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CERTAIN CLASSES OF ANALYTIC FUNCTIONS BY USING SA˘LA˘GEAN CARLSON-SHAFFER OPERATOR

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ABSTRACT

In this paper, we define the subclass of analytic function by using Sălăgean Carlson-Shaffer Operator. The objective of this article is to obtain the result concerning the coefficient estimates of the class $M_\lambda(a, c, m, \alpha)$.

Key words and phrases: *Analytic functions, Carlson-Shaffer operator, Sălăgean operator and Inclusion theorem.*

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1. INTRODUCTION

Let A denote the class of analytic functions f of the form

$$f(z) = z + \sum_{m=2}^{\infty} a_m z^m \tag{1.1}$$

which are analytic in the open unit disc $U = \{z; |z| < 1\}$. Let $M(\alpha)$ be the subclass of A consisting of functions f which satisfies the inequality,

$$\mathbb{R}\left\{\frac{zf'}{f}\right\} < \alpha, for\ some\ \alpha > 1.$$

Let $N(\alpha)$ be the subclass of A consisting of functions f which satisfies the inequality,

$$\mathbb{R}\left\{1 + \frac{zf''}{f'}\right\} < \alpha, for\ some\ \alpha > 1.$$

Then we observed that $f \in N(\alpha)$ if and only if $zf' \in M(\alpha)$. For $n \in \mathbb{N}_0$ and $\lambda \geq 0, a, c, \in \mathbb{R} \setminus \mathbb{Z}$, let a linear operator [2] defined by

$$SL_\lambda f(z) = (1 - \lambda)[(k * k * \cdots * k) * f] + \lambda[\phi(a, c) * f](z), \qquad z \in U, \tag{1.2}$$

where $k(z) = z(1 - z)^{-2}$ is the Koebe function and

$$\phi(a, c; z) = \sum_{m=2}^{\infty} \frac{(a)_{m-1}}{(c)_{m-1}} z^m, \ |z| < 1, \ a, c \neq 0, -1, -2, \cdots,$$

is the incomplete beta function. For functions $f \in A$ of the form (1.1), we have

$$SL_\lambda f(z) = z + \sum_{m=2}^{\infty} B_\lambda(a, c, m, n)\, a_m z^m, \tag{1.3}$$

where

$$B_\lambda(a, c, m, n) = \left[(1 - \lambda)m^n + \lambda \frac{(a)_{m-1}}{(c)_{m-1}}\right]. \tag{1.4}$$

Here $(a)_m$ is the Pochhammer symbol defined interms of the Gamma function by,

$$(a)_m = \frac{\Gamma(a + m)}{\Gamma(a)} = \begin{cases} 1, & for\ m = 0 \\ a(a + 1)(a + 2) \ldots \ldots (a + m - 1) & for\ m \in \mathbb{N}. \end{cases}$$

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Now using the linear operator $\boldsymbol{SL_\lambda}$ we define the class $\boldsymbol{M_\lambda(a,c,m,\alpha)}$ consisting functions of the form (1.1) satisfying the condition

$$\mathbb{R}\left\{\frac{z[SL_\lambda f]'}{SL_\lambda f}\right\} < \alpha. \tag{1.5}$$

Note that $a = c = 1, and\ \lambda = 1$ the class reduces to $M(\alpha)$ and $a = 2, c = 1, and\ \lambda = 1$ the class reduces to $N(\alpha)$ defined by Owa and Nishwaki [3].

2. INCLUSION THEOREM INVOLVING COEFFIEICIENT INEQUALITIES

Theorem 2.1: If $f \in A$ satisfies

$$\sum_{m=2}^{\infty}\{(m-k)+|m+k-2\alpha|\}B_\lambda(a,c,m,n)|a_m| \le 2(\alpha-1), \tag{2.1}$$

for some $0 \le k \le 1$, then $f \in M_\lambda(a,c,m,\alpha)$.

Proof: Let us suppose that

$$\sum_{m=2}^{\infty}\{(m-k)+|m+k-2\alpha|\}B_\lambda(a,c,m,n)|a_m| \le 2(\alpha-1),\ \ f \in A.$$

It suffices to show that,

$$\left|\frac{\frac{z[SL_\lambda f]'}{SL_\lambda f}-k}{\frac{z[SL_\lambda f]'}{SL_\lambda f}-(2\alpha-k)}\right| < 1, (z \in U).$$

We note that,

$$\left|\frac{\frac{z[SL_\lambda f]'}{SL_\lambda f}-k}{\frac{z[SL_\lambda f]'}{SL_\lambda f}-(2\alpha-k)}\right| \le \left|\frac{(1-k)+\sum_{m=2}^{\infty}(m-k)B_\lambda(a,c,m,n)a_m z^{m-1}}{(1+k-2\alpha)+\sum_{m=2}^{\infty}(m+k-2\alpha)B_\lambda(a,c,m,n)a_m z^{m-1}}\right|$$

$$\le \frac{(1-k)+\sum_{m=2}^{\infty}(m-k)B_\lambda(a,c,m,n)|a_m|\,|z^{m-1}|}{(2\alpha-1-k)-\sum_{m=2}^{\infty}|(m+k-2\alpha)B_\lambda(a,c,m,n)||a_m|\,|z^{m-1}|}$$

$$< \frac{(1-k)+\sum_{m=2}^{\infty}(m-k)B_\lambda(a,c,m,n)|a_m|}{(2\alpha-1-k)-\sum_{m=2}^{\infty}(m+k-2\alpha)B_\lambda(a,c,m,n)|a_m|}$$

This expression is bounded above by 1 if,

$$(1-k)+\sum_{m=2}^{\infty}(m-k)B_\lambda(a,c,m,n)|a_m| < (2\alpha-1-k)-\sum_{m=2}^{\infty}(m+k-2\alpha)B_\lambda(a,c,m,n)|a_m|$$

which is equivalent to condition (2.1). Hence the proof.

