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THERMOELASTIC RESPONSE TO SURFACE TEMPERATURE ASYMMETRY AND INTERNAL HEAT GENERATION PARAMETERS IN A COMPOSITE SPHERICAL BODY

S. P. PAWAR[1], N. J. WANGE*[2] AND M. N. GAIKWAD[3]

**[1]Department of Mathematics,
S.N. Mor and Smt. G.D. Saraf Science College, Tumsar (MS), India.**

**[2]Department of Mathematics,
Datta Meghe Institute of Engineering,
Technology & Research, Sawangi (Meghe), Wardha (M.S.), India.**

**[3]Principal, Gopikabai Sitaram Gawande Mahavidyalaya,
Umarkhed, Yavatmal (M.S.), India.**

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ABSTRACT

The thermoelastic response to surface temperature asymmetry and internal Heat Generation parameters with steady state temperature field in the context of uncoupled thermoelasticity studied over a composite spherical domain. The thermal stresses are computed analytically and presented numerically and graphically. The mathematical model for three layered hollow sphere of an Aluminum, Copper and Iron is prepared and observations are illustrated.

Keywords: *Composite hollow sphere, Thermal stresses, Heat generation, Temperature asymmetry.*

INTRODUCTION

The temperature distribution and its effect on thermal stresses in the composite regions consisting of several layers have numerous applications in engineering and manufacturing fields. The increasing use of composite materials in engineering applications has resulted in considerable research activity in this area in recent years. The use of composite materials of multilayer type has been tremendous in many engineering fields such as aerospace, automobiles, chemical and energy, civil and infrastructure, sports and recreation, biomedical engineering and so on.

An understanding of thermally induced stresses in multilayer isotropic bodies is essential for a comprehensive study of their response due to an exposure to a temperature field, which may in turn occur in service or during the manufacturing stages. Numerical methods are a common method for such problems; however, analytic approaches can provide greater insight into the physical processes and can be used to validate numerical methods.

The Laplace transform technique is used by Carslaw and Jaeger [1], they discussed the infinite composite of two different medium and obtained the temperature distribution. They solved the transient boundary value problem of heat conduction in solids consisting of many parallel layers. In practice if the number of layers is more than two, the inverse of Laplace transform becomes quite difficult. The Adjoin-solution technique which has been introduced by Goodman [2] provides a method of solution to large class of heat conduction problems in composite slabs from the solution but one adjoin problem. The primary disadvantage of the Adjoin-solution method is that only the solutions of the boundaries (i.e. interface) of the layers can be determined.

Recently Tittle [3] introduced a technique for orthogonal expansion of functions over a one dimensional multilayer region. The method essentially is an extension of Sturm-Liouville problem to the case of one dimensional multilayer region and it has the advantage on other analytic methods is that its application to the solution of the boundary value problem of heat conduction is relatively simple. Bulavin and Kashcheev [4] used the method of separation of variables and of orthogonal expansion of functions over a one dimensional multilayer region to solve the transient heat conduction problem involving distributed volume heat sources in a multilayer region.

Corresponding Author: N. J. Wange*[2]

Recently, Vollbrech [6]discussed the stress in cylindrical and spherical walls subjected to internal pressure stationary heat flow. Kandil [7] has studied the effect of steady state temperature and pressure gradient on compound cylinder under high pressure and temperature. Ghosn and Sabbaghian [8] investigated a one dimensional axisymmetric quasi-static coupled thermoelasticity problem. The solution technique uses Laplace transform. The inversion to the real domain is obtained by means of Cauchy's theorem of residues. Sherif and Anwar [10] discussed the problem of infinitely long elastic circular cylinder whose inner and outer surfaces are subjected to known temperature and are traction free. They have neglected both the inertia term and relaxation effects. Chen and Yang [9] discussed the thermal response of one dimensional quasi-static coupled thermoelastic problem of an infinite long cylinder composed of two different materials. They applied the Laplace transform with respect to time and used the Fourier series and matrix operation to obtain the solution. Jane and Lee [11] considered the solution by using the Laplace transform and the finite difference method. The cylinder was composed of multilayer of different materials. They obtained solution for the temperature and thermal stress distributions in a transient state. Zong-Yi Lee [13] studied the one dimensional quasi-static coupled thermoelastic problem of multilayered sphere with time dependent boundary conditions is considered. The medium is without body forces and heat generation. Laplace transform and finite difference methods are used to obtain the solution of wide range of transient thermal stresses.

