International Journal of Mathematical Archive-9(3), 2018, 225-231
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SOLUTION OF CONSTANTLY INCLINED ROTATING TWO PHASE MAGNETOHYDRODYNAMIC FLOWS THROUGH POROUS MEDIA

SAYANTAN SIL[1], MANOJ KUMAR[2] AND SAHENDRA SINGH[3]

[1]Department of Physics,
P. K. Roy Memorial College, Dhanbad-826004, Jharkhand, India.

[2]Department of Physics,
Ram Lakhan Singh Yadav College, Ranchi-834001, Jharkhand, India.

[3]Department of Applied Geology,
IIT(ISM), Dhanbad-826004, Jharkhand, India.

(Received On: 12-02-18; Revised & Accepted On: 13-03-18)

ABSTRACT

Steady, plane, viscous, incompressible, constantly inclined two-phase magnetohydrodynamic (MHD) fluid in a rotating frame through porous media is considered. A second order partial differential equation is obtained and by applying hodograph transformation technique exact solution for vortex flow is found out. Result is summarised in the form of a theorem and streamline pattern is shown for the solution.

***Key Words**: Two phase flow, MHD, exact solution, rotating frame, hodograph transformation.*

***2010 Mathematics Subject Classification:** 76W05.*

1. INTRODUCTION

In fluid mechanics, two-phase flow occurs in a system containing gas and liquid with a meniscus separating the two phases [19]. Two-phase flow is a particular example of multiphase flow. The interest in the problems of mechanics of systems with more than one phase has developed rapidly during the past few years [13]. Considerable amount of work has been devoted to dusty fluid flows due to the importance of such studies in many physical applications, ranging from fluidization problems to high-speed and dust supersonic flows [36]. Saffman [24] pioneered the study of the fluid-particle system. He derived the equations describing the motion of a gas carrying small dust particles and the equations satisfied by small disturbances of a steady laminar flow. Saffman formulated these equations of motion of dusty fluid which is represented in terms of large number density $N(x,t)$ of very small spherical inert particles whose volume concentration is small enough to be neglected. It is assumed that the density of the dust particles is large when compared with the fluid density so that the mass concentration of the particles is an appreciable fraction of unity. In this formulation, Saffman also assumed that the individual particles of dust are so small that Stoke's law of resistance between the particles and the fluid remains valid. Using the model of Saffman, several authors [12, 20, 22, 29] investigated various aspects of hydrodynamics and hydromagnetic two-phase fluid flows. M.H. Hamdan and R.M. Barron [7] developed the differential equations governing the dusty fluid flow in porous media based on Saffman's [24] dusty gas flow equations. Brent E. Sleep [14], M. H. Hamdan and K.D. Sawalha [15], Fathi M. Allam *et al.* [1], K.R. Madhura *et al.* [21] studied the dusty gas flow through porous medium. C. Thakur and R. B. Mishra [35] applied hodograph transformation in constantly inclined MFD flow. Manoj Kumar, Sayantan Sil and C. Thakur [18] studied two phase MFD flows through porous media.

Various transformation techniques involving inverse or semi-inverse methods are used for reformulation of equations in solvable form in order to get exact solutions. The hodograph transformation method is one of such methods. Ames [2] has given an excellent survey of the method. Many authors including Chandna *et al.* [6, 8, 9, 10, 11, 25] have applied hodograph and Legendre transform to investigate steady plane viscous flows, non-Newtonian flows and constantly inclined, aligned, transverse and orthogonal MHD non-Newtonian flows.

Corresponding Author: Sayantan Sil[1],
[1]Department of Physics, P. K. Roy Memorial College, Dhanbad-826004, Jharkhand, India.