Example: The function f given by

$$f(z) = z + \sum_{m=2}^{\infty}\frac{4(\alpha-1)}{m(m+1)\{(m-k)+|m+k-2\alpha|\}B_\lambda(a,c,m,n)}\, z^m \text{ belong to the class } M_\lambda(a,c,m,\alpha).$$

Now we discuss the coefficient estimates of functions $f \in M_\lambda(a,c,m,\alpha)$.

Theorem 2.2: $f \in M_\lambda(a,c,m,\alpha)$, then

$$|a_m| \le \frac{\prod_{j=1}^{m}(j+2\alpha-4)}{B_\lambda(a,c,m,n)(m-1)!} \tag{2.2}$$

Proof: Let us define the function $p(z)$ by,

$$p(z) = \frac{\alpha-\frac{z[SL_\lambda f]'}{SL_\lambda f}}{\alpha-1}$$

for $f \in M_\lambda(a,c,m,\alpha)$. Then $p(z)$ is analytic in U, $p(0) = 1\ and\ \mathbb{R}\{p(z)\} > 0$. If

$$p(z) = 1+\sum_{m=1}^{\infty}p_m z^m,\quad then\ |p_m| \le 2,\quad (m \ge 1).$$

Since,

$$\alpha SL_\lambda f - z[SL_\lambda f]' = (\alpha-1)p(z)SL_\lambda f$$

We obtain that,

$$(1-m)\boldsymbol{B_\lambda(a,c,m,n)a_m} = (\alpha-1)\{\boldsymbol{p_{m-1}} + \boldsymbol{p_{m-2}B_\lambda(a,c,2,n)a_2} + \cdots + \boldsymbol{p_1B_\lambda(a,c,m-1,n)a_{m-1}}\}.$$

If $m = 2, then\ \boldsymbol{B_\lambda(a,c,2,n)a_2} \le (\alpha-1)p_1 implies\ that$

$$|a_2| \le \frac{(\alpha-1)|p_1|}{\boldsymbol{B_\lambda(a,c,2,n)}} \le \frac{2(\alpha-1)}{\boldsymbol{B_\lambda(a,c,2,n)}}.$$

Hence the coefficient estimate for (2.2) is true for $m = 2$. Let us suppose that the coefficient estimate,

$$|a_k| \le \frac{\prod_{j=2}^{k}(j+2\alpha-4)}{\boldsymbol{B_\lambda(a,c,k,n)(k-1)!}}$$

is true for all $k = 2,3,4,\cdots,m$. Then we have,

$$-m\boldsymbol{B_\lambda(a,c,m+1,n)a_{m+1}} = (\alpha-1)\{\boldsymbol{p_m} + \boldsymbol{p_{m-2}B_\lambda(a,c,2,n)a_2} + \cdots + \boldsymbol{p_1B_\lambda(a,c,m,n)a_m}\}$$

So that,

$$\begin{aligned} m\boldsymbol{B_\lambda(a,c,m+1,n)}|\boldsymbol{a_{m+1}}| &\le (\boldsymbol{2\alpha-2})(\boldsymbol{1} + \boldsymbol{B_\lambda(a,c,2,n)}|\boldsymbol{a_2}| + \cdots + \boldsymbol{B_\lambda(a,c,m,n)}|\boldsymbol{a_m}|) \\ &\le (\boldsymbol{2\alpha-2})\left(\boldsymbol{1} + (\boldsymbol{2\alpha-2}) + \frac{(\boldsymbol{2\alpha-2})(\boldsymbol{2\alpha-1})}{\boldsymbol{2}!} + \cdots + \frac{\prod_{j=2}^{m}(\boldsymbol{j+2\alpha-4})}{(\boldsymbol{m-1})!}\right) \\ &= (\boldsymbol{2\alpha-2})\left(\frac{(\boldsymbol{2\alpha-1})\boldsymbol{2\alpha}(\boldsymbol{2\alpha+1})\cdots(\boldsymbol{2\alpha+m-4})}{(\boldsymbol{m-2})!} + \frac{(\boldsymbol{2\alpha-2})(\boldsymbol{2\alpha-1})\boldsymbol{2\alpha}\cdots(\boldsymbol{2\alpha+m-4})}{(\boldsymbol{m-1})!}\right) \\ &= \frac{\prod_{j=2}^{m+1}(j+\boldsymbol{2\alpha-4})}{(\boldsymbol{m-1})!}. \end{aligned}$$

This implies

$$|\boldsymbol{a_{m+1}}| \le \frac{\prod_{j=2}^{m+1}(j+\boldsymbol{2\alpha-4})}{\boldsymbol{B_\lambda(a,c,m,n)m}!}.$$

Hence the coefficient estimate (2.2) holds true for the class $k = m + 1$. Hence the theorem.

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