The observations and study of all above cited papers and other referred literature on multilayer composites with different geometries reveals that results appearing in the different articles are with complexities such as space and time dependent properties. In most of the articles authors have discussed the heat conduction. Therefore it is to clarify, that how internal heat generation and surface temperature asymmetry affect the temperature and thermal stresses. In view of these findings, there is need to quantify the conclusions regarding the effect of internal heat generation and temperature asymmetry in fundamental problems where the layers of composites are homogeneous and isotropic. Recently Pawar *et al.* [16] discussed the problem where the temperature and thermal stresses are discussed under surface temperature asymmetry and heat generation and obtained analytic solution.

In this study, an exact analytic solution of steady temperature distribution and stress distribution function for one dimensional three layered sphere subjected to asymmetric surface temperature and internal heat generation is presented. The solutions are obtained and the effects on thermal stresses due to heat generation and surface temperature asymmetryparameter in the sphere are analyzed and results are illustrated graphically. This is a novel work to study the thermal stresses under changing source in each layer and surface temperature asymmetry parameter. The analysis is made on the basis of uncoupled thermoelasticity. On determining the temperature distribution function from heat conduction equation, it is used as a known function and introduced in thermoelastic equations to obtain the stress function. The Results presented here could not be found in the open literature despite of extensive search.

FORMULATION OF THE PROBLEM

A three layered hollow sphere contains an inner region $r_0 = a \le r \le r_1$, middle region $r_1 \le r \le r_2$ and an outer region $r_2 \le r \le r_3 = b$ which are in perfect contact and uniform volumetric heat with the rate $q\left(W/m^3\right)$is generated for $t > 0$. The inner and outer surfaces are maintained at constant temperatures $T^{(a)}$ and $T^{(b)}$ respectively. The layers of the multilayer sphere are homogeneous and isotropic, $k^{(i)}, (i = 1,2,3)$ are the thermal conductivities of material of these layers.

The geometry of the problem for steady state heat conduction in three layered hollow sphere has been considered as following figure.

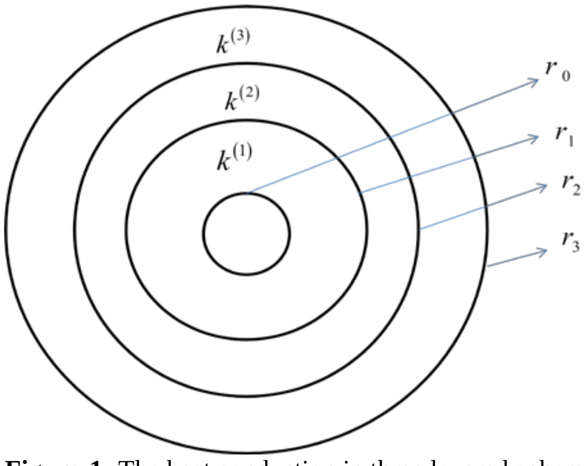

Figure-1: The heat conduction in three layered sphere

The inner and outer radii of multilayer sphere and the radius at the interface between generic first and third phases have been denoted by r_0, r_3 and r_i respectively. The mechanical and thermal properties of each layer have been assumed to be homogeneous and isotropic and are denoted with apex (i).

HEAT CONDUCTION PROBLEM

Assume one dimensional steady state radial temperature field. The heat conduction equation in the i^{th} layer of the spherical composite is given as [5],

$$\frac{1}{r}\left[\frac{d^2\left(rT^{(i)}\right)}{d\,r^2}\right]+\frac{q}{k^{(i)}}=0\,,r_0\le r\le r_3,\ i=1,2,3\,,t>0 \tag{1}$$

subjected to the following boundary conditions,

$$T^{(1)}=T^{(a)},r=r_0=a \tag{2}$$

$$T^{(3)}=T^{(b)},\ r=r_3=b \tag{3}$$

Interface conditions

$$T^{(1)}(r_1)=T^{(2)}(r_1) \tag{4}$$

$$k^{(1)}\frac{dT^{(1)}}{dr}=k^{(2)}\frac{dT^{(2)}}{dr} \tag{5}$$

$$T^{(2)}(r_2)=T^{(3)}(r_2) \tag{6}$$

$$k^{(2)}\frac{dT^{(2)}}{dr}=k^{(3)}\frac{dT^{(3)}}{dr} \tag{7}$$

NONDIMENSIONALIZATION

For convenience, we recast the above system of governing equations and auxiliary conditions into a dimensionless form. Redefining the variables as follows,