As the theory of rotating fluids has become very important because of its occurrence in many natural phenomena for its application in various technological solutions many studies have been carried out on various types of flows both non-MHD and MHD in a rotating system. Several authors [3, 4, 17, 30, 31, 32, 33, 34, 37] studied rotating MHD flow and found out exact solutions. Krishan Dev Singh [28] studied unsteady MHD Poiseuille flow in a rotating system. M.A. Imran *et al.* [16] found out exact solutions for the MHD second grade fluid in a porous medium using integral transformation technique. A.M. Rashid [23] studied unsteady MHD flow of a rotating fluid from stretching surface in porous medium and effects of radiation and variable viscosity on it. Sayantan Sil and Manoj Kumar [26] studied rotating orthogonal plane MHD flow through porous media using complex variable technique. Also Sayantan Sil and Manoj Kumar [27] obtained exact solutions of a second grade rotating fluid.

In this paper hodograph transformation is employed for steady, plane, rotating, viscous, incompressible, constantly inclined two-phase magnetohydrodynamic (MHD) flows through porous media and partial differential equation of second order is obtained which is used to find the solution for vortex flow.

2. BASIC EQUATIONS

The basic equations of motion governing the steady flow of a dusty, incompressible, viscous fluid with infinite electrical conductivity through porous media in a rotating frame in the presence of magnetic field are given by [24]

For Fluid Phase:

$$\nabla\cdot\mathbf{V}=0, \qquad \text{(Continuity)} \qquad (1)$$

$$\rho\left[\left(\mathbf{V}\cdot\nabla\right)\mathbf{V}+2\boldsymbol{\Omega}\times\mathbf{V}\right]=-\nabla P+\mu\left(\nabla\times\mathbf{H}\right)\times\mathbf{H}+KN(\mathbf{U}-\mathbf{V})+\eta\nabla^2\mathbf{V}-\frac{\eta}{k}\mathbf{V}, \qquad \text{(Linear Momentum)} \qquad (2)$$

$$\nabla\times(\mathbf{V}\times\mathbf{H})=\mathbf{0}\,. \qquad \text{(Diffusion)} \qquad (3)$$

For Dust Phase:

$$\nabla\cdot(N\mathbf{U})=0, \qquad \text{(Continuity)} \qquad (4)$$

$$m\left[(\mathbf{U}\cdot\nabla)\mathbf{U}+2\boldsymbol{\Omega}\times\mathbf{U}\right]=K(\mathbf{V}-\mathbf{U}), \qquad \text{(Linear Momentum)} \qquad (5)$$

$$\nabla\cdot\mathbf{H}=0, \qquad \text{(Solenoidal)} \qquad (6)$$

where **V**, **U**, **H**, $\boldsymbol{\Omega}$ P, ρ, η, k are fluid velocity vector, dust velocity vector, magnetic field vector, constant angular velocity vector, fluid pressure, fluid density, kinematic coefficient of viscosity, magnetic permeability and permeability of the porous medium respectively; m is the mass of each dust particle, N the number density of dust particles and $K=6\pi a\eta$ -Stoke's resistance (drag coefficient) for the particles, a is spherical radius of dust particles.

The situation for which the velocity of fluid and dust particles are everywhere parallel, is defined as [5]

$$\mathbf{U}=\frac{\alpha}{N}\mathbf{V}, \qquad (7)$$

where α is some scalar satisfying

$$\mathbf{V}\cdot\nabla\alpha=0, \qquad (8)$$

which implies that α is a constant on the fluid streamlines.

Introducing vorticity function, current density function and Bernoulli function defined, respectively, by

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}, \qquad (9)$$

$$Q=\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial y}, \qquad (10)$$

$$B=P'+\frac{1}{2}\rho\left|\boldsymbol{\Omega}\times\mathbf{r}\right|^2+\frac{1}{2}\rho V^2, \qquad (11)$$

where $V^2=u^2+v^2$, P' is the reduced pressure given by $P'=P-\frac{1}{2}\rho\left|\boldsymbol{\Omega}\times\mathbf{r}\right|^2$ and the last term being the centrifugal contribution of pressure. The system of equations (1)-(6) can be replaced by the following system:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (12)$$

$$\eta\frac{\partial\omega}{\partial y}-\rho\omega v-2\rho\Omega v+\mu QH_2-K(\alpha-N)u=-\frac{\partial B}{\partial x}-\frac{\eta u}{k} \tag{13}$$

