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OPERATION ON EDGE REGULAR INTUITIONISTIC FUZZY GRAPHS

S. RAVI NARAYANAN*[1] AND S. MURUGESAN[2]

[1]Department of Mathematics
Don Bosco College of Arts and Science, Keela Eral, Thoothukudi, Tamil Nadu, India.

[2]Department of Mathematics
Sri S. Ramasamy Naidu Memorial College, Sattur, Tamil Nadu, India.

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ABSTRACT

In this paper, degree of an edge in cartesian product, composition and alpha product of two intuitionistic fuzzy graphs are defined. Edge regular property of cartesian product, composition and alpha product of two intuitionistic fuzzy graphs are discussed.

1. INTRODUCTION

Some operations on fuzzy graphs were introduced by John N. Modernson and Peng [4], R. Parvathi , M.G. Karunambigai and K.T. Atanassov[10] introduced operations on intuitionistic fuzzy graphs. Operations on Regular intuitionistic fuzzy graphs were studied by Ismail Mohaideen and *et al.*, [3]. M.G. Karunambigai et.al.,[5] introduced the concept of edge regular intuitionistic fuzzy graph. A. Nagoor Gani and B. Fathima Kani [6] introduced alpha product in fuzzy graphs. S. Ravi Narayanan and S. Murugesan[12] discussed the properties of edge irregular intuitionistic fuzzy graphs. S. Ravi Narayanan and S. Murugesan[13] introduced pseudo degree of a vertex, pseudo total degree of a vertex in an intuuitionistic fuzzy graphs. These motivates us to discuss some properties of cartesian product, composition and alpha product of edge regular intuitionistic fuzzy graphs.

Throughout this paper, the Graph G_1 takes the membership value $((\mu_1, \gamma_1), (\mu_2, \gamma_2))$ and the Graph G_2 takes the membership value $((\mu_1', \gamma_1'), (\mu_2', \gamma_2'))$

2. PRELIMINARIES

In this section we see the definitions related to fuzzy graphs and intuitionistic fuzzy graphs

Definition 2.1: [1] An intuitionistic fuzzy graph with an underlying set V is defined to be a pair $G = (V, E)$ where
- (i) $V = \{v_1, v_2, v_3, \ldots, v_n\}$ such that $\mu_1: V \to [0, 1]$ and $\gamma_1: V \to [0, 1]$ denote the degree of membership and non membership of the element $v_i \in V$, $(i = 1, 2, 3, \ldots, n)$, such that $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$
- (ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0, 1]$ and $\gamma_2: V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \le \min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\gamma_2(v_i, v_j) \le \max\{\gamma_1(v_i), \gamma_1(v_j)\}$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$, $(i, j = 1, 2, \ldots, n)$.

Definition 2.2: [4] Let $G: (A,B)$ be an intuitionistic fuzzy graph on $G^*(V,E)$ .Then the degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d\mu_1(v_i), d\gamma_1(v_i))$,where $d\mu_1(v_i) = \sum \mu_2(v_i, v_j)$ and $d\gamma_1(v_i) = \sum \gamma_2(v_i, v_j)$, for $(v_i, v_j) \in E$ and $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for $(v_i, v_j) \notin E$.

Definition 2.3: [3] Let G : (A,B) be an intuitionistic fuzzy graph and let $e_{ij} \in B$ be an edge of G. Then the degree of an edge e_{ij} is defined as $d_\mu(e_{ij}) = d_\mu(v_i) + d_\mu(v_j) - 2\mu_2(e_{ij})$ and $d_\gamma(e_{ij}) = d_\gamma(v_i) + d_\gamma(v_j) - 2\gamma_2(e_{ij})$ and the edge degree of G is $d(e_{ij}) = (d_\mu(e_{ij}), d_\gamma(e_{ij}))$.

Definition 2.4: [3] Let G : (A,B) be an intuitionistic fuzzy graph and let $e_{ij} \in B$ be an edge of G. G is said to be edge regular intuitionistic fuzzy graph if all the edges of G have the same degree.