$$\left(\vartheta^{(i)},\vartheta^{(a)},\vartheta^{(b)}\right)=\left(T^{(i)},T^{(a)},T^{(b)}\right)/T^{(a)}$$

$$\left(R,R_{0,}R_1,R_2,R_3\right)=\left(r,r_0,r_1,r_2,r_3\right)/r_0$$

$$K^{(i)}=\frac{k_i}{k_1} \tag{8}$$

Introducing these new variables into the governing and auxiliary equations (1-7) to obtain the problem in more concise form as

$$\frac{1}{R}\left[\frac{d^2R\vartheta^{(i)}}{dR^2}\right]+Q^{(i)}=0 \tag{9}$$

where the internal heat generation in dimensionless form is written as

$$Q^{(i)}=\frac{2qr_0^2}{k^{(i)}T^{(a)}} \tag{10}$$

The boundary conditions are expressed as

$$\vartheta^{(a)}=1\ ,R=R_0=1$$

$$\vartheta^{(b)}=\frac{T^{(b)}}{T^{(a)}},\ R=R_3=\frac{b}{a} \tag{11}$$

$$\vartheta^{(1)}(R_1)=\vartheta^{(2)}(R_1)$$

$$K^{(1)}\frac{d\vartheta^{(1)}}{dR}=K^{(2)}\frac{d\vartheta^{(2)}}{dR},\quad R=R_1 \tag{12}$$

$$\vartheta^{(2)}(R_2)=\vartheta^{(3)}(R_2)$$

$$K^{(2)}\frac{d\vartheta^{(2)}}{dR}=K^{(3)}\frac{d\vartheta^{(3)}}{dR} \tag{13}$$

THERMOELASTICITY PROBLEM

In this section, thermal stress for steady state Temperature field is analyzed on the basis of uncoupled Thermoelasticity. For one dimensional problem in the spherical coordinate system, which means spherically symmetric problem, the displacement technique is extensively used. The properties in spherical coordinates ϕ and θ direction are identical and u denotes the displacement in the radial direction, the strain displacement relations for the i^{th} layer as [12,295]

$$\varepsilon_{rr}^{(i)} = \frac{du^{(i)}}{dr}, \ \varepsilon_{\theta\theta}^{(i)} = \frac{u^{(i)}}{r} \tag{14}$$

The corresponding thermoelastic stress strain relation or Hooke's relations are

$$\sigma_{rr}^{(i)} = \lambda^{(i)} e + 2\mu^{(i)} \varepsilon_{rr}^{(i)} - (3\lambda^{(i)} + 2\mu^{(i)}) \alpha^{(i)} \vartheta^{(i)}(R) \tag{15}$$

$$\sigma_{\theta\theta}^{(i)} = \sigma_{\phi\phi}^{(i)} = \lambda^{(i)} e + 2\mu^{(i)} \varepsilon_{\theta\theta}^{(i)} - (3\lambda^{(i)} + 2\mu^{(i)}) \alpha^{(i)} \theta^{(i)}(R) \tag{16}$$

where $\sigma_{rr}^{(i)}$, $\sigma_{\theta\theta}^{(i)}$ and $\sigma_{\phi\phi}^{(i)}$ are the stresses in the radial and tangential direction, $\varepsilon_{rr}^{(i)}$ and $\varepsilon_{\theta\theta}^{(i)}$ are strains in radial and tangential direction in the i^{th} layer of the composite hollow sphere. $\vartheta^{(i)}(R)$ is the temperature change obtained from the heat conduction equation (9), $a^{(i)}$ is the coefficient of thermal expansion, $e^{(i)}$ is the strain dilation and $\lambda^{(i)}$ and $\mu^{(i)}$ are the lame constants related to the modulus of elasticity $E^{(i)}$ and the Poisson's ratio $\nu^{(i)}$ as,

$$\lambda^{(i)} = \frac{\nu^{(i)} E^{(i)}}{\left(1+\nu^{(i)}\right)\left(1-2\nu^{(i)}\right)}, \ \mu^{(i)} = \frac{E^{(i)}}{2\left(1+\nu^{(i)}\right)} \tag{17}$$