$$\eta\frac{\partial\omega}{\partial x}-\rho\omega u-2\rho\Omega u+\mu QH_1+K(\alpha-N)v=\frac{\partial B}{\partial y}+\frac{\eta v}{k}, \tag{14}$$

$$uH_2-vH_1=f \text{ , (arbitrary constant)} \tag{15}$$

$$\frac{m\alpha}{N}\left[\frac{\alpha}{N}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-2\Omega v+u\left\{u\frac{\partial}{\partial x}\left(\frac{\alpha}{N}\right)+v\frac{\partial}{\partial y}\left(\frac{\alpha}{N}\right)\right\}\right]=K\left(\frac{\alpha}{N}-1\right)u, \tag{16}$$

$$\frac{m\alpha}{N}\left[\frac{\alpha}{N}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)+2\Omega u+v\left\{u\frac{\partial}{\partial x}\left(\frac{\alpha}{N}\right)+v\frac{\partial}{\partial y}\left(\frac{\alpha}{N}\right)\right\}\right]=K\left(\frac{\alpha}{N}-1\right)v, \tag{17}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0. \tag{18}$$

The advantage of this system over the original system is that the order of partial differential equation is reduced from two to one.

We now consider constantly inclined plane flows and let θ_0 denote the constant non-zero angle between **V** and **H**. The vector and scalar product of **V** and **H**, using the diffusion equation (15), gives

$$uH_2-vH_1 = VH\sin\theta_0 = f,$$
$$uH_1-vH_2 = VH\cos\theta_0 = f\cot\theta_0, \tag{19}$$

where $H=\sqrt{H_1^2+H_2^2}$.

Solving (19), we get

$$H_1=\frac{f}{V^2}\left(Cu-v\right), H_2=\frac{f}{V^2}\left(Cv+u\right), \tag{20}$$

where $C=\cot\theta_0$ is a known constant for a prescribed constantly inclined non-aligned flow.

Using (20) in the system of equations (9)-(18), we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{21}$$

$$\eta\frac{\partial\omega}{\partial y}-\rho\omega v-2\rho\Omega v+\mu Q\frac{f}{V^2}\left(Cv+u\right)-K(\alpha-N)u=-\frac{\partial B}{\partial x}-\frac{\eta u}{k} \tag{22}$$

$$\eta\frac{\partial\omega}{\partial x}-\rho\omega u-2\rho\Omega u+\mu Q\frac{f}{V^2}\left(Cu-v\right)+K(\alpha-N)v=\frac{\partial B}{\partial y}+\frac{\eta v}{k}, \tag{23}$$

$$\frac{m\alpha}{N}\left[\frac{\alpha}{N}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-2\Omega v+u\left\{u\frac{\partial}{\partial x}\left(\frac{\alpha}{N}\right)+v\frac{\partial}{\partial y}\left(\frac{\alpha}{N}\right)\right\}\right]=K\left(\frac{\alpha}{N}-1\right)u, \tag{24}$$

$$\frac{m\alpha}{N}\left[\frac{\alpha}{N}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)+2\Omega u+v\left\{u\frac{\partial}{\partial x}\left(\frac{\alpha}{N}\right)+v\frac{\partial}{\partial y}\left(\frac{\alpha}{N}\right)\right\}\right]=K\left(\frac{\alpha}{N}-1\right)v, \tag{25}$$

$$\left(v^2-u^2+2uv\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(Cv^2-Cu^2+2uv\right)\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)=0, \tag{26}$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\omega, \tag{27}$$

$$\frac{\partial}{\partial x}\left(\frac{Cv+u}{V^2}\right)-\frac{\partial}{\partial y}\left(\frac{Cu-v}{V^2}\right)=\frac{Q}{f}. \tag{28}$$

Let the flow variables $u(x,\ y),\ v(x,\ y)$ be such that, in the flow region under consideration, the Jacobian

$$J = \frac{\partial(u,v)}{\partial(x,y)} \quad \text{satisfies } 0 < |J| < \infty .$$