Corresponding Author: S. Ravi Narayanan[1], [1]Department of Mathematics
Don Bosco College of Arts and Science, Keela Eral, Thoothukudi, Tamil Nadu, India.*

Definition 2.5: [5] Let $G_1 : (A_1,B_1)$ and $G_2 : (A_2,B_2)$ where $A_1 = (\mu_1, \gamma_1), B_1 = (\mu_2, \gamma_2)$, $A_2 = (\mu_1', \gamma_1')$, $B_2= (\mu_2', \gamma_2')$ be two intuitionistic fuzzy graph on $G^* : (V,E)$. Then the cartesian product of G_1 and G_2 is defined as $G = G_1 \times G_2 : (V,E)$ where $V = V_1\times V_2$ and $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1$ & $u_2v_2 \in E_2$ or $u_2 = v_2$ & $u_1v_1 \in E_1\}$ with
$((\mu_1 \times \mu_1')(\gamma_1 \times \gamma_1'))(u_1,u_2) = \{min(\mu_1(u_1), \mu_1'(u_2)), \max(\gamma_1(u_1),\gamma_1'(u_2))\}$, for every $(u_1,u_2) \in E$

$$(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(v_1,v_2)) = \begin{cases} \{min(\mu_1(u_1),\mu_2'(u_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,v_2))\} & u_1 = v_1, u_2v_2 \in E_2 \\ \{\min \mu_1'(u_2),\mu_2(u_1,v_1)), \max(\gamma_1'(u_2),\gamma_2(u_1,v_1))\} & u_2 = v_2, u_1v_1 \in E_1 \\ 0 & otherwise \end{cases}$$

Definition 2.5: [5] Let $G_1 : (A_1,B_1)$ and $G_2 : (A_2,B_2)$ where $A_1 = (\mu_1, \gamma_1), B_1 = (\mu_2, \gamma_2)$, $A_2 = (\mu_1', \gamma_1')$, $B_2= (\mu_2', \gamma_2')$ be two intuitionistic fuzzy graph on $G^* : (V,E)$. Then the composition of two intuitionistic fuzzy graphs G_1 and G_2 is defined as $G = G_1 \circ G_2 : (V,E)$ where $V = V_1\times V_2$ and $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1$ & $u_2v_2 \in E_2$ or $u_2 = v_2$ & $u_1v_1 \in E_1\}$ with
$((\mu_1 \times \mu_1')(\gamma_1 \times \gamma_1'))(u_1,u_2) = \{min(\mu_1(u_1), \mu_1'(u_2)), \max(\gamma_1(u_1),\gamma_1'(u_2))\}$, for every $(u_1,u_2) \in E$

$$(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(v_1,v_2)) = \begin{cases} \{min(\mu_1(u_1),\mu_2'(u_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,v_2))\} & u_1 = v_1, u_2v_2 \in E_2 \\ \{\min(\mu_1'(u_2),\mu_2(u_1,v_1)), \max(\gamma_1'(u_2),\gamma_2(u_1,v_1))\} & u_2 = v_2, u_1v_1 \in E_1 \\ \{\min(\mu_1'(u_2),\mu_1'(v_2),\mu_2(u_1,v_1)), \max(\gamma_1'(u_2),\gamma_1'(v_2),\gamma_2(u_1,v_1))\} & u_2 \neq v_2, u_1v_1 \in E_1 \end{cases}$$

Definition 2.5: [5] Let $G_1 : (\sigma_1,\mu_1)$ and $G_2 : (\sigma_2,\mu_2)$ be two fuzzy graphs. Then the alpha product of two fuzzy graphs G_1 and G_2 is denoted by $G_1 \times_\alpha G_2$ and defined as
$(\sigma_1 \times_\alpha \sigma_2)(u_1,u_2) = \sigma_1(u_1)\Lambda\sigma_2(u_2)$