The equilibrium equation in radial direction excluding the body forces and inertia term is,

$$r\frac{d\sigma_{rr}^{(i)}}{dr} + 2\left(\sigma_{rr}^{(i)} - \sigma_{\theta\theta}^{(i)}\right) = 0 \tag{18}$$

Assuming the traction free surface is as

$$\sigma_{rr}^{(0)}(r) = 0 \ \text{, at } r = r_0 = a \text{ and } r = r_3 = b \tag{19}$$

Then stress components for i^{th} layer are obtains as [12]

$$\sigma_{rr}^{(i)} = \frac{a^{(i)} E^{(i)}}{1-\nu^{(i)}} \left[\frac{2\left(r^3 - r_{i-1}^3\right)}{r^3\left(r_3^3 - r_{i-1}^3\right)} \int_{r_{i-1}}^{r_3} T^{(i)}(r) r^2 dr - \frac{2}{r^3} \int_{r_{i-1}}^{r} T^{(i)}(r) r^2 dr \right] \tag{20}$$

Assuming the interface conditions as follows

$$\sigma_{\theta\theta}^{(i)} = \sigma_{\phi\phi}^{(i)} = \frac{a^{(i)} E^{(i)}}{1-\nu^{(i)}} \left[\frac{(2r^3 + r_{i-1}^3}{r^3\left(r_3^3 - r_{i-1}^3\right)} \int_{r_{i-1}}^{r_3} T^{(i)}(r) r^2 dr + \frac{1}{r^3} \int_{r_{i-1}}^{r} T^{(i)}(r) r^2 dr - T^{(i)}(r) \right] \tag{21}$$

$$i = 1,2,3$$

Assuming the interface conditions as

$$\sigma_{rr}^{(1)}(r_1) = \sigma_{rr}^{(2)}(r_1)$$
$$\sigma_{rr}^{(2)}(r_2) = \sigma_{rr}^{(3)}(r_2) \tag{22}$$

Using the dimensionless coordinates (8) one can obtain following relations for stress function in following dimensionless form as,

$$\sigma_{RR}^{(i)^*} = \frac{\sigma_{rr}^{(i)}\left(1-\nu^{(i)}\right)}{2a^{(i)} E^{(i)} T^{(a)}} = \frac{1}{R^3} \left[\frac{\left(R^3 - 1\right)}{\left(R_b^3 - 1\right)} \int_1^{R_b} \vartheta^{(i)}(R) R^2 dR - \int_1^{R} \vartheta^{(i)}(R) R^2 dR \right] \tag{23}$$

$$\sigma_{\theta\theta}^{(i)^*} = \sigma_{\phi\phi}^{(i)^*} = \frac{\sigma_{\theta\theta}^{(i)}\left(1-\nu^{((i))}\right)}{a^{(i)} E^{(i)} T^{(a)}} = \frac{1}{R^3} \left[\frac{\left(2R^3 + 1\right)}{(R_b^3 - 1)} \int_1^{R_b} \vartheta^{(i)}(R) R^2 dR + \int_1^{R} \vartheta^{(i)}(R) R^2 dR - \vartheta^{(i)}(R) R^3 \right] \tag{24}$$

Boundary conditions are expressed as,

$$\sigma_{RR}^{(i)^*}=0,\ R=R_a=1\ and\ R=R_b=\frac{b}{a} \tag{25}$$

$$\sigma_{RR}^{(1)^*}\left(R_1\right)=\sigma_{RR}^{(2)^*}\left(R_1\right)_3$$

$$\sigma_{RR}^{(2)*}\left(R_2\right)=\sigma_{RR}^{(3)*}\left(R_2\right) \tag{26}$$

The equations (9-13) and (23-26) constitutes the Mathematical formulation of the problem in dimensionless variables

Solution:
The heat conduction equation (9) is Cauchy's homogeneous linear equation expressed as

$$R^2\frac{d^2\vartheta^{(i)}}{dR^2}+2R\frac{d\vartheta^{(i)}}{dR}=-Q^{(i)}R^2 \tag{27}$$

The solution of this equation is obtained by integral method using boundary conditions and expressed with interface conditions as

$$\vartheta^{(i)}(R)=1+\frac{Q^{(i)}}{6}-\frac{Q^{(i)}R^2}{6}-\frac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\frac{(1-R)}{R} \tag{28}$$

with interface conditions (12) and (13)

