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ON CONTRA J – ČECH CONTINUOUS MAPPINGS

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ABSTRACT

The purpose of this paper is to introduce the concept of Contra J – Čech Continuous Mappings in closure spaces. The properties and relationships with other types of mappings in closure spaces are obtained with examples.

***Key ward:** Contra – Čech Continuous map, Contra J – Čech Continuous map , Čech clopen, J – Čech locally indiscrete, Almost contra Čech continuous map, almost contra J-Čech continuous map.*

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1. INTRODUCTION

Norman Levine [15] introduced the concept of generalised closed sets as a generalisation of closed sets ,to investigate some topological properties. The concept of closure space is the generalization of a topological space. E. Čech [2] introduced the concept of Čech - closure spaces. Closure functions that are more general than the topological ones have been studied already by many authors [1, 9, 15]. A thorough discussion is due to Hammer, [13, 14] and Gnilka [11, 12].

Let (X, k) or simply X denote a Čech - closure space. For any subset $A \subseteq X$, $int_k(A)$ and k (A) denote the Čech interior and Čech closure of a set A with respect to the function k. The ideas about the concepts of a continuous mapping and of a set endowed with continuous operations (Compositions) play a fundamental role in general mathematical analysis. Dontchev [10] introduced the notions of Contra –continuity and strong S-closedness in topological spaces. In [10], he obtained very interesting and important results concerning contra continuity, compactness, S-closedness and strong S – closedness.In this paper, the present work has as its purpose to investigate some fundamental properties of Contra J – Čech Continuous maps in the light of J – Čech open sets.

2. PRELIMINARIES

A map $k: P(X) \rightarrow P(X)$ defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, k) is called a closure space if the following axioms are satisfied.

(i) $k(\phi) = \phi$,
(ii) $A \subseteq k(A)$ for every $A \subseteq X$
(iii) $k(A \cup B) = k(A) \cup k(B)$ for all $A,B \subset X$

A closure operator k on a set X is called idempotent if $k(A)=k[k(A)]$ for all $A \subseteq X$.

Definition 2.1: A subset A of a Čech – closure (X, k) will be called Čech closed if k (A) =A and Čech – open if its complement is closed .i.e., if k(X-A) = X-A.

Definition: 2.2 A subset A in Čech – closure space X is called

(i) Čech regular open if $A= int(k(A))$
(ii) Čech regular closed if $A = k(int(A))$
(iii) Čech pre open if $A \subseteq int(k(A))$
(iv) Čech pre closed if $k(int(A)) \subseteq A$
(v) Čech semi open if $A \subseteq k(int(A))$
(vi) Čech α- open if $A \subseteq int(k(int(A)))$
(vii) Čech α- closed if $k(int(k(A))) \subseteq A$

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(viii) Čech β-open if $A \subseteq k(\, int\,(k(A))\,)$
(ix) Čech β- closed if $int\,(\,k\,(\,int\,(A))) \subseteq A$
(x) Čech g-closed if $k(A) \subseteq U$ whenever $A \subseteq U$, U is Čech – open .
(xi) Čech g-open if X-A is Čech g-closed.

Definition: 2. 3 [1] Let (X, k) be a closure space. A subset $A \subseteq X$ is called a **J - Čech Closed Set,** if $k_{\alpha}(A) \subseteq U$, whenever $A \subseteq U$, U is an semi-open subset of (X, k). $k_{\alpha}(A)$ is the smallest α - closed set containing A

Definition: 2.4 [1] A function $f : (X, u) \rightarrow (Y, v)$ is said to be J – Čech Continuous if $f^{-1}(V)$ is J – Čech closed set of (X,u) ,for every Čech closed set V in (Y, v).

3. CONTRA J – ČECH CONTINUOUS MAPS

Definition: 3.1 Let (X, u) and (Y, v) be two Closure spaces and let $f : (X, u) \rightarrow (Y, v)$ be a function. Then f is said to be **Contra – Čech Continuous** if $f^{-1}(V)$ is Čech closed set of (X ,u) for every Čech open set V in (Y, v).