$$(\mu_1 \times_\alpha \mu_2)(u_1,u_2)(v_1,v_2) = \begin{cases} \sigma_1(u_1)\Lambda\mu_2(u_2,v_2) & u_1 = v_1, u_2v_2 \in E_2 \\ \sigma_2(u_2)\Lambda\mu_1(u_1,v_1) & u_2 = v_2, u_1v_1 \in E_1 \\ \mu_1(u_1,v_1)\Lambda\sigma_2(u_2)\Lambda\sigma_2(v_2) & u_1v_1 \in E_1, u_2v_2 \notin E_2 \\ \mu_2(u_2,v_2)\Lambda\sigma_1(u_1)\Lambda\sigma_1(v_1) & u_1v_1 \notin E_1, u_2v_2 \in E_2 \end{cases}$$

2. EDGE REGULAR PROPERTY OF CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY GRAPHS

In this section, degree of an edge in cartesian product of two intuitionistic fuzzy graphs is defined and we see about the edge regular property of cartesian product of two intuitionistic fuzzy graph.

Degree of an edge

By definition for any $((u_1,u_2),(v_1,u_2)) \in E$, we have

$$d_{G_1\times G_2}((u_1,u_2),(v_1,v_2)) = \sum_{(w_1,w_2)\neq(v_1,v_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(w_1,w_2)) + \sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,w_2)(v_1,v_2))$$

If $u_1 = v_1$, $(u_2,v_2) \in E_2$

$$\begin{aligned} d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) &= \sum_{(w_1,w_2)\neq(v_1,u_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(w_1,w_2)) \\ &+ \sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,w_2)(v_1,u_2)) \\ &= \sum_{u_1=w_1,w_2\neq v_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(u_1,w_2)) \\ &+ \sum_{w_1\neq v_1,u_2=w_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(w_1,u_2)) \\ &+ \sum_{w_1=u_1,w_2\neq u_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,w_2)(u_1,v_2)) \\ &+ \sum_{w_1\neq u_1,w_2=v_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,v_2)(u_1,v_2)) \\ &= \sum_{u_1=w_1,w_2\neq v_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\} \\ &+ \sum_{w_1\neq v_1,u_2=w_2}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\} \\ &+ \sum_{w_1=u_1,w_2\neq u_2}\{min(\mu_1(u_1),\mu_2'(w_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(w_2,v_2))\} \\ &+ \sum_{w_1\neq u_1,w_2=v_2}\{\min(\mu_2(w_1,u_1),\mu_1'(v_2)), \max(\gamma_2(w_1,u_1),\gamma_1'(v_2))\} \\ &= \sum_{w_2\neq v_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\} \\ &+ \sum_{w_1\in V_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\} \\ &+ \sum_{w_2\neq u_2}\{min(\mu_1(u_1),\mu_2'(w_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(w_2,v_2))\} \\ &+ \sum_{w_1\in V_1}\{\min(\mu_2(w_1,u_1),\mu_1'(v_2)), \max(\gamma_2(w_1,u_1),\gamma_1'(v_2))\} \end{aligned}$$

If $u_2 = v_2$, $(u_1, v_1) \in E_1$

$$d_{G_1\times G_2}((u_1,u_2),(v_1,u_2)) = \sum_{(w_1,w_2)\neq(v_1,u_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(w_1,w_2))$$
$$+ \sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,w_2)(v_1,u_2))$$
$$= \sum_{u_1=w_1}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(u_1,w_2))$$
$$+ \sum_{w_1\neq v_1,u_2=w_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((u_1,u_2)(w_1,u_2))$$
$$+ \sum_{w_1=v_1}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,u_2)(v_1,u_2))$$
$$+ \sum_{w_1\neq u_1,w_2=u_2}(\mu_2 \times \mu_2')(\gamma_2 \times \gamma_2')((w_1,u_2)(v_1,u_2))$$
$$= \sum_{u_1=w_1}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$$
$$+ \sum_{w_1\neq v_1,u_2=w_2}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$$
$$+ \sum_{u_1=w_1}\{min(\mu_1(v_1),\mu_2'(w_2,u_2)), \max(\gamma_1(v_1),\gamma_2'(w_2,u_2))\}$$
$$+ \sum_{w_1\neq u_1,u_2=w_2}\{\min(\mu_2(w_1,v_1),\mu_1'(u_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(u_2))\}$$
$$= \sum_{w_2\in V_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$$
$$+ \sum_{w_1\neq v_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$$
$$+ \sum_{w_2\in V_2}\{min(\mu_1(v_1),\mu_2'(w_2,u_2)), \max(\gamma_1(v_1),\gamma_2'(w_2,u_2))\}$$
$$+ \sum_{w_1\neq u_1}\{\min(\mu_2(w_1,v_1),\mu_1'(u_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(u_2))\}$$

