (τ_i, τ_j)- πsg –cl(A) iff V ∩ A ≠ ϕ for every (τ_i, τ_j)- πsg open set V containing x.

Proof: Let A be a subset of X and x∈X, suppose there exists, a (τ_i, τ_j)- πsg –open set V containing x such that V ∩ A = ϕ,since A ⊂ X / V,(τ_i, τ_j)- πsg –cl(A) ⊂ X / V and then x ∉ (τ_i, τ_j)- πsg –cl(A),which is a contradiction, therefore V ∩ A ≠ ϕ.

Conversely, suppose that x ∉ (τ_i, τ_j)- πsg cl(A), then there exists a (τ_i, τ_j)- πsg closed set F containing A such that x ∉ F, since x ∈X / F and X/F is (τ_i, τ_j)- πsg open. Therefore (X/F) ∩ A = ϕ, which is a contradiction.

Therefore x ∈ (τ_i, τ_j)- πsg cl(A).

Lemma: 4.5 Let A and B be subsets of (X, τ_i, τ_j).Then we have,
(a). (τ_i, τ_j)- πsg –cl(ϕ)=ϕ and (τ_i, τ_j)- πsg –cl(X)=X.
(b).If A ⊆ B ,then (τ_i, τ_j)- πsg –cl(A) ⊆ (τ_i, τ_j)- πsg –cl(B)
(c). (τ_i, τ_j)- πsg –cl(A) = (τ_i, τ_j)- πsg –cl((τ_i, τ_j)- πsg–cl(A))
(d). (τ_i, τ_j)-πsg –cl(A∪B) ⊃(τ_i, τ_j)- πsg –cl(A)∪(τ_i, τ_j)-πsg –cl(B)
(e). (τ_i, τ_j)-πsg –cl(A∩B) ⊂(τ_i, τ_j)- πsg –cl(A)∩(τ_i, τ_j)-πsg –cl(B)

Remark: 4.6
(1) If A is (τ_i,τ_j)-πsg closed in (X, τ_i, τ_j) then (τ_i,τ_j)-πsg-cl(A)= A.

(2) If πSGC(X, τ_i,τ_j) is closed under finite unions, then (τ_i,τ_j)-πsg-cl(A ∪ B) = (τ_i,τ_j)-πsg-cl(A) ∪ (τ_i,τ_j)-πsg-cl(B).

Proof: Since πSGC(X, τ_i, τ_j) is closed,=> (τ_i,τ_j)-πsg-cl(A) = A and (τ_i,τ_j)-πsg-cl(B) = B.(τ_i,τ_j)-πsg-cl(A∪B) ⊂A∪B = (τ_i,τ_j)-πsg-cl(A)∪(τ_i,τ_j)-πsg-cl(B).Hence,(τ_i,τ_j)-πsg-cl(A ∪ B) = (τ_i,τ_j)-πsg-cl(A) ∪ (τ_i,τ_j)-πsg-cl(B).

(3) If sc(X, τ_i,τ_j) is closed under finite union, then πSGC(X, τ_i,τ_j) is closed under finite unions. Since sc(X, τ_i,τ_j) is closed => τ_j -scl(A) =A. Since every semi closed set is (τ_i, τ_j)-πsg-closed.=> πSGC(X, τ_i,τ_j) is closed under finite unions.

Definition :4.7 [4] For a subset A of (X, τ_i, τ_j), (τ_i, τ_j)-g closure of A is defined by (τ_i, τ_j)C*(A) = $\cap$ { F : F is (τ_i, τ_j)-g closed ; A $\subset$ F}

Definition :4.8[4] For a subset A of (X, τ_i, τ_j), (τ_i, τ_j)-sg closure of A is defined by (τ_i, τ_j)scl*(A) = $\cap$ {F : F is (τ_i, τ_j)-sg closed ; A $\subset$ F}

Definition: 4.9 For a subset A of (X, τ_i, τ_j), (τ_i, τ_j)-πg closure of A is defined by (τ_i, τ_j)-πg-cl(A) = $\cap$ {F : F is (τ_i, τ_j)-πg closed ; A $\subset$ F}

Definition: 4.10
(a).For a bitopological space (X, τ_i, τ_j), (τ_i, τ_j) $_s\tau^*$ = { U $\subset$ X ; (τ_i, τ_j)-scl* (X / U) = X / U}
(b) For a bitopological space (X, τ_i, τ_j), (τ_i, τ_j) $\tau^*_{\pi g}$ = { U $\subset$ X ; (τ_i, τ_j)- πg-cl* (X / U) = X / U}
(c) For a bitopological space (X, τ_i, τ_j), (τ_i, τ_j) $\tau^*_{\pi sg}$ = { U $\subset$ X ; (τ_i, τ_j)- πsg-cl* (X / U) = X / U}

Theorem: 4.11 If SO (X, τ_i, τ_j) is closed under finite intersections, then (τ_i, τ_j) $\tau^*_{\pi sg}$ is a bitopology for X.

