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ZERO SUFFIX METHOD FOR THE TRANSSHIPMENT PROBLEM WITH MIXED CONSTRAINTS

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ABSTRACT

A method, namely zero suffix method is proposed to solve the transshipment problem with mixed constraints. Standard transshipment model in which the origin and destination constraints consists not only of the equality sign but also of greater than or equal to, or less than or equal to type constraints, is termed as transshipment problem with mixed constraints. Transshipment problem with mixed constraints is transformed into an equivalent classical transportation problem by introducing an additional row and column. The classical transportation problem is then solved by zero suffix method. The solution tested through modi-indices declares an optimal solution. Method is structured in the form of algorithm and illustrated through a numerical example.

Keywords: *Transportation problem, Transshipment problem, Mixed constraints, Zero suffix method, Optimal solution.*

INTRODUCTION

In a transportation problem shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or transshipment points. Each of these points in turn supply to other points. Thus when the shipments also pass from destination to destination and from source to source, the transportation problem is termed as the transshipment problem.

Since transshipment problem is a particular case of transportation problem hence to solve transshipment problem, we firstly convert it into equivalent transportation problem and then obtain optimal solution using zero suffix method.

Orden, A. (1956) [4], has developed a method for solving transshipment problem with equality constraints by converting it into a classical transportation problem. A method has been developed by Bridgen (1974) [1**]**, for solving the transportation problem with mixed constraints. He solved this problem by considering a related standard transportation problem having two additional supply points and two additional destination points. Khurana, Arora (2011) [2]**,** considered the transshipment problem with mixed constraints. They changed it to the classical transportation problem. Kumari, N. and Kumar, R. (2016) [3], developed a method for finding Initial Basic feasible solution for the Transshipment Problem with mixed constraints. Solution obtained by this method is the nearest to the optimal solution, some times exactly optimal solution. It is a method for getting a better solution (some times optimal solution).

We propose a method for getting a good basic feasible solution for the transshipment problem with mixed constraints.

MATHEMATICAL FORMULATION OF THE TRANSSHIPMENT PROBLEM

To formulate the transshipment problem we consider a transportation table given below:

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Transportation Table

	D_1	D_2	......	D_j	......	D_n	supply
O_1	x_{11} c_{11}	x_{12} c_{12}	......	x_{1j} c_{1j}	......	x_{1n} c_{1m}	a_1
O_2	x_{21} c_{21}	x_{22} c_{22}	......	x_{2j} c_{2j}	......	x_{2n} c_{2m}	a_2
......	......	......	......	......	......	......	......
O_i	x_{i1} c_{i1}	x_{i2} c_{i2}	......	x_{ij} c_{ij}	......	x_{in} c_{im}	a_i
......	......	......	......	......	......	......	......
O_m	x_{m1} c_{m1}	x_{m2} c_{m2}	......	x_{mj} c_{mj}	......	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	......	b_j	......	b_n	

In the transportation table $O_1, O_2, \ldots O_i, \ldots O_m$ are sources from where goods are to be transported to destinations $D_1, D_2, \ldots D_j, \ldots D_n$. Any of the sources can transport to any of the destinations. C_{ij} is per unit transporting cost of goods from i^{th} source O_i to j^{th} destination D_j for all i=1,2,..m and j=1,2,..n. x_{ij} is the amount of goods transported from i^{th} source O_i to j^{th} destination D_j. a_i be the amount of goods available at the source O_i and b_j, the demand at the destination D_j. The corresponding transportation problem is

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \ \sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots n$$

$$x_{ij} \geq 0, \forall i = 1, 2, \ldots m \ \& \ j = 1, 2, \ldots n.$$

Since in a transshipment problem, any origin or destination can ship to any other origin or destination it would be convenient to number them successively so that the origins are numbered from 1 to m denoted by $T_1, T_2, \ldots, T_m$ and the destinations from m + 1 to m + n denoted by $T_{m+1}, T_{m+2}, \ldots, T_{m+n}$.