Theorem 2.1 Let G_1 and G_2 be two intuitionistic fuzzy graph such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$

1. If $((u_1,u_2),(u_1,v_2)) \in E$, then
   $d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$
2. If $((u_1,u_2),(v_1,u_2)) \in E$, then
   $d_{G_1\times G_2}((u_1,u_2),(v_1,u_2)) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2)$

Proof:

1. For any $((u_1,u_2),(u_1,v_2)) \in E$, we have

$$d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) = \sum_{w_2\neq v_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$$
$$+ \sum_{w_1\in V_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$$
$$+ \sum_{w_2\neq u_2}\{min(\mu_1(u_1),\mu_2'(w_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(w_2,v_2))\}$$
$$+ \sum_{w_1\in V_1}\{\min(\mu_2(w_1,u_1),\mu_1'(v_2)), \max(\gamma_2(w_1,u_1),\gamma_1'(v_2))\}$$
$$= \sum_{w_2\neq v_2}(\mu_2'(u_2,w_2)),\gamma_2'(u_2,w_2)) + \sum_{w_1\in V_1}(\mu_2(u_1,w_1),\gamma_2(u_1,w_1))$$
$$+ \sum_{w_2\neq u_2}(\mu_2'(w_2,v_2)),\gamma_2'(w_2,v_2)) + \sum_{w_1\in V_1}(\mu_2(w_1,u_1),\gamma_2(w_1,u_1))$$

$$d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$$

2. For any $((u_1,u_2),(v_1,u_2)) \in E$, we have

$$d_{G_1\times G_2}((u_1,u_2),(v_1,u_2)) = \sum_{w_2\in V_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$$
$$+ \sum_{w_1\neq v_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$$
$$+ \sum_{w_2\in V_2}\{min(\mu_1(v_1),\mu_2'(w_2,u_2)), \max(\gamma_1(v_1),\gamma_2'(w_2,u_2))\}$$
$$+ \sum_{w_1\neq u_1}\{\min(\mu_2(w_1,v_1),\mu_1'(u_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(u_2))\}$$
$$= \sum_{w_2\in V_2}(\mu_2'(u_2,w_2)),\gamma_2'(u_2,w_2)) + \sum_{w_1\neq v_1}(\mu_2(u_1,w_1),\gamma_2(u_1,w_1))$$
$$+ \sum_{w_2\in V_2}(\mu_2'(w_2,u_2)),\gamma_2'(w_2,u_2)) + \sum_{w_1\neq u_1}(\mu_2(w_1,v_1),\gamma_2(w_1,v_1))$$

$$d_{G_1\times G_2}((u_1,u_2),(v_1,u_2)) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2)$$

Theorem 2.2: Let G_1 and G_2 be two intuitionistic fuzzy graph such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$, $\mu_1(u) = c_1$, $\gamma_1(u) = c_2$, for all u $\in$V. Then

1. For $((u_1,u_2),(u_1,v_2)) \in E$, then
   $d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) = 2d_{G_1}(u_1) + (c_1,c_2)\ (d^*_{G_2}(u_2) + d^*_{G_2}(v_2) - 2)$
2. For $((u_1,u_2),(v_1,u_2)) \in E$, then
   $d_{G_1\times G_2}((u_1,u_2),(v_1,u_2)) = d_{G_1}(u_1,v_1) + 2(c_1,c_2)\ (d^*_{G_2}(u_2))$