DETERMINATION OF THERMAL STRESS

Using equations (23), (24) and (28), the expressions for radial and tangential stresses are obtained as

$$\sigma_{RR}^{(i)^*}=\frac{1}{R^3}\left[\begin{array}{l}\dfrac{\left(R^3-1\right)}{\left(R_3^3-1\right)}\left\{\begin{array}{l}\left(\dfrac{1}{3}+\dfrac{Q^{(i)}}{18}\right)\left(R_3^3-1\right)+\dfrac{Q^{(i)}}{30}\left(1-R_3^5\right)+\dfrac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\\ \left[\dfrac{1-3R_3^2+2R_3^3}{6}\right]\end{array}\right\}\\ -\left\{\begin{array}{l}\left(\dfrac{1}{3}+\dfrac{Q^{(i)}}{18}\right)\left(R^3-1\right)+\dfrac{Q^{(i)}}{30}\left(1-R^5\right)+\dfrac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\\ \dfrac{1-3R^2+2R^3}{6}\end{array}\right\}\end{array}\right] \tag{29}$$

$$\sigma_{\theta\theta}^{(i)^*}=\frac{1}{R^3}\left[\begin{array}{l}\dfrac{\left(2R^3-1\right)}{\left(R_3^3-1\right)}\left\{\begin{array}{l}\left(\dfrac{1}{3}+\dfrac{Q^{(i)}}{18}\right)\left(R_3^3-1\right)+\dfrac{Q^{(i)}}{30}\left(1-R_3^5\right)-\dfrac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\\ \left(\dfrac{1-3R_3^2+2R_3^3}{6}\right)\end{array}\right\}\\ +\dfrac{1}{R^3}\left\{\begin{array}{l}\left(\dfrac{1}{3}+\dfrac{Q^{(i)}}{18}\right)\left(R^3-1\right)+\dfrac{Q^{(i)}}{30}\left(1-R^5\right)+\dfrac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\\ \left(\dfrac{1-3R^2+2R^3}{6}\right)\end{array}\right\}\\ -R^3\left\{1+\dfrac{Q^{(i)}}{6}-\dfrac{Q^{(i)}R^2}{6}-\dfrac{R_3\left[6\left(1-\vartheta^{(b)}\right)+Q^{(i)}\left(1-R_3^2\right)\right]}{6\left(1-R_3\right)}\left(\dfrac{1-R}{R}\right)\right\}\end{array}\right] \tag{30}$$

with interface conditions (26)

VALIDATION

If, in the governing expression or solutions obtained for temperature distribution, radial and tangential stressdistribution functions, one substitutes Q for $Q^{(i)}$ and excludes interface conditions, one gets the expression for temperature and stresses for homogenous and isotropic hollow sphere and the results are validated with results obtained by Pawar *et al.* [16] or the results obtained will reduce to special case of isotropic and homogeneous material hollow sphere. This article deals with the thermoelastic response to heat generation and temperature asymmetry parameter in three layered composite hollow sphere. No one has discussed such problem for a multilayered composite spherical domain. Hence, the correctness and accuracy is verified by above validation.

NUMERICAL AND GRAPHICAL DISCUSSION

A three layer composite hollow sphere was considered for the numerical calculation purpose. Its schematic picture is demonstrated in fig. 1.The radius of the hollow sphere varies from $0.02\,m\,to\,0.30\,m$ .The material layers and its thermal properties are given in following table.

Layer	1	2	3
Material properties	Aluminum	Copper	Iron
$k_i\left(W/mK\right)$ $(Thermal\ \ conductivity)$	204.2	386	72.7
$\nu_i\ \left(Poisson's\ Ratio\right)$	0.35	0.33	0.30
$E_i\ \left(GPa\right)$	70	117	100
$a_i\left(1/K\right)$(coeff. Thermal expansion)	2.3x10^{-6}	16.5x10^{-6}	6.7x10^{-6}
$r_i\left(m\right)$	0.02 to 0.10	0.10 to 0.20	0.20 to 0.3

Table-1

In dimensionless variables radius R varies in layers as (8)

R_i (1 to 15)	1 to 5	5 to 10	10 to 15

Table-2

Dimensionless source parameter $Q^{(i)}$ varies in layers as (10)

g	$Q^{(i)}$	$Q^{(2)}$	$Q^{(i)}$
500	3	2	7
1000	5	3	14
1500	7	4	21

Table-3

The numerical calculations and graphs are obtained by using the MATLAB software. One dimensional steady state temperature and thermal stress variation with radius is discussed under this analysis. The main approach of the analysis is to investigate the variation in above quantities with internal heat generation and surface temperature asymmetry parameter. The uniform volumetricsource inside the sphere changes due to varied conductivity of layer material and hence heat generation parameter. Thus it is very interesting to observe the effect of these parameters on thermal stresses inside the body.