We now extend this transportation problem to permit transshipment with the additional feature that shipments may go via any sequence of points rather than being restricted to direct connections from one origin to one of the destination. The unit cost of shipment from a point considered as a shipper to the same point considered as receiver is set equal to zero. For the terminals $T_1, T_2, \ldots, T_m$ the total out shipments exceeds the total in shipments by amounts equal to $a_1, a_2, \ldots, a_m$ respectively and at the remaining n terminals $T_{m+1}, T_{m+2}, \ldots, T_{m+n}$, the total in shipment exceeds the out shipment by amount $b_{m+1}, b_{m+2}, \ldots, b_{m+n}$ respectively. If the total in shipment at terminals $T_{m+1}, T_{m+2}, \ldots, T_{m+n}$ be $t_1, t_2, \ldots, t_m$ respectively and the total out shipment at the terminals $T_{m+1}, T_{m+2}, \ldots, T_{m+n}$ be $t_1, t_2, \ldots, t_{m+n}$ respectively, the classical transhipment problem can be written as

Thus the transshipment problem may be written as

$$\text{Minimize } z = \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij}$$

subject to constraints

$$\sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} = a_i, i = 1, 2, ...m$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j, j = m+1, m+2, ...m+n.$$

$$x_{ij} \geq 0, i = 1, 2, ...m+n, \text{ j} \neq \text{i}$$

The above formulation is a linear programming problem, which is similar to a classical transportation problem but not exactly the equal since some coefficients of x_{ij}'s are also -1. The problem however easily be converted to a standard transportation problem.

$$\text{let } t_i = \sum_{j=1, j\neq i}^{m+n} x_{ji}, i = 1, 2, ...m,$$

$$i.e. \qquad t_i + x_{ii} = \sum_{j=1}^{m+n} x_{ji}, i = 1, 2, ...m$$

$$and \ t_j = \sum_{i=1, i\neq j}^{m+n} x_{ji}, j = m+1, m+2, ...m+n$$

$$i.e. \qquad t_j + x_{jj} = \sum_{i=1}^{m+n} x_{ji}, j = m+1, m+2, ...m+n$$

Where t_i represents the total amount of transshipment through the i^{th} origin and t_j represents the total amount shipped put from the j^{th} destination as transshipment.

Let T>0 be sufficiently large number so that $t_i \leq T$, for all i and $t_j \leq T$ for all j.

We now write $t_i + x_{ii} = T$ .Then the non negative slack variable x_{ii} represents the difference between T and the actual amount of transshipment through the i^{th} origin.
Similarly, if we let $t_j + x_{jj} = T$, then the non negative slack variable x_{jj} represents the difference between T and the actual amount of transshipment through the j^{th} destination.
Note that T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of goods transshipped at any point cannot exceed the amount produced or received and hence we take

$$T = \sum_{i=1}^{m} a_i$$

The transshipment problem then reduces to

$$\min \text{ z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij} \ \ x \in \mathbb{R}$$

$$\text{s.t.} \qquad \sum_{j=1}^{m+n} x_{ij} = a_i + T, i = 1, 2, ...m,$$

$$\sum_{j=1}^{m+n} x_{ij} = T, i = m+1, m+2, ...m+n,$$

$$\sum_{i=1}^{m+n} x_{ij} = T, j = 1, 2, ...m,$$

$$\sum_{i=1}^{m+n} x_{ij} = b_j + T, j = m+1, m+2, ...m+n,$$

$$x_{ij} \geq 0, i = 1, 2, ...m+n \text{ and } j = 1, 2, ...m+n, \text{where } c_{ii} = 0, i = 1, 2, ...m+n.$$

The above mathematical model represents a standard transportation problem with (m+n) origins and (m+n) destinations.

The solution of the problem contains 2m+2n-1 basic variables. However, m+n of these variables appearing in the diagonal cells represent the remaining buffer stock and if they are omitted. We have m+n-1 basic variables of our intrest.

TRANSSHIPMENT PROBLEM WITH MIXED CONSTRAINTS

The substantially increase or decrease of the capacity of a factory will affect the overall production and transportation cost. Similarly, the substantially increase or decrease of the demand of a destination will affect the overall production and transportation cost.

Suppose that the source $O_i, i \in \alpha_1$ supplies exactly fixed amount a_i, source $O_i, i \in \alpha_2$ supplies at least amount a_i and source $O_i, i \in \alpha_3$ supplies at most an amount a_i. Similarly, the destination $D_j, j \in \beta_1$ demands exactly the fixed amount b_j, the destination $D_j, j \in \beta_2$ demands at least an amount b_j, the destination $D_j, j \in \beta_3$ demands at most an amount b_j.