Definition 3.2: A function $f : (X, u) \rightarrow (Y, v)$ is said to be **Contra J –Čech Continuous** if $f^{-1}(V)$ is J –Čech closed set of (X, u), for every Čech open set V in (Y, v).

Definition: 3.3 A function $f:(X, u) \rightarrow (Y, v)$ is said to be **Contra Čech α- Continuous** if $f^{-1}(V)$ is Čech α- closed set of (X, u), for every Čech open set V in (Y, v).

Definition: 3.4 A function $f:(X, u) \rightarrow (Y, v)$ is said to be **Contra Čech gα -Continuous** if $f^{-1}(V)$ is Čech gα- closed set of (X, u), for every Čech open set V in (Y, v).

Definition: 3.5 Let (X, u) and (Y, v) be two Closure spaces and let $f:(X, u) \rightarrow (Y, v)$ be a function. Then f is said to be **Contra Čech αg- Continuous** if $f^{-1}(V)$ is Čech αg- closed set of (X, u), for every Čech open set V in (Y , v).

Definition: 3.6 A subset A of a space (X, u) is called **Čech clopen** if it is both Čech open and Čech closed.

Proposition: 3.7

1. Every Contra Čech Continuous map is Contra J – Čech Continuous.
2. Every Contra Čech α - Continuous map is Contra J – Čech Continuous.
3. Every Contra J – Čech Continuous map is Contra Čech αg-Continuous map.
4. Every Contra J –Čech Continuous map is Contra Čech gα -Continuous map.

Proof: Let $f:(X, u) \rightarrow (Y, v)$ be a Contra Čech Continuous map. Let V be a Čech open set in (Y, v). Since f is Contra Čech Continuous map, $f^{-1}(V)$ is Čech closed set of (X, u).

Every Čech closed set is J – Čech closed set.Hence $f^{-1}(V)$ is J – Čech closed set of (X, u), for every Čech open set V in (Y, v). So f is Contra J–Čech Continuous. Therefore Every Contra Čech Continuous map is Contra J–Čech Continuous.

Similarly we can prove the other propositions.

The converses are not true as can be seen from the following examples:

Example: 3.8 Let X = {a, b, c} and Y = {p, q}. Define a function $f :(X, u) \rightarrow (Y, v)$ such that f(a) = q, f(b) = q, f(c) = p. Let u,v be closure operators of X,Y defined as u(ϕ) = ϕ, u{a} ={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=u{X}=X and v(ϕ) = ϕ, v{p}={p}, v{q} = v{Y}= Y**.** Here {q} is Čech open set in (Y, v), $f^{-1}\{q\}$ = {a, b} is J - Čech Closed Set in X but not Čech Closed Set in X. Therefore, f is Contra J – Čech Continuous but not Contra Čech Continuous.

Example: 3.9 Let X = {a, b, c} and Y= {p, q}. A function $f : (X, u) \rightarrow (Y, v)$ is defined by f(a) = p , f(b) = q, f(c) =q . Let u, v be closure operators of X, Y defined as u(ϕ) = ϕ, u{a} =u{a, b}={a, b} , u{b}={b}, u{c}={a, c}, u{a ,c}= u{a, b}= u{b, c}= u{X} = X and v(ϕ) = ϕ v{p}={p} ,v{q} = v{Y}= Y .Here {q} is Čech open set in (Y, v), $f^{-1}\{q\}$ = {b, c} is J - Čech Closed Set in X but not Čech α-Closed Set in X.

Example: 3.10 Let X = {a, b, c} and Y= {p, q}. Define a function $f : (X, u) \rightarrow (Y, v)$ such that f(a) = q , f(b) = q , f(c) = p. Let u ,v be closure operators of X,Y defined as u(ϕ) = ϕ, u{a} ={a} , u{b} = u{b, c} = {b, c} ,u{c} = {c}, u{a, c } ={a, c} , u{a, b} = u(X) = X and v(ϕ) = ϕ v{p}={p} ,v{q} = v{Y}= Y .Here {q} is Čech open set in (Y, v), f^{-1} {q }= {a, b} is Čech gα - Closed Set in X . But not J- Čech Closed Set.