Theorem: 4.12 For a subset A of (X, τ_i, τ_j), the following statements hold :
(a)A $\subset$ (τ_i, τ_j) -πsg -cl(A) $\subset$ (τ_i, τ_j) -πg-cl(A)
(b)A $\subset$ (τ_i, τ_j) -πsg -cl(A) $\subset$ (τ_i, τ_j) -scl*(A)
(c) (τ_i, τ_j) $\tau^*_{\pi g} \subset$ (τ_i, τ_j) $\tau^*_{\pi sg}$
(d) (τ_i, τ_j) $_s\tau^* \subset$ (τ_i, τ_j) $\tau^*_{\pi sg}$

Theorem: 4.13 Let (X, τ_i, τ_j) be a bitopological space . Then every (τ_i, τ_j)-πsg closed set is τ_j –semi closed (ie. (X, τ_i, τ_j) is (τ_i, τ_j)-πsg $T^2{}_{1/2}$) iff (τ_i, τ_j)-$\tau^*_{\pi sg}$ = τ_j-so(X,τ_i, τ_j)

Proof: Let A $\in$ (τ_i, τ_j) $\tau^*_{\pi sg}$,then (τ_i, τ_j)-πsg -cl (X / A) = X / A. By hypothesis, τ_j -scl(X / A) = (τ_i, τ_j)-πsg -cl(X/A) = X / A. => τ_j -scl(X / A) = X / A =>X / A is τ_j -semi closed ,hence A $\in$ τ_j –so(X, τ_i, τ_j).

Converse: Let A be a (τ_i, τ_j)-πsg -closed set. Then (τ_i, τ_j)-πsg -cl(A) = A. Hence X / A $\in$ (τ_i, τ_j) $\tau^*_{\pi sg}$ = τ_j-so (X, τ_i, τ_j)=> X / A $\in$ τ_j- so (X, τ_i, τ_j), So A is τ_j-semi closed.

Theorem: 4.14 Let (X, τ_i, τ_j) be a bitopological space . Then every (τ_i, τ_j)-πsg closed set is τ_j-closed iff (τ_i, τ_j) $\tau^*_{\pi sg}$ = τ_j .

Theorem:**4.15** Let (X, τ_i, τ_j) be a bitopological space . Then every (τ_i, τ_j)-πsg closed set is (τ_i, τ_j)-sg closed then (τ_i, τ_j) $\tau^*_{\pi sg}$ = (τ_i, τ_j) $_s\tau^*$

Proof: Obvious.

Theorem: 4.16 Let (X, τ_i, τ_j) be a bitopological space . Then every (τ_i, τ_j) -πsg closed set is (τ_i, τ_j) -πg closed then (τ_i, τ_j) $\tau^*_{\pi sg}$ = (τ_i, τ_j) $\tau_{\pi g}{}^*$

Proof: Obvious.

5. πsg CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACE

Definition: 5.1 A function f:(X,τ_1,τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (τ_i, τ_j)-σ_k-πsg-continuous if the inverse image of every σ_k –closed set in Y is (τ_i, τ_j)-πsg-closed in (X, τ_i, τ_j).

Remark: 5.2
(i) Suppose that $\tau_1 = \tau_2 = \tau$ and $\sigma_1 = \sigma_2 = \sigma$,then the above definition of (τ_i, τ_j)- σ_k -πsg-continuous function coincides with the πsg- continuous function.

(ii) It follows from the definition of (τ_i, τ_j) - σ_k -πsg-continuous function that f is (τ_i, τ_j)- σ_k -πsg-continuous iff for every σ_k-open set in Y, its inverse image is (τ_i, τ_j)-πsg open in X.

The following is an example for (τ_i, τ_j) - σ_k -πsg-continuous function.