Considering this fact, the transportation problem with mixed constraints may be written as

$$Min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} = a_i \forall i \in \alpha_1$$

$$\sum_{j=1}^{n} x_{ij} \geq a_i \forall i \in \alpha_2$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \forall i \in \alpha_3$$

$$\sum_{i=1}^{m} x_{ij} = b_j \forall j \in \beta_1$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \forall j \in \beta_2$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j \forall j \in \beta_3$$

$$where\ \mathrm{I}_1 = \{1, 2, ...m\} = \alpha_1 \cup \alpha_2 \cup \alpha_3\}$$

$$I_2 = \{1, 2, ...n\} = \beta_1 \cup \beta_2 \cup \beta_3\}$$

The corresponding transshipment problem is as follows:

Find the values of x_{ij} such that

Minimize $z = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij}$

Subject to constraints

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = a_i, \forall i \in \alpha_1$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} \geq a_i, \forall i \in \alpha_2$$

$$\sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} \leq a_i, i \in \alpha_3$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j, j \in \beta_1$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} \geq b_j, j \in \beta_2$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} \leq b_j, j \in \beta_3$$

$$x_{ij} \geq 0, i, j = 1, 2, ...m+n, i \neq j$$

where $\alpha_1 \cup \alpha_2 \cup \alpha_3 = I_1 = \{1, 2, ...m\}$,

$\beta_1 \cup \beta_2 \cup \beta_3 = I_2 = \{m+1, m+2, ...m+n\}$.

The problem is said to be the transshipment problem with mixed constraints.

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ then the problem is said to be a balanced transshipment problem with mixed constraints.

If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ then the problem is said to be an unbalanced transshipment problem with mixed constraints. In this case a dummy origin/destination can be introduced to make it a balanced transshipment problem with mixed constraints.

Now supposing large number T for $\sum_{j=1}^{m+n} x_{ji}, \forall i \in I_1$ and also for $\sum_{i=1}^{m+n} x_{ij}, \forall j \in I_2$,the above transshipment problem can be reduced to

Minimize $z = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij}$

Subject to constraints

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} = a_i + T, \forall i \in \alpha_1$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} \geq a_i + T, \forall i \in \alpha_2$$

$$\sum_{j=1, j\neq i}^{m+n} x_{ij} \leq a_i + T, i \in \alpha_3$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} = T, \forall i \in \alpha_1 \cup \alpha_2 \cup \alpha_3$$

Which comes out to be the transportation problem with mixed constraints.

The idea given by Khurana and Arora, to convert the transportation problem with mixed constraints to an equivalent standard transportation problem is used to convert the transshipment problem with mixed constraints into standard transportation problem. The corresponding standard transportation problem is as follows:

$$\min z = \sum_{i=1}^{m+n+1} \sum_{j=1}^{m+n+1} c_{ij}' x_{ij}$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{m+n+1} x_{ij} = a_i', \forall i \in I_1, \\ a_i' = a_i + T, \forall i \in I_i, \\ \sum_{j=1}^{m+n+1} x_{ij} = T, \forall i \in I_2, \\ \sum_{i=1}^{m+n+1} x_{ij} = b_j', \forall j \in I_2, \\ b_j' = b_j + T, \forall j \in I_2, \\ \sum_{i=1}^{m+n+1} x_{ij} = T, \forall j \in I_1, \\ a_i' = T, \forall i \in I_2, \\ b_j' = T, \forall j \in I_1, \\ a'_{m+n+1} = \left(\sum_{i=1}^{m} a_i' + nT \right), \\ b'_{m+n+1} = \left(\sum_{j=m+1}^{m+n} b'_j + mT \right), \\ c'_{ij} = c_{ij}, \forall i, j \in I_1 \cup I_2, \\ (A) = \begin{cases} c'_{i,m+n+1} = \min_{j \in \beta_2} \{c_{ij}\}, \forall i \in \alpha_1 \cup \alpha_2, \\ \qquad = 0, \quad \forall \mathrm{i} \in \alpha_3 \\ c'_{i,m+n+1} = \min_{j \in \alpha_2} \{c_{ij}\}, \forall i \in \beta_1 \cup \beta_2, \\ \qquad = 0, \quad \forall i \in \beta_3, \\ c'_{m+n+1,j} = \min_{i \in \beta_2} \{\mathrm{c}_{ij}\}, \forall \mathrm{j} \in \alpha_1 \cup \alpha_2, \\ \qquad = 0, \quad \forall j \in \alpha_3, \\ \mathrm{c}'_{m+n+1,j} = \min_{i \in \alpha_2} \{\mathrm{c}_{ij}\}, \forall \mathrm{j} \in \beta_1 \cup \beta_2, \\ \qquad = 0, \quad \forall j \in \beta_3, \\ \mathrm{c}'_{m+n+1,m+n+1} = 0, \\ x_{ij} \geq 0, \mathrm{i}=1,2,\ldots,\mathrm{m+n}. \end{cases} \end{cases}$$

Where A is the inlarged cost matrix.