Lemma: 3.11 If (X, u) be a closure space and A be a subset of X then the following conditions are equivalent
(1) A is Čech clopen .
(2) A is J – Čech closed and Čech regular open.

Proof: (1) ⇒ (2) Assume that A is Čech clopen .Then A is both Čech open and Čech closed.

But every Čech closed set is J- Čech closed .Therefore A is J- Čech closed and X -A = k(X-A). This implies that A = X - k(X-A) = int A = int k(A), Since A is Čech closed.

Hence A is Čech regular open.

(2) ⇒ (1) By (2) A is Čech regular open. Čech regular open implies that A is Čech open.

⇒ A is Čech closed .Thus A is Čech clopen.

Proposition 3.12: If $f : (X, u) \to (Y, v)$ and $g : (Y, v) \to (Z ,w)$ are Contra J – Čech Continuous then $g \circ f : (X, u) \to (Z ,w)$ is not Contra J – Čech Continuous.

Example 3.13: Let X = {a, b, c}, Y= {l, m, n} and Z= {p, q}. Define a function $f : (X, u) \to (Y, v) \ni f(a) =n$, $f(b) = l$, $f(c) = m$. and $g : (Y, v) \to (Z, w) \ni g(l) = q$, $g(m) = q$, $g(n) = p$.Let u ,v and w be closure operators of X ,Y and Z defined as $u (\phi) = \phi$, u {a} = {a},u{b} = {b, c } , u{c}= u{a, c}={a, c}, u{a, b} = u {b, c } = u(X) =X. $v(\phi) = \phi$, v{l} = v{m}=v{l, m}={l, m} , v{n} ={n} , v{l, n}=v{m, n}=v(Y)=Y. $w (\phi) = \phi$,w{p}={p} , w{q}=w(Z)=Z . Here f and g are Contra J – Čech Continuous and {q} is Čech open set of Z, $(g \circ f)^{-1} \{q\}= f^{-1}(g^{-1}(q)) =\{b,c\}$ is not J – Čech Closed set in (X ,u).

Definition 3.14:A closure space (X ,u) is called J – Čech locally indiscrete if every J – Čech open set is closed .

Theorem 3.15: If a function $f : (X ,u) \to (Y , v)$ is J – Čech Continuous and (X ,u) is J – Čech locally indiscrete then f is Contra Čech Continuous.

Proof: Let V be Čech open in (Y, v).Since f is J – Čech Continuous, f^{-1} (V) is J – Čech open in (X, u).Since X is J – Čech locally indiscrete , f^{-1} (V) is J – Čech closed in X. Hence f is Contra Čech Continuous.

Theorem 3.16: If a function $f : (X ,u) \to (Y , v)$ is J – Čech Continuous and (X ,u) is J – Čech locally indiscrete then f is Contra J - Čech Continuous.

Proof: Assume that V is a Čech open set in (Y, v). Since f is J–Čech Continuous, f^{-1}(V) is J–Čech open in (X, u).Since X is J – Čech locally indiscrete, f^{-1} (V) is J – Čech closed in X. But every Čech closed set is J – Čech closed. Hence f is Contra J- Čech Continuous.

Theorem 3.17: Suppose J -Čech open sets of (X, u) are closed under arbitrary union .Let (X, u) and (Y, v) be closure spaces .Then the following statements are equivalent for a function
$f: (X ,u) \to (Y , v)$.
(1) f is Contra J -Čech continuous .
(2) f^{-1}(F) is a J--Čech open set of (X, u) for every Čech closed set F of (Y, v).
(3) For each $x \in X$ and each Čech closed set F of (Y,f(x)) ,there exist a J -Čech open set U of (X, x) such that $f(U) \subset F$.
(4) For each $x \in X$ and each Čech open set V in Y not containing f(x) there exist a J -Čech closed set A in X not containing x such that $f^{-1}(V) \subset A$.