In such a case, we consider x and y as function of u and v such that the following relations hold true:

$$\frac{\partial u}{\partial x} = J\frac{\partial y}{\partial v}\ , \quad \frac{\partial u}{\partial y} = -J\frac{\partial x}{\partial v}\ ,$$
$$\frac{\partial v}{\partial x} = -J\frac{\partial y}{\partial u}\ , \frac{\partial v}{\partial y} = J\frac{\partial x}{\partial u}\ . \tag{29}$$

Employing transformation equation (29) in (21) and (26), we get

$$\frac{\partial x}{\partial u} + \frac{\partial y}{\partial v} = 0\ , \tag{30}$$
$$\left(Cu^2 - Cv^2 - 2uv\right)\left(\frac{\partial x}{\partial u} - \frac{\partial y}{\partial v}\right) + \left(u^2 - v^2 - 2Cuv\right)\left(\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\right) = 0 . \tag{31}$$

The equation of continuity implies the existence of stream function $\psi(x,y)$ so that

$$\frac{\partial \psi}{\partial x} = -v\ , \quad \frac{\partial \psi}{\partial y} = u\ . \tag{32}$$

Likewise, equation (30) implies the existence of a function $L(u,v)$ called the Legendre transform of the stream function $\psi(x,y)$ such that

$$\frac{\partial L}{\partial u} = -y\ , \quad \frac{\partial L}{\partial v} = x\ . \tag{33}$$

Employing (33) in (31), we have

$$\left(v^2 - u^2 - 2Cuv\right)\frac{\partial^2 L}{\partial u^2} + \left(2Cu^2 - 2Cv^2 - 4uv\right)\frac{\partial^2 L}{\partial u\,\partial v} + \left(u^2 - v^2 - 2Cuv\right)\frac{\partial^2 L}{\partial v^2} = 0 . \tag{34}$$

Now introducing the polar coordinate (V, θ) in the hodograph plane i.e. the $(u,\ v)$ plane through the relation :

$$u = V\cos\theta\ , \quad v = V\sin\theta\ ,$$

equation (34) gets transformed into

$$\frac{\partial^2 L}{\partial V^2} - \frac{2C}{V}\frac{\partial^2 L}{\partial V\partial\theta} - \frac{1}{V^2}\frac{\partial^2 L}{\partial\theta^2} - \frac{1}{V}\frac{\partial L}{\partial V} + \frac{2C}{V^2}\frac{\partial L}{\partial\theta} = 0 \tag{35}$$

where θ is the inclination of vector field $\mathbf{V}$.

3. VORTEX FLOW

A solution of (35) is given by

$$L = B_2V^2 + \left(A_1\cos\theta + B_1\sin\theta\right)V\ ,$$
$$= B_2\left(u^2 + v^2\right) + A_1u + B_1v\ , \tag{36}$$

where A_1, B_1 and B_2 are arbitrary constants and $B_2 \neq 0$. In this case,

$$x = \frac{\partial L}{\partial v} = 2B_2v + B_1,\ y = -\frac{\partial L}{\partial u} = -(2B_2u + A_1)\ , \tag{37}$$

and therefore the velocity field is given by

$$u = -\frac{y + A_1}{2B_2},\quad v = \frac{x - B_1}{2B_2}\ . \tag{38}$$

These relations represent a circulatory flow.