METHOD FOR CHANGING TRANSSHIPMENT PROBLEM WITH MIXED CONSTRAINTS INTO A CLASSICAL TRANSPORTATION PROBLEM-

The method is given in the form of an algorithm as follows:

Algorithm

Step-1. Given the linear transhipment problem. If $\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j$, then the transhipment problem is balanced, take $T = \sum_{i=1}^{m} a_i$ else take $T = \max\left(\sum_{i=1}^{m} a_i, \sum_{j=1}^{m+n} b_j \right)$ and go to step 2.

Step-2. Construct a transportation tableau as follows. A row in the tableau will be needed for each supply point and transhipment point, and a column will be needed for each demand point and transhipment point.

Step-3. Add a dummy demand point/column with a demand $=\sum_{i=1}^{m} a_i' + nT$ or a dummy supply point/row with a supply $=\sum_{j=m+1}^{m+n} b_j' + mT$ having costs as defined in (A) of equation Shipments to the dummy and from a point to itself are taken as zero.

Step-4. Each transhipment point will have a supply equal to its original supply $a_i(i=1,2,...m)+T$ and will have a demand equal to its original demand $b_j(j=m+1,m+2,...m+n)+T$. Also, each supply point will have supply equal to original supply, T(for i=m+1,m+2,...m+n) and each demand point will have its demand equal to original demand, T(for j=1,2,...m). This ensures that any transhipment point that is a net supplier will have a net outflow equal to points original supply and a net demander will have a net inflow equal to points original demand. Although we don't know how much will be shipped through each transhipment point, we can be sure that the total amount will not exceed T.

Solution of classical transportation problem is obtained by zero suffix method which is clearly stated through the algorithm as follows:

ZERO SUFFIX ALGORITHM

Step-1. Change transshipment problem into transportation problem.

Step-2. Subtract the minimum element of each row/column from all the elements of the corresponding row/column to ensure that each row/column contains at least one zero.

Step-3. Obtain the suffix value S of each zero by S=average of minimum elements of corresponding rows and minimum elements of corresponding columns.

Step-4. Search the greatest suffix value of zeros. If it is unique then assign the minimum of demand and supply to the corresponding cell and delete the row/column having supply/demand are exhausted and find the reduced table. Then go to step 2. If the greatest suffix value of zeros is not unique, then go to step 5.

Step-5. Find the second suffix values for that zeros whose first suffix values are greatest but not unique by S1=average of the minimum and next minimum elements of corresponding row and minimum and next minimum elements of corresponding column. Find the cell which has greatest second suffix value and assign the minimum of demand and supply to the corresponding cell. Delete the row /column whose supply/demand are exhausted and find the reduced table. Go to step 2. If the second suffix value is not unique then find the third suffix value and so on and do accordingly.

Step-6. If all the demands and supplies are exhausted then stop. The optimal basic feasible solution of the transshipment problem with mixed constraints is obtained.

Step-7. Ignoring the allocations in the diagonal cells, the solution obtained is the optimal basic feasible solution for the transhipment problem with mixed constraints.

Step-8. Continue the process from Step 2 to Step 7 until all the demand and supply are fulfilled.

NUMERICAL EXAMPLE

Balanced transhipment problem with mixed constraints.

Consider the balanced transhipment problem with mixed constrints involving three origins and three destinations. The availabilities at the origins, the requirements at the destinations and the costs of transportation are given below in the Table.

Table-1

	O_1 (j=1)	O_2 (j=2)	O_3 (j=3)	D_1 (j=4)	D_2 (j=5)	D_3 (j=6)	a_i
O_1 (i=1)	0	1	1	5	4	7	$=4$
O_2 (i=2)	1	0	1	2	6	5	≥ 6
O_3 (i=3)	1	1	0	4	8	3	≤ 5
D_1 (i=4)	5	2	4	0	2	2	...
D_2 (i=5)	4	6	8	2	0	2	...
D_3 (i=6)	7	5	3	2	2	2	...
b_j	...	...	...	$=5$	≥ 6	≤ 4	

Since $T=\sum_{i=1}^{3}a_i=\sum_{j=4}^{6}b_j=15$, we convert the problem into a linear transportation problem by adding 15 units to each a_i and b_j.

Table-2

	O_1 (j=1)	O_2 (j=2)	O_3 (j=3)	D_1 (j=4)	D_2 (j=5)	D_3 (j=6)	a_i
O_1 (i=1)	0	1	1	5	4	7	19
O_2 (i=2)	1	0	1	2	6	5	21
O_3 (i=3)	1	1	0	4	8	3	20
D_1 (i=4)	5	2	4	0	2	2	15
D_2 (i=5)	4	6	8	2	0	2	15
D_3 (i=6)	7	5	7	2	2	2	15
b_j	15	15	15	20	21	19	

Also, we add a dummy column D_4 with demand equal to $\sum_{i=1}^{m}a_i'+nT=105$ and a dummy row O_4 with availability equal to $\sum_{j=m+1}^{m+n}b_j'+mT=105$. We have the following transportation table.