Proof:
(1) ⇒ (2) Let F be Čech closed set of (Y, v), then Y-F is Čech open set in (Y, v).By (1) $f^{-1}(Y-F) = X - f^{-1}(F)$ is J – Čech closed set in (X, u).Therefore f^{-1}(F) is J – Čech open set in (X, u). **(2) ⇒ (1)** Let F be a Čech open set in (Y, v), then Y-F is Čech closed set in (Y, v).By (2) $f^{-1}(Y-F) = X – f^{-1}(F)$ is J- Čech open set in (X, u).Hence f^{-1}(F) is J- Čech closed set in (X, u).

(2) ⇒ (3) Let F be any Čech closed set in Y containing f(x). By (2) f^{-1}(F) is J- Čech open set in (X, u) and $x \in f^{-1}(F)$ Take $U = f^{-1}(F)$ .Thus f (U) = F.

(3) ⇒ (2) Let F be a Čech closed set in (Y, v) and $x \in f^{-1}(F)$ .From (3) there exist a J – Čech open set U_x in X containing x such that $U_x \subset f^{-1}(F)$. We have $f^{-1}(F) = \cup \{Ux / x \in f^{-1}(F)\}$ is J- Čech open in X.

(3) ⇒ (4) Let V be any open set in Y not containing f(x) .Then Y-V is a Čech closed set containing f(x).By (3) there exist a J – Čech open set U in X containing x such that f(U) ⊂ Y-V.

Hence U ⊂ f^{-1}(Y-V) ⊂ X - f^{-1}(U) and f^{-1}(U) ⊂ X – U .Take A = X – U.

Therefore we get J – Čech closed set A in X not containing x.

(4) ⇒(3) Let F be a Čech closed set in Y containing f(x) .By (4) there exist a J – Čech closed set A in X not containing x such that f^{-1}(Y - F) ⊂ A . This implies that X - f^{-1}(F) ⊂ A.

Therefore f(X – A) ⊂ F .Take U = X – A. Then U is J – Čech open set in X containing x such that f (U) ⊂ F.

4. ALMOST CONTRA J- ČECH CONTINUOUS MAPS

Definition 4.1:A function f : (X ,u) → (Y, v) is said to be almost contra Čech continuous if f^{-1} (V) is Čech closed set in (X, u) for every Čech regular open set V of (Y, v).

Definition 4.2:A function f : (X ,u) → (Y, v) is said to be almost contra J- Čech continuous if f^{-1} (V) is J- Čech closed set in (X, u) for every Čech regular open set V of (Y, v).

Definition 4.3:A function f : (X ,u) → (Y, v) is said to be Perfectly Čech continuous if f^{-1} (V) is Čech clopen set in (X, u) for each Čech open set V of (Y, v).

Theorem4.4: Let (X ,u) and (Y, v) be closure spaces .Then the following statements are equivalent for a function f: (X ,u) → (Y, v) .

(1) F is almost contra J - Čech continuous.
(2) f^{-1} (F) is a J – Čech open sets of (X, u) ,for every F in Čech regular closed set of (Y, v).
(3) for each x ∈ X and each F ∈ Čech regular closed set of (Y, f(x)) ,there exist J - Čech open set of (X ,x) such that f (U) ⊂ F .
(4) For each x ∈ X and each Čech regular open set V in Y not containing f(x) there exist a J- Čech closed set A in X not containing x such that f^{-1} (U) ⊂ A.
(5) f^{-1} (int (cl(G))) is a J – Čech closed set of (X ,u) for every open subset G of Y .
(6) f^{-1}(cl (int F))) is a J – Čech open set of X, for every Čech closed subset F of Y.