From (20), we get

$$H_1 = \frac{-2B_2 f\left[(x-B_1)+C(y+A_1)\right]}{(x-B_1)^2+(y+A_1)^2},$$

and $$H_2 = \frac{2B_2 f\left[C(x-B_1)-(y+A_1)\right]}{(x-B_1)^2+(y+A_1)^2}. \tag{39}$$

The vorticity ω and current density Q can be expressed as

$$\omega = \frac{1}{B_2}, \; Q = 0. \tag{40}$$

From the integrability condition for B with the use of (13) and (14) and (38)-(40), we obtain

$$(x-B_1)\frac{\partial}{\partial x}(N-\alpha) + (y+A_1)\frac{\partial}{\partial y}(N-\alpha) + 2(N-\alpha) = -\frac{2\eta}{kK} \tag{41}$$

Solving (41), the number density of dust particles N(x, y) is given by

$$N = \frac{C_1}{(x-B_1)(y+A_1)} + \alpha - \frac{\eta}{kK}, \tag{42}$$

where C_1 is an arbitrary constant. From equation (8) and (38), we obtain

$$\alpha = C_2\left[(x-B_1)^2+(y+A_1)^2\right], \tag{43}$$

where C_2 is an arbitrary constant.

Hence $$N = \frac{C_1}{(x-B_1)+(y+A_1)} + C_2\left[(x-B_1)^2+(y+A_1)^2\right] - \frac{\eta}{kK}. \tag{44}$$

Using (38) – (40) and (42) in (22) and (23) and solving, we get

$$B = \frac{\rho(1+2B_2\Omega)}{4B_2^2}\left[(x-B_1)^2+(y+A_1)^2\right] + \frac{KC_1}{2B_2}\ln\frac{x-B_1}{y+A_1} + C_3, \tag{45}$$

where C_3 is an arbitrary constant. The pressure P is given by

$$P = \frac{\rho(1+4B_2\Omega)}{8B_2^2}\left[(x-B_1)^2+(y+A_1)^2\right] + \frac{KC_1}{2B_2}\ln\frac{x-B_1}{y+A_1} + C_3, \tag{46}$$

In this case the streamlines are given by $(x-B_1)^2+(y+A_1)^2 = \text{constant}$, which are concentric circles.

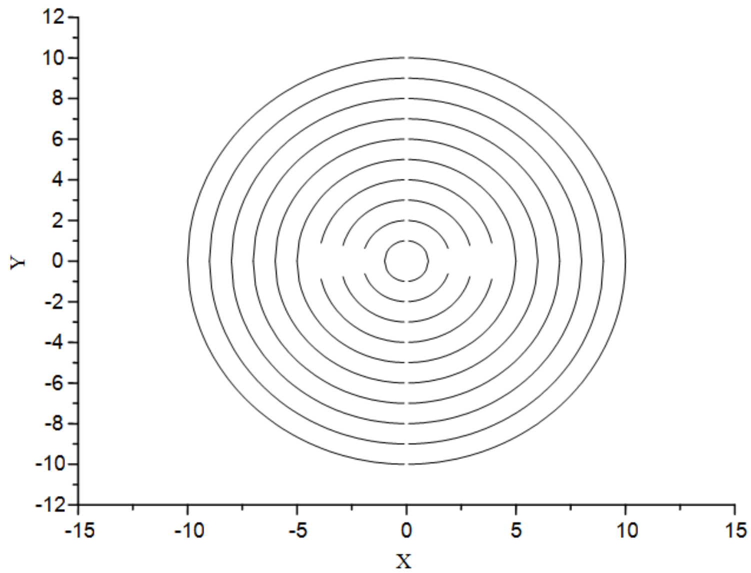

Figure-1: Concentric circular streamlines taking A_1=0, B_1=0

Summing up, we have:

Theorem 1: If the dust particle is everywhere parallel to fluid velocity in the steady, plane, constantly inclined Magnetohydrodynamic flow of an incompressible, viscous, two phase fluid through porous media in a rotating frame, then the streamlines are concentric circles and the dust particle number density is given by (44). Also the velocity, the magnetic field, the vorticity, the current density and the pressure are given by (38), (39), (40) and (46) respectively.

In the absence of rotating reference frame i.e. $\mathbf{\Omega} = \mathbf{0}$ we recover the results of Manoj Kumar, Sayantan Sil and C. Thakur [18]. Also when porous media is absent i.e. the term $\frac{\eta}{k} \to 0$ our result will tally with C. Thakur and R. B. Mishra [35].