Table-3

	O_1	O_2	O_3	D_1	D_2	D_3	dD_4	a_i
O_1	0_{1}	1	1	5	4	7	4	19
O_2	1	0_{1}	1	2	6	5	6	21
O_3	1	1	0_{0}	4	8	3	0_{0}	20
D_1	5	2	4	$0_{2,2.4}$	2	2	2	15
D_2	4	6	8	2	[15] $0_{2,2.7}$	2	6	$15\times$
D_3	7	5	3	2	2	2	$0_{1.2}$	15
dO_4	4	6	0_{0}	2	6	$0_{1.2}$	0_{0}	105
b_j	15	15	15	20	$21\times$ 6	19	105	

This is a balanced transportation problem.

To obtain its optimal basic feasible solution, we solve it by zero point method. Since its each rows and each column contains at least one zero, the suffix value 'S' of each zero is obtained by S= average of the least values in its row and column. That is suffix value of zero in cell (O_1, O_1) is equal to (1+1+1+1)/4 =1. This suffix value 1 of zero is mentioned by the subscript of zero in the table 6.3. Similarly suffix values of all the zeros are obtained and attached to the corresponding zeros by the subscript in table 3.

Now the maximum suffix value is 2 for two cells (D_1, D_1), (D_2, D_2) . That is, the maximum suffix value is not unique for these two cells. The second suffix value S1of these two cells are obtained as follows:
S1 = average of the minimum and the next minimum elements of its row and column. That is, second suffix value of zero of cell (D_1, D_1) is equal to (2+2+2+2+4+2+2+2+2+4)/10=2.4 which is mentioned by the second subscript of the corresponding zero in the table 3. Similarly the second suffix value of cell (D_2, D_2) is 2.7.

Now the second suffix value of the cell (D_2, D_2) is the greatest. We assign the cell minimum (D_2, D_2)m of (15,21) i.e., 15 in table 3. As the supply of D_2 row is exhausted so after deleting that row from table 3., we have reduced table 4 given below.

Table-4

	O_1	O_2	O_3	D_1	D_2	D_3	dD_4	a_i
O_1	0	1	1	5	4	7	4	19
O_2	1	0	1	2	6	5	6	21
O_3	1	1	0	4	8	3	0	20
D_1	5	2	4	0	2	2	2	15
D_3	7	5	3	2	2	2	0	15
dO_4	4	6	0	2	6	0	0	105
b_j	15	15	15	20	6	19	105	

Repeating the process
We have optimal solution of balanced transhipment problem with mixed constraints given in table

Table-5

	O_1	O_2	O_3	D_1	D_2	D_3	dD_4	a_i	
O_1	15 0	1	4 1	5	4	7	4	19	$u_1 = 0$
O_2	1	15 0	1 1	5 2	6	5	6	21	$u_2 = 0$
O_3	1	1	10 0	4	8	3	10 0	20	$u_3 = -1$
D_1	5	2	4	15 0	2	2	2	15	$u_4 = -2$
D_2	4	6	8	2	15 0	2	6	15	$u_5 = -3$
D_3	7	5	3	2	6 2	2	9 0	15	$u_6 = -1$
dO_4	4	6	0	2	6	19 0	86 0	105	$u_7 = -1$
b_j	15	15	15	20	21	19	105		
	$v_1 = 0$	$v_2 = 0$	$v_3 = 1$	$v_4 = 2$	$v_5 = 3$	$v_6 = 1$			$v_7 = 1$

Corresponding transhipment cost is

$$\text{minimum z} = 15\times0+4\times1+15\times0+1\times1+5\times2+10\times0+10\times0+15\times0+15\times0+6\times2+9\times0+19\times0+86\times0$$
$$= 4+1+10+12 = 27.$$

Table 5 contains allotment and dual variable (Modi Indices), we have $c_{ij}-(u_i+v_j)\geq 0$ for all non basic cell contains an optimal solution.

Note that unbalanced transhipment problem with mixed constraints can also be solved using the same way as illustrated above.

CONCLUSION

A simple algorithm for solving a transshipment problem with mixed constraints has been developed. Unbalanced transshipment problem may be solved. It is converted into a standard transportation problem by adding an additional row and an additional column.

A zero suffix method is developed, which finds optimal solution of standard transportation problem directly without obtaining basic feasible solution. Optimality of the solution is tested by modi-indices. Optimal solution of the transhipment problem with mixed constraints is the optimal solution of related equivalent standard transportation problem.

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