Proposition: 5.3 If a function f : (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is τ_j - σ_k continuous then f is (τ_i, τ_j)- σ_k - πsg continuous.[(i.e)., The inverse image of every σ_k –closed set is τ_j–closed]

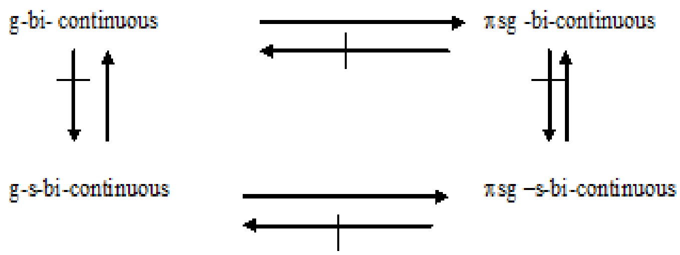
Proof: It follows from the fact that every τ_j -closed set is (τ_i, τ_j)-πsg closed. However the converse need not hold.

Example:5.4 Let X =Y ={ a, b, c},τ_1 = {ϕ,X,{a},{b},{a, b}},τ_2 ={ϕ,X, {a},{a, b},{a, c}}, σ_1 ={ ϕ,Y, {a},{b},{a, b}}, σ_2 ={ ϕ,Y, {a}}.Then f is (τ_1 , τ_2)- σ_2 -πsg continuous. i, j, k $\in$ {1,2},i $\neq$ j. But not τ_2- σ_1- continuous

Definition: 5.5 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called πsg-bi-continuous if f is (τ_1, τ_2)- σ_2 -πsg-continuous and (τ_2, τ_1)- σ_1 -πsg-continuous.

Definition: 5.6 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called πsg-strongly -bi-continuous (briefly πsg-s-bi-continuous) if f is πsg -bi-continuous,(τ_2, τ_1)- σ_2 -πsg-continuous and (τ_1, τ_2)- σ_1 -continuous.

Remark: 5.7 Every g-bi-continuous function is πsg -bi-continuous and every g-s-bi continuous function is πsg -s-bi-continuous. But the reverse implications need not be true.



Example: 5.8 Let X = Y = {a, b, c}, τ_1 = { ϕ,X, {c},{a, b}},τ_2 = { ϕ,X, {a, b}}, σ_1 ={ ϕ,Y, {a},{b},{a, b}}, σ_2 ={ ϕ,Y, {a, b}}, f(a) = a, f(b)=c, f(c)=b. Then f is (τ_2, τ_1)- σ_1 -g-continuous, but not (τ_1, τ_2)- σ_2 -g-continuous.

Hence f is not g-bi-continuous. But f is both (τ_1, τ_2) - σ_2 -πsg-continuous and (τ_2, τ_1) - σ_1 -πsg-continuous and so it is πsg-bi-continuous. Also f is (τ_1, τ_2) - σ_1 -πsg-continuous and (τ_2, τ_1) - σ_2 -πsg-continuous and hence it is πsg-s-bi-continuous where as it is not g-s-bi-continuous.

Theorem: 5.9 A function f:(X,τ_1,τ_2) $\rightarrow$(Y,σ_1,σ_2) is called (τ_i ,τ_j)-πsg-σ_k-continuous, then f((τ_i, τ_j)- πsg-cl(A)) $\subset$ σ_k –cl(f(A)) holds for every subset A of X.

Proof: For every subset A of X, σ_k –cl(f(A)) is σ_k –closed in Y and A$\subset$ f^{-1}(σ_k –cl(f(A))).Since f is (τ_i , τ_j)- σ_k -πsg -continuous,(τ_i , τ_j)- σ_k -πsg-cl(A) $\subset$ f^{-1}(σ_k -cl(f(A)) (or)f((τ_i , τ_j) -πsg-cl(A)) $\subset$ (σ_k –cl(f(A)).

Proposition: 5.10 The following statements are equivalent. Let f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a function.

(i).For each point x$\in$X and every σ_k-open set V containing f(x)there exists a (τ_i, τ_j)- πsg –open set U containing X such that f(U) $\subset$ V.

(ii).For every subset A of X, f((τ_i, τ_j)- πsg-cl(A)$\subset$ (σ_k –cl(f(A)) holds.

Definition:5.12 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (τ_i , τ_j)- (σ_k, σ_e)-πsg-continuous if f^{-1} (A) $\in$(τ_i , τ_j)-πsg-closed set for every A$\in$(σ_k, σ_e)-πsg-closed set. If i =j and k = e simultaneously, then f becomes a πsg-irresolute function.

Definition:5.13 A function f is (τ_i , τ_j)- (σ_k, σ_e)-πsg-continuous iff, for every (σ_k, σ_e)-πsg-open set A of Y, the inverse image f^{-1}(A) is (τ_i , τ_j)-πsg-open in X.