Proof: 1. For any $((u_1,u_2),(u_1,v_2)) \in E$, we have

$$d_{G_1\times G_2}((u_1,u_2),(u_1,v_2)) = \sum_{w_2\neq v_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$$
$$+ \sum_{w_1\in V_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$$
$$+ \sum_{w_2\neq u_2}\{min(\mu_1(u_1),\mu_2'(w_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(w_2,v_2))\}$$
$$+ \sum_{w_1\in V_1}\{\min(\mu_2(w_1,u_1),\mu_1'(v_2)), \max(\gamma_2(w_1,u_1),\gamma_1'(v_2))\}$$
$$= \sum_{w_2\neq v_2}((\mu_1(u_1),\gamma_1(u_1)) + \sum_{w_1\neq v_1}(\mu_2(u_1,w_1),\gamma_2(u_1,w_1))$$

$+ \sum_{w_2 \neq u_2}((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq u_1}(\mu_2(w_1, u_1), \gamma_2(w_1, u_1))$
$= \sum_{w_2 \neq v_2}(c_1, c_2) + \sum_{w_1 \in V_1}(\mu_2(u_1, w_1), \gamma_2(u_1, w_1))$
$+ \sum_{w_2 \neq u_2}(c_1, c_2) + \sum_{w_1 \in V_1}(\mu_2(w_1, u_1), \gamma_2(w_1, u_1))$
$= (c_1, c_2)\ (d^*_{G_2}(u_2)-1) + d_{G_1}(u_1) + (c_1, c_2)(\ (d^*_{G_2}(v_2)-1) + d_{G_1}(u_1)$

$d_{G_1 \times G_2}\big((u_1, u_2), (u_1, v_2)\big) = 2d_{G_1}(u_1) + (c_1, c_2)\ (d^*_{G_2}(u_2) + d^*_{G_2}(v_2)\ -2)$

2. For any $\big((u_1, u_2), (v_1, u_2)\big) \in E$, we have

$d_{G_1 \times G_2}\big((u_1, u_2), (v_1, u_2)\big) = \sum_{w_2 \in V_2}\{min(\mu_1(u_1), \mu_2'(u_2, w_2)), \max(\gamma_1(u_1), \gamma_2'(u_2, w_2))\}$
$+ \sum_{w_1 \neq v_1}\{\min(\mu_2(u_1, w_1), \mu_1'(u_2)), \max(\gamma_2(u_1, w_1), \gamma_1'(u_2))\}$
$+ \sum_{w_2 \in V_2}\{min(\mu_1(v_1), \mu_2'(w_2, u_2)), \max(\gamma_1(v_1), \gamma_2'(w_2, u_2))\}$
$+ \sum_{w_1 \neq u_1}\{\min(\mu_2(w_1, v_1), \mu_1'(u_2)), \max(\gamma_2(w_1, v_1), \gamma_1'(u_2))\}$
$= \sum_{w_2 \in V_2}((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq v_1}(\mu_2(u_1, w_1), \gamma_2(u_1, w_1))$
$+ \sum_{w_2 \in V_2}((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq u_1}(\mu_2(w_1, v_1), \gamma_2(w_1, v_1))$
$= \sum_{w_2 \in V_2}(c_1, c_2) + d_{G_1}(u_1, v_1) + \sum_{w_2 \in V_2}(c_1, c_2)$
$= d_{G_1}(u_1, v_1) + 2\ (c_1, c_2)\ (d^*_{G_2}(u_2))$

$d_{G_1 \times G_2}\big((u_1, u_2), (v_1, u_2)\big) = d_{G_1}(u_1, v_1) + 2(c_1, c_2)\ (d^*_{G_2}(u_2))$

Remark 2.3: If G_1 and G_2 are two edge regular intuitionistic fuzzy graph, then $G_1 x G_2$ need not be edge regular intuitionistic fuzzy graph.