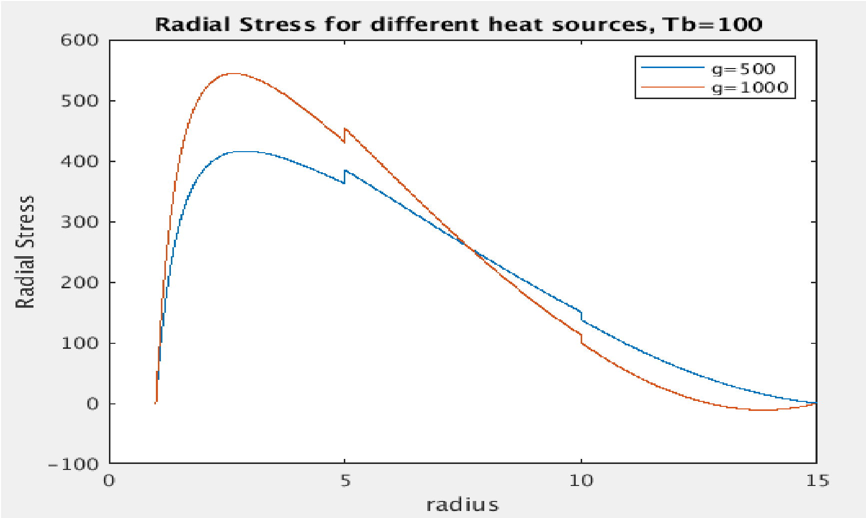

Figure-2: Radial stress variation for different values of heat generation ($\vartheta^{(b)} > 1$)

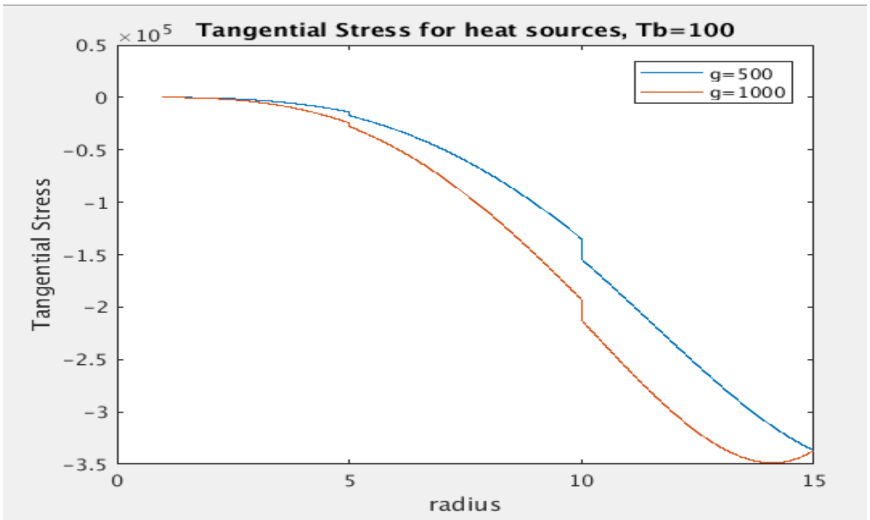

Figure-3: Tangential stress variation for different values of heat generation ($\vartheta^{(b)} > 1$)