Definition:5.14 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called τ_i -σ_k -π open if the image of every τ_i-open set in X is σ_k -π open in Y.

Definition: 5.15 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (τ_i, τ_j) -σ_e -πsg-semi continuous if the image of every σ_e-semi closed set in Y is (τ_i, τ_j) -πsg- closed in X.

Definition: 5.16 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is termed as τ_i -σ_k -π-continuous if the inverse image of every σ_k-open set in Y is τ_i - π-open in X.

Definition: 5.17 A function f: (X, τ_1, τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called τ_i -σ_e^* semi closed function if the image of a τ_i–semi closed set in X is σ_e semi closed in Y.

Proposition: 5.18 If a function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1,\sigma_2)$ is bijective, τ_i-σ_k-π-open and (τ_i, τ_j)-σ_k-πsg semi-continuous, then it is (τ_i, τ_j)–(σ_k, σ_e)-continuous.

Proof: Let A be (σ_k , σ_e)-πsg-closed in Y. Let U be a τ_i-π-open set in X. Such that $f^{-1}(A) \subset U$. Then $A \subset f(U)$, where f(U) is σ_k-π-open.As A is (σ_k ,σ_e)-πsg-closed, σ_e-scl(A) $\subset$ f(U) (or) $f^{-1}(\sigma_e\text{-scl}(A)) \subset U$. Since σ_e-scl(A) is σ_e-Semiclosed in Y,τ_j –scl($f^{-1}(\sigma_e$-scl(A))) $\subset$ U.(i.e)., τ_j –scl(f^{-1}(A)) $\subset$ U.

Hence, $f^{-1}(A)$ is (τ_i, τ_j)-πsg closed set in X.

Proposition: 5.19 Let f: $(X, \tau_1, \tau_2) \to (Y,\sigma_1,\sigma_2)$ be a τ_i-σ_k-π–continuous function and τ_j-σ_e^*-semi closed function. Then for every (τ_i, τ_j)-πsg closed set B in X the image f(B) is (σ_k ,σ_e)-πsg-closed in Y.

Theorem:5.20 Let f: $(X, \tau_1, \tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. If (X, τ_1, τ_2) is a (τ_i, τ_j)-semi π-$T_{½}$ space and f is bijective, τ_i-σ_k-π–open, τ_j-σ_e^*-semi closed and (τ_i,τ_j)-σ_e-semi continuous, then (Y,σ_1,σ_2) is a (σ_k, σ_e)semi π-$T_{½}$ space.

Proof: Let V be (σ_k ,σ_e)-πsg-closed in Y. Then by proposition,$f^{-1}(V)$ is (τ_i, τ_j)-πsg closed. Since X is (τ_i, τ_j)-semi π-$T_{½}$ space, $f^{-1}(V)$ is τ_j-semi closed in X. F is bijective => $f(f^{-1}(V))=V$ and f is τ_j-σ_e^*-semi closed => V is -σ_e-semi closed in Y. In other words, (Y,σ_1,σ_2) is a (σ_k,σ_e)semi π-$T_{½}$ space.

Lemma:5.21 Suppose that $B \subset A \subset X$,B is a (τ_i, τ_j)-πsg closed set relative to A and A is τ_i-open and τ_j-semi closed subset of X .Then B is (τ_i, τ_j)-πsg closed set in X.

Theorem: 5.22 Let f: $(X, \tau_1, \tau_2) \to (Y,\sigma_1,\sigma_2)$ be a (τ_i, τ_j)-σ_k-πsg continuous function and H be τ_j – open, τ_j-semi closed and τ_i –clopen in X then its restriction f | H :(H, τ_1 | H, τ_2| H) $\to (Y,\sigma_1,\sigma_2)$ is (τ_i, τ_j)-σ_k-πsg continuous.

Proof: Let F be a σ_k – closed subset of Y .Then $f^{-1}(F)$ is (τ_i, τ_j)-πsg-closed in (X, τ_1, τ_2).Then $f^{-1}(F) \cap H = (f \mid H)^{-1}(F)$ (Say H_1) is (τ_i, τ_j)-πsg-closed in X. Let $H_1 \subset G_1$,where G_1 is τ_i-π open in H. Then $G_1 = H \cap V$ Where V is -π open in X.As H_1 is (τ_i, τ_j)-πsg-closed in X and $H_1 \subset V$, τ_j-scl(H_1) $\subset$ V holds. As H is semi closed τ_j-$scl_H(H_1) = \tau_j$-$scl_X(H_1) \cap H$. Then τ_j-$scl_H(H_1) \subset V \cap H = G_1$.Therefore f|H is (τ_i, τ_j)-σ_k-πsg-continuous

Definition: 5.23 Let X =A ∪ Band let f:A→ Y and h:B→ Y be two functions. We say that f & h are compatible if f | (A∩B) = h | (A∩B) .