Remark 2.4: If $G_1 x G_2$ is an edge regular intuitionistic fuzzy graph, then G_1 and G_2 need not be edge regular intuitionistic fuzzy graph

Theorem 2.5: Let G_1:(A_1,B_1) and G_2 :(A_2,B_2) be two regular intuitionistic fuzzy graph of same degree on G_1^*: (V,E) and G_2^*: (V,E)such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$. Then G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree if and only if $G_1 x G_2$ is an edge regular intuitionistic fuzzy graph.

Proof: Let G_1 and G_2 be two regular intuitionistic fuzzy graph of degree (c_1, c_2). Then

$d_{G_1}(u_1) = d_{G_2}(u_2)$ = (c_1, c_2), for all $u_1 \in V_1$, $u_2 \in V_2$.

Assume that G_1 and G_2 are (k_1, k_2)-edge regular intuitionistic fuzzy graph. Then

$d_{G_1}(u_1, v_1) = d_{G_2}(u_2, v_2)$ = (k_1, k_2), for all (u_1, v_1) ∈ E_1, (u_2, v_2) ∈ E_2.

By theorem 2.1 we have for any $\big((u_1, u_2), (v_1, v_2)\big) \in E$,

When $u_1 = v_1$, $(u_2, v_2) \in E_2$

$d_{G_1 \times G_2}\big((u_1, u_2), (u_1, v_2)\big) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2)$ = 2(c_1, c_2) + (k_1, k_2)

When $u_2 = v_2$, $(u_1, v_1) \in E_1$

$d_{G_1 \times G_2}\big((u_1, u_2), (v_1, u_2)\big) = 2d_{G_2}(u_2) + d_{G_1}(u_1, v_1)$ = 2(c_1, c_2) + (k_1, k_2)

Hence $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph.

Conversely suppose $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph.

Let (u_1, v_1), (w_1, x_1) ∈ E_1 be two edges of G_1 and let $u_2 \in V_2$. Then $\big((u_1, u_2), (v_1, u_2)\big)$, $\big((w_1, u_2), (x_1, u_2)\big) \in E$

Also $d_{G_1 \times G_2}\big((u_1, u_2), (v_1, u_2)\big) = d_{G_1 \times G_2}\big((w_1, u_2), (x_1, u_2)\big)$
$d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) = d_{G_1}(w_1, x_1) + 2d_{G_2}(u_2)$
$d_{G_1}(u_1, v_1)$ + 2(c_1, c_2) = $d_{G_1}(w_1, x_1)$ + 2(c_1, c_2).
$d_{G_1}(u_1, v_1) = d_{G_1}(w_1, x_1)$

So, G_1 is an edge regular intuitionistic fuzzy graph.

Similarly G_2 is an edge regular intuitionistic fuzzy graph.

Now, suppose that G_1 is (s_1, s_2)-edge regular intuitionistic fuzzy graph and G_2 is (s_3, s_4)-edge regular intuitionistic fuzzy graph with (s_1, s_2) ≠ (s_3, s_4)

Then $d_{G_1\times G_2}\big((u_1,u_2),(u_1,v_2)\big) = 2d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$ = 2(c_1, c_2) + (s_3, s_4).

Also $d_{G_1\times G_2}\big((u_1,u_2),(v_1,u_2)\big) = 2d_{G_2}(u_2) + d_{G_1}(u_1,v_1)$ = 2(c_1, c_2) + (s_1, s_2)

So, $d_{G_1\times G_2}\big((u_1,u_2),(u_1,v_2)\big) \neq d_{G_1\times G_2}\big((u_1,u_2),(v_1,u_2)\big)$

which is a contradiction to $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph. Hence G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree.

Theorem 2.6: Let G_1:(A_1,B_1) and G_2 :(A_2,B_2) be two intuitionistic fuzzy graphs such that G_1^*: (V,E) and G_2^*: (V,E) are regular graphs with $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$ and B_1 is constant function. Then G_1xG_2 is an edge regular intuitionistic fuzzy graph.