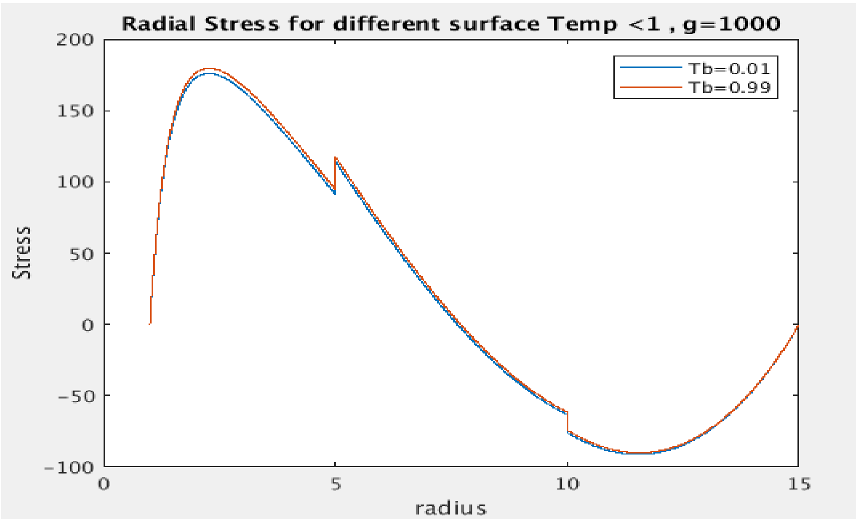

Figure-4: Radial stress variation for surface temperature asymmetry ($\vartheta^{(b)} < 1$)

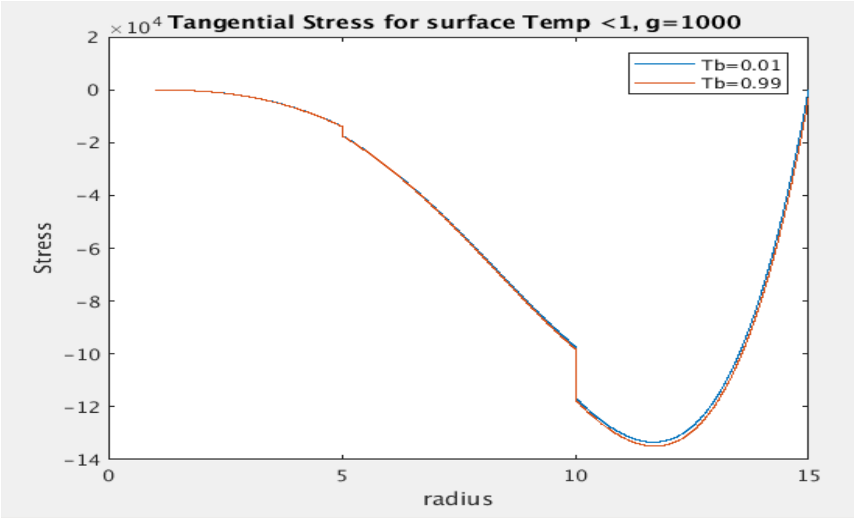

Figure-5: Tangential stress variation for surface temperature asymmetry ($\vartheta^{(b)} < 1$)

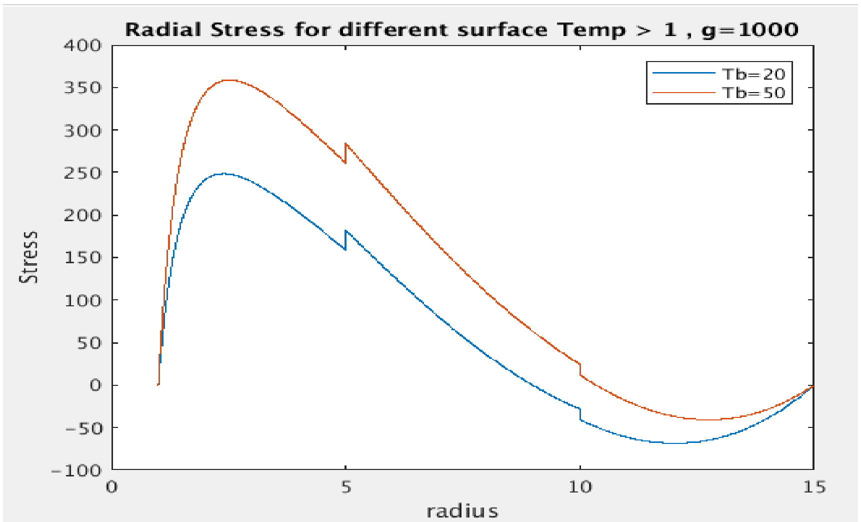

Figure-6: Radial stress variation for different surface temperature ($\vartheta^{(b)} > 1$)