4. CONCLUSION

In this paper, the analytical solution of steady, plane, viscous, incompressible, constantly inclined two-phase Magnetohydrodynamic fluid through porous media in a rotating frame is obtained using hodograph transformation technique. The expressions for velocity profile, magnetic field, number density of dust particles, streamline and pressure distributions are found out. Streamline pattern is also plotted. The present analysis is more general and several results of various authors (as already mentioned in the text) can be recovered in the limiting cases.

REFERENCES

1. Allan F. M., Qatanani N., Barghouthi I. and Takatka K. M., Dusty gas model of flow through naturally occurring porous media, App. Math. Comp. 148 (2004), 809-821.
2. Ames W. F. Non-linear Partial Differential Equation in Engineering, Academic press, New York (1965).
3. Bagewadi C. S. and Siddabasappa, Study of variably inclined rotating MHD flows in magnetograph plane, Bull. Cal. Math. Soc. 85 (1993), 93-106.
4. Bagewadi C. S. and Siddabasappa, The plane rotating viscous MHD flows, Bull. Cal. Math. Soc. 85 (1993), 513-520.
5. Barron R. M. Steady plane flow of a viscous dusty fluid with parallel velocity fields, Tensor N.S. 31 (1977) 271-274.
6. Barron R. M. and Chandna O. P., Hodograph transformations and solutions in constantly inclined MHD plane flows, J. Engng. Math. 15(3) (1981), 211-220.
7. Brent E. Sleep, Modelling transient organic vapour transport in porous media with the dusty gas model, Adv. Water Resour. 22(3) (1998), 247-256.
https://pdfs.semanticscholar.org/c38f/633aa862516458ec4278830c1444c7d60761.pdf
8. Chandna O. P., Barron R. M. and Smith A. C., Rotational plane steady flow of viscous fluid, SIAM J. Appl. Math. 42 (1982), 1323-1336.
9. Chandna O. P. and Garg M. R. On steady plane Magnetohydrodynamic flows with orthogonal magnetic and velocity field, Int J. Engng. Sci. 17 (1979), 251-257.
10. Chandna O. P. and Nguyen P. V, Hodograph method in non-Newtonian MHD transverse fluid flows, J. Engng. Math. 23(2) (1989) 119-139.
11. Chandna O. P. and Nguyen P. V, Hodograph study of MHD constantly inclined fluid flows, Int. J. Engng. Sci. 30 (1992), 69-82.
12. Debnath L. and Basu U., Unsteady slip flow in an electrically conducting two-phase fluid under transverse magnetic fields, II Nuvo Cimento B Series 11. 28(2) (1975), 349-362.
13. Gireesha B. J, Madura K. R. and Bagewadi C. S., Flow of an unsteady dusty fluid through porous media in a uniform pipe with sector of a circle as cross-section, Int. J. Pure and Appl. Math.76(1) (2012), 29-47.
14. Hamdan M. H. and Barron R. M., A dusty gas flow model in porous media, J. Comput. Appl. Math. 30 (1990), 21-37.
https://ac.els-cdn.com/037704279090003I/1-s2.0-037704279090003I-main.pdf?_tid=1e6dcfda-0efe-11e8-ad7a-00000aab0f26&acdnat=1518334703_4640331fc83698941f337de4e1c36b84
15. Hamdan M. H. and Sawalha K. D., Dusty gas flow through porous media, App. Math. Comp.75 (1996), 59-73.
16. Imran M. A., Imran M. and Fetecau C., MHD oscillating flows of a rotating second grade fluid in porous medium, Commun. in Numerical Anal. 2014 (2014), 1-12.
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.896.6749&rep=rep1&type=pdf
17. Jana R. N. and Dutta N., Couette flow and heat transfer in a rotating system, Acta Mech. 26 (1977), 301-306.
18. Kumar M., Sil S. and Thakur C., Hodograph transformation in constantly inclined two-phase MFD flows through porous media, IJMA. 4(7), (2013), 42-47.
www.ijma.info/index.php/ijma/article/view/2206/1333
19. Levy S., Two-phase flow in complex systems, New York, Wiley (1999).
20. Liu J. T. C., Flow induced by an oscillating infinite flat plate in dusty gas, Phys. Fluids. 9 (1966) 1716-1720.
21. Madhura K. R., Gireesha B. J. and Bagewadi C. S., Exact solutions of unsteady dusty fluid flow through porous media in an open rectangular channel, Adv. Theor. Appl. Mech. 2(1), (2009), 1 – 17.