Proof: Let G_1 and G_2 be two regular intuitionistic fuzzy graph on regular graph and B_1(u, v) = (c_1, c_2). Then G_1 is both regular and edge regular intuitionistic fuzzy graph.

Let $d_{G_1}(u_1,v_1)$ = (k_1, k_2) and $d_{G_1}(u_1)$ = (m_1,m_2), $d^*_{G_2}(u_2)$ = n, for all u_1, v_1 ∈V_1 ,u_2 ∈ V_2.

Case-(i): When $u_1 = v_1$, (u_2,v_2) ∈E_2, by theorem 2.2,

$$\begin{aligned} d_{G_1\times G_2}\big((u_1,u_2),(u_1,v_2)\big) &= 2d_{G_1}(u_1) + (c_1, c_2)(d^*_{G_2}(u_2) + d^*_{G_2}(v_2) - 2) \\ &= 2(m_1,m_2) + (c_1, c_2)(n + n - 2) \\ &= 2((m_1,m_2) + (c_1, c_2)(n - 1)) \end{aligned}$$

Case-(ii): When $u_2 = v_2$, (u_1,v_1) ∈E_1

$$\begin{aligned} d_{G_1\times G_2}\big((u_1,u_2),(v_1,u_2)\big) &= d_{G_1}(u_1,v_1) + 2(c_1, c_2)\, d^*_{G_2}(u_2) \\ &= (k_1, k_2) + 2(c_1, c_2)n \\ &= 2(m_1,m_2) - 2(c_1, c_2) + 2(c_1, c_2)n \\ &= 2((m_1,m_2) + (c_1, c_2)(n - 1)). \end{aligned}$$

Hence $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph.

Corollary 2.7: Let G_1 and G_2 be two intuitionistic fuzzy graph with $\mu_1 \leq \mu_2'$, $\gamma_1 \geq \gamma_2'$ and A_1 is a constant function with A_1(u) = (c_1, c_2) for all u ∈ V_1. Let G_1^* and G_2^* be two regular graphs. If G_1 is strong, then $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph.

Proof: Let A_1(u) = (c_1, c_2) for all u ∈ V_1. Since G_1 is strong, we have B_1(uv) = (c_1, c_2) for all (u, v) ∈ E_1. Also given $\mu_1 \leq \mu_2'$, $\gamma_1 \geq \gamma_2'$ Hence by theorem 2.6, the result follows.

Theorem 2.8: Let G_1: (A_1, B_1) and G_2 :(A_2,B_2) be two intuitionistic fuzzy graphs such that G_1^*: (V, E) and G_2^*: (V, E) are regular graphs with $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$ and B_2 is constant function. Then G_1xG_2 is an edge regular intuitionistic fuzzy graph.

Proof: Proof is similar to Theorem 2.6

Corollary 2.9: Let G_1 and G_2 be two intuitionistic fuzzy graph with $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$ and A_2 is a constant function with A_2(u) = (c_1, c_2) for all u ∈ V_2. Let G_1^* and G_2^* be two regular graphs. If G_2 is strong, then $G_1 \times G_2$ is an edge regular intuitionistic fuzzy graph.

Proof: Let A_2(u) = (c_1, c_2) for all u ∈ V_2. Since G_2 is strong, we have B_2(uv) = (c_1, c_2), for all (u, v) ∈ E. Also given $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$ Hence by theorem 2.8, the result follows.

3. EDGE REGULAR PROPERTY OF COMPOSITION OF INTUITIONISTIC FUZZY GRAPHS

In this section, degree of an edge in composition product of two intuitionistic fuzzy graphs is defined and we see about the edge regular property of composition product of two intuitionistic fuzzy graph.