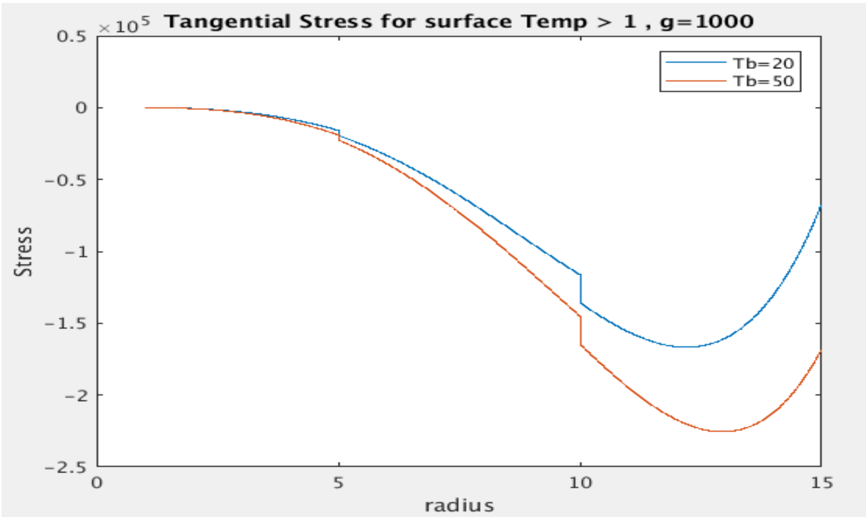

Figure-7: Tangential stress variation for different surface temperature ($\vartheta^{(b)} > 1$)

EFFECT OF INTERNAL HEAT GENERATION

Fig. 2shows the radial stress distribution in the layers for different values of heat generation parameter $Q^{(i)}$ ($g = 500, 1000$) and $\vartheta^{(b)} = 100$ (>1). Due to induced conditions, the radial stresses at the inner and outer surfaces of the hollow sphere are zero. As g increases, $Q^{(i)}$ have different values in the layers, as shown in the table 3. In the inner layer, there is a tension which increases with increase in g , in middle region the tension gradually changes to compression, while in outer layer stress changes completely to compression and it increases with heat generation. Thus the internal heat generation parameter takes different values with respect to conductivity of material and accordingly significantly affects the nature of radial stress in that layer. It is also found that there is a radial position where radial stress switches from tension to compression.

The effect of heat generation in multilayer sphere on the tangential stress distribution in fig. 3 with $\vartheta^{(b)} = 100$ and $g = 500, 1000$, the surface of inner layer experiences tangential tension and this increases with increase in the value of heat generation parameter while outer surface of experience tangential compression, while in the middle layer of copper, tangential stress changes from tensile to compression along the radial direction and tangential stress values coincides on outer surface.

EFFECT OF TEMPERATURE ASYMMETRY

The effect of temperature asymmetry on the radial stresses in the layered hollow sphere is illustrated in Fig. 4 for $\vartheta^{(b)} = 0.01, 0.99\ (<1)$, $g = 1000$. The value $\vartheta^{(b)} = 1$ corresponds to zero temperature gradient inside the sphere and consequently no thermally induced stresses. The temperature at the outer surface is lower than that of lower surface. The lower the value of $\vartheta^{(b)}$, greater the temperature asymmetry and larger the temperature gradient. This fact is reflected by the stress curves in Fig. 4, where stresses occur at these values. The maximum part of the hollow sphere is under compression in all layers. It is observed that the inner layer is under tension and then in middle and outer layers the radial stresses becomes more compressive. As expected, on inner and outer surface the radial stress are null. As $\vartheta^{(b)}$ increases, the stresses tension as well as compression decreases.

Fig. 5 depicts the effect of temperature asymmetry on tangential stress distribution. The inner layer experiences tension, middle region experiences tension and compression and outer layer experiences compression. It gives greater variation and curves touches at some point in the sphere.

Now the analysis is presented when the temperature of the outer surface is higher than inner surface, $\vartheta^{(b)} > 1$. Fig. 6 shows the radial stress distribution $\vartheta^{(b)} = 20, 50$ with $g = 1000$. This fig is exactly same as that of Fig. 4 but values are greater but nature is same and graphs do not touch anywhere in any layer. The tangential stress distribution is shown in Fig 7.

CONCLUSION

The exact analytical solutions have been developed for thermal stresses in a three layered hollow sphere experiencing internal heat generation and subjected to asymmetric temperature on its surface. The layers were homogeneous and isotropic and solutions with interface conditions are analyzed by Mathematical software MATLAB. The results were discussed numerically and graphically layer wise and observations are presented. The radial and tangential stress components have different nature in different layers, tensile or compressive depending on the level of heat generation and the degree of temperature asymmetry. There are radial locations are obtained where stress switches from tensile to compression that they are neutral radii.

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