Proof:

(1) ⇒ (2)
Let F be a Čech regular closed set of (Y,v) .This implies that Y – F is Čech regular open in (Y ,v) .Since f is almost Contra J - Čech continuous, f^{-1}(Y –F) = X - f^{-1}(F) is J-Čech closed in (X, u). Hence f^{-1} (F) is J- Čech open set in (X, u).

(2) ⇒(1)
Let V be Čech regular open set in (Y, v) ,then Y – V is Čech regular closed set in (Y, v).
By (2) f^{-1} (Y – V) = X - f^{-1} (V) is J - Čech open set in (X, u). Thus f^{-1} (V) is J- Čech closed set in (X, u).

(2) ⇒ (3)
Let F be any Čech regular closed set in Y containing f(x).
By (2) f^{-1} (F) is J- Čech open in (X, u) and x ∈ f^{-1} (F).Take U = f^{-1} (F) then U is J- Čech open set in X containing x such that f(U) ⊆ F.

(3) ⇒ (2)
Let F is Čech regular closed set U_x ⊂ f^{-1}(F), we have f^{-1}(F) = ∪{U_x / x ∈ f^{-1}(F)} is J- Čech open set in X .Therefore f^{-1} (F) is J- Čech open.

(3) ⇒ (4)
Let V be any Čech regular open in Y not containing f(x) .Then Y-V is a Čech regular closed set containing f(x) .By (3) there exist a J- Čech open set U in X containg x such that f(U) ⊂ Y-V .Hence U ⊂ f^{-1} (Y – V) = X - f^{-1} (V). And f^{-1} (V) ⊂ X-U. Take A = X-U.

Therefore we get J- Čech closed set A in X not containing x.

(1) ⇒ (3)
Let F be a Čech regular closed set Y containing f(x) .Then Y-F is a Čech regular set in Y not containing f(x).By (4) there exist a J- Čech closed set A in X not containing x such that f^{-1} (Y - F) ⊂ A.⇒ X - f^{-1} (F) ⊂ A. Therefore f (X-A) ⊂ F .Take U = X-A.Then U is J- Čech open set in X containing x such that f(U) ⊂ F**.**

(1)⇒(5)
Let G be an Čech open subset of Y .Since int(cl(G)) is Čech regular open ,then by (1) f^{-1} (int (cl(G))) is J- Čech closed set of X.

(5) ⇒ (1)
Let V be a Čech regular open set in Y , then V is Čech open in Y.Therefore by (5) f^{-1} (int (cl(V))) is J- Čech closed set in (X,u).⇒ f^{-1} (V) is J- Čech closed set in X .Therefore
f is almost contra J- Čech continuous **.**

(1)⇒ (6)
Let F be Čech closed subset of Y .Since cl (int (F)) is Čech regular closed , then by (2) f^{-1} (cl (int (F))) is J- Čech open set in (X ,u).

(6) ⇒(2)
Let F be Čech regular closed set of (Y, v).Then F is Čech closed set in Y. By (6) f^{-1} (cl (int (F))) is J- Čech open set in X.

Theorem 4.5: If f: (X, u) → (Y, v) is Contra J – Čech continuous, then it is almost contra J – Čech Continuous.

Proof: Let V be Čech regular open in X .Then V is Čech open in Y. By assumption f^{-1} (V) is J- Čech closed in (X, u). Thus f is almost contra J- Čech continuous.

Definition 4.6: A function f: (X, u) → (Y, v) is called Čech regular set connected if f^{-1} (V) is Čech clopen in (X ,u) for each regular open set V of (Y, v).

Proposition 4.7:
1. If f: (X, u) → (Y, v) be Čech regular set connected then it is almost contra Čech continuous.
2. If f: (X, u) → (Y, v) be Čech regular set connected then it is almost contra J - Čech continuous.
3. If f: (X, u) → (Y, v) is Contra Čech continuous then it is almost Contra Čech continuous.
4. If f: (X, u) → (Y, v) is Contra Čech continuous then it is almost Contra J- Čech continuous.

The proof is obvious.

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