Then we define a function f ∇ h : X → Y as (f ∇ h) (x) = f(x) for x∈ A and (f ∇ h) (x) = h(x) for x∈ B. The function f ∇ h is called the combination of f and h.

Theorem: 5.24 (Pasting Lemma) Let X=A∪B, where A & B are both τ_i-open and τ_j–semiclosed subsets of X and let f: (A, τ_1 | A, τ_2| A) $\to (Y,\sigma_1,\sigma_2)$ and h :(B, τ_1 | B,τ_2| B) $\to (Y,\sigma_1,\sigma_2)$ be compatible functions. If f and h are (τ_i | A, τ_j| A)-σ_k-πsg continuous and (τ_i | B, τ_j| B)- σ_k-πsg continuous respectively and (X, τ_j) is submaximal, then the combination f ∇ h :
$(X, \tau_1, \tau_2) \to (Y,\sigma_1,\sigma_2)$ is(τ_i, τ_j)-σ_k-πsg-continuous

Proof: Let F be any σ_k –closed set in Y, then $(f \nabla h)^{-1}(F) = f^{-1}(F) \cup h^{-1}(F)$.By using Lemma[5.21],we have that $f^{-1}(F)$ and $h^{-1}(F)$ are (τ_i, τ_j)-πsg-closed in A and B respectively. Therefore their union $(f \nabla h)^{-1}(F)$ is also (τ_i, τ_j)-πsg-closed by proposition [3.14]. Hence (f ∇ h) is (τ_i, τ_j)-σ_k-πsg-continuous.

Corollary: 5.25 Let X = A ∪ B where A and B are τ_1 open, τ_2 open, τ_1 semi closed and τ_2 semiclosed. Let let f: (A, τ_1 | A, τ_2| A) $\to (Y, \sigma_1, \sigma_2)$ and h :(B, τ_1 | B, τ_2| B) $\to (Y,\sigma_1,\sigma_2)$ be compatible functions. If f and h are πsg-bi-continuous (respectively πsg-s-bi-continuous) functions and (X,τ_i), i∈{1,2} are submaximal, then the combination (f ∇ h) : $(X, \tau_1, \tau_2) \to (Y,\sigma_1,\sigma_2)$ is πsg-bi-continuous, (respectively πsg-s-bi-continuous).The proof follows from the above theorem.

References:

[1] Arockia rani and K. Mohana, πg-closed sets and π-normal spaces in bitopological settings, Accepted, Acta Ciencia Indica.

[2] S. P. Arya and T. M. Nour, Quasi semi-components in bitopological spaces, Indian J. Pure ppl.Math, 19(1988), 42-50.

[3] P. Bhattacharya and B. K. K Lahiri,"Semi generalized closed sets in topology", Indian J. Math.29(1987),375-382.

[4] R. Devi, H. Maki and K. Balachandran,"Semi-generalized closed maps and generalized semi-closed maps", Mem.

Fac. Sci. Kochi Univ (Math) 14(1993), 41-54.

[5] Dontchev. J,"On sub maximal spaces"J. Math 26(1995); 253-60.

[6] Dontchev. J, Noiri. T," Quasi normal spaces and πg-closed sets, Acta Math Hungar 89(3) (2000):211-9.

[7] Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. Part III, 35(1986), 19-28.

[8] Kelly. J. C –Bitopological spaces, Proc. London Math.soc., 13(1963), 71-99.

[9] N. Levine,"Semi-open sets and Semi-continuity in topological space", Am. Math. Monthly,70(1963)(36-41).

[10] N. Levine,"Semi-open sets and Semi-continuity in topological space",Am. Math. Monthly, 19(1963)(89-96).

[11] H. Maki, P. Sundaram and K. Balachandran, on generaized Continuous maps and Pasting lemma in bitopological spaces, Bull. Fukuoka. Univ. Ed. Part III, 40(1991), 23-31.

[12] Patty, C. W-Bitopological spaces, Duke Math.J., 34(1967), 303-307.

[13] Reilly. I. L-On bitopological separation properties, Nanta Math., 5(1972), 14-25.

[14] M. Sheik John and P. Sundaram – g^* closed sets in bitopological spaces –Indian J. Pure. Appl. Math., 35(1) (2004):71-80,

****