www.m-hikari.com/atam/atam2009/atam1-4-2009/gireeshaATAM1-4-2009.pdf
22. Michael D. H. and Miller D. A., Plane parallel flow of a dusty gas, Mathematika. 13 (1966), 97 – 109.
23. Rashid A. M., Effects of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous media, J. Egyptian Math. Soc. 2(1) (2014) 134-142.
24. Saffman P. G., On the stability of laminar flow of a dusty gas, J. Fluid Mech. 13, (1962), 120 – 128.
25. Siddiqui A. M., Kaloni P. N. and Chandna O. P., Hodograph transformation methods in non-Newtonian fluid flows. J. Engng. Math. 19(3), (1985), 203-216. https://docslide.com.br/download/link/hodograph-transformation-methods-in-non-newtonian-fluids
26. Sil S. and Kumar M., A Class of solution of orthogonal plane MHD flow through porous media in a rotating frame, Global J. of Sci. Frontier Research: A Physics and Space Science. 14(7) (2014), 17-26. https://www.google.co.in/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahUKEwjQu87FwJ3ZAhVBpY8KHeBIAywQFggzMAI&url=https%3A%2F%2Fjournalofscience.org%2Findex.php%2FGJSFR%2Farticle%2Fdownload%2F1864%2F1725%2F&usg=AOvVaw1ffe70XqTKEdxGFaWN01Ta
27. Sil S. and Kumar M., Exact solution of second grade fluid in a rotating frame through porous media using hodograph transformation method. J. Appl. Math. Phys. 3 (2015), 1443-1453. https://file.scirp.org/pdf/JAMP_2015112615043840.pdf
28. Singh K. D., Rotating oscillatory MHD Poisseuille flow: An exact solution. Kragujevac J. Sci. 35, (2013), 15-25. http://www.pmf.kg.ac.rs/kjs/images/volumes/vol35/kjs35devsingh15.pdf
29. Singh S. N. and Babu R., Some geometric properties of a dusty fluid in MFD flows. Ap&SS. 104(2) (1984), 285-292.
30. Singh K. K. and Singh D. P., Steady plane MHD flows through porous media with constant speed along each stream line, Bull. Cal. Math. Soc. 85 (1993) 255-262.
31. Singh S. N., Singh H. P. and Rambabu, Hodograph transformations in steady plane rotating hydromagnetic flow. Astrophys. Space Sci. 106 (1984), 231-243.
32. Singh H. P. and Tripathi D. D., A class of exact solutions in plane rotating MHD flows. Indian J. Pure Appl. Math. 19(7) (1988), 677-687.
33. Soundalgekar V. M. and Pop I., On hydromagnetic flow in a rotating fluid past an infinite porous wall. Jour. Appl. Math. Mech. ZAMM. 53 (1973), 718-719.
34. Thakur C. and Kumar M., Plane rotating viscous MHD flows through porous media. Pure and Appl. Mathematika Sci. LXVII (1-2) (2008), 113-124.
35. Thakur C. and Mishra R. B., Hodograph transformation in constantly inclined two-phase MFD flows, Indian J. pure appl. Math. 20(7) (1989) 728-735.
36. Thakur C. and Mishra R. B., Two-Phase MFD Flows in Orthogonal Curvilinear Coordinate System. Int. J. Theor. Phys. 29 (9), (1990), 1029-1037.
37. Vidyanidhu V. and Nigam S. D., Secondary flow in a rotating channel, Jour. Mech. Phys. Sci.1, (1967), 85-100.

Source of support: Nil, Conflict of interest: None Declared.