Degree of an edge

By definition for any ((u_1,u_2)(v_1,v_2)) ∈E, we have

$$\begin{aligned} d_{G_1\circ G_2}\big((u_1,u_2),(u_1,v_2)\big) &= \textstyle\sum_{(w_1,w_2)\neq(v_1,v_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((u_1,u_2),(w_1,w_2)\big) \\ &\quad + \textstyle\sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((w_1,w_2),(v_1,v_2)\big) \end{aligned}$$

If $u_1=v_1$, $u_2 \neq v_2$ then

$$\begin{aligned}
d_{G_1\circ G_2}\big((u_1,u_2),(u_1,v_2)\big) &= \textstyle\sum_{(w_1,w_2)\neq(u_1,v_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((u_1,u_2),(w_1,w_2)\big)\\
&+ \textstyle\sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((w_1,w_2),(v_1,v_2)\big)\\
&= \textstyle\sum_{w_2\neq v_2,w_2u_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(u_2,w_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(u_2,w_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,w_2v_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(w_2,v_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(w_2,v_2)\big)\}\\
&+ \textstyle\sum_{w_2=v_2,w_1u_1\in E_1}\{\min\big(\mu_2(w_1,u_1),\mu_1'(v_2)\big),\max\big(\gamma_2(w_1,u_1),\gamma_1'(v_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq v_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(w_2),\mu_1'(v_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(w_2),\gamma_1'(v_2)\big)\}
\end{aligned}$$

If $u_1 \neq v_1$, $u_2 = v_2$ then

$$\begin{aligned}
d_{G_1\circ G_2}\big((u_1,u_2),(v_1,u_2)\big) &= \textstyle\sum_{(w_1,w_2)\neq(v_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((u_1,u_2),(w_1,w_2)\big)\\
&+ \textstyle\sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((w_1,w_2),(v_1,v_2)\big)\\
&= \textstyle\sum_{u_1=w_1,w_2u_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(u_2,w_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(u_2,w_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,w_1u_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2)\big)\}\\
&+ \textstyle\sum_{u_1=w_1,w_2u_2\in E_2}\{\min\big(\mu_1(v_1),\mu_2'(w_2,u_2)\big),\max\big(\gamma_1(v_1),\gamma_2'(w_2,u_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,w_1v_1\in E_1}\{\min\big(\mu_2(w_1,v_1),\mu_1'(u_2)\big),\max\big(\gamma_2(w_1,v_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,w_1v_1\in E_1}\{\min\big(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(u_2)\big),\max\big(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(u_2)\big)\}
\end{aligned}$$

If $u_1 \neq v_1$, $u_2 \neq v_2$ then

$$\begin{aligned}
d_{G_1\circ G_2}\big((u_1,u_2),(v_1,v_2)\big) &= \textstyle\sum_{(w_1,w_2)\neq(v_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((u_1,u_2),(w_1,w_2)\big)\\
&+ \textstyle\sum_{(w_1,w_2)\neq(u_1,u_2)}(\mu_2\circ\mu_2')(\gamma_2\circ\gamma_2')\big((w_1,w_2),(v_1,v_2)\big)\\
&= \textstyle\sum_{u_1=w_1,w_2u_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(u_2,w_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(u_2,w_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,w_1u_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2)\big)\}\\
&+ \textstyle\sum_{u_1=w_1,w_2v_2\in E_2}\{\min\big(\mu_1(v_1),\mu_2'(w_2,v_2)\big),\max\big(\gamma_1(v_1),\gamma_2'(w_2,v_2)\big)\}\\
&+ \textstyle\sum_{w_2=v_2,w_1v_1\in E_1}\{\min\big(\mu_2(w_1,v_1),\mu_1'(v_2)\big),\max\big(\gamma_2(w_1,v_1),\gamma_1'(v_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,w_1v_1\in E_1}\{\min\big(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(v_2)\big),\max\big(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(v_2)\big)\}
\end{aligned}$$

Theorem 3.1: Let G_1 and G_2 be two intuitionistic fuzzy graph such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$, then

a) $d_{G_1\circ G_2}\big((u_1,u_2),(u_1,v_2)\big) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$

b) $d_{G_1\circ G_2}\big((u_1,u_2),(v_1,u_2)\big) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))$

c) $d_{G_1\circ G_2}\big((u_1,u_2),(v_1,v_2)\big) = d_{G_1}(u_1,v_1) + (p_2-1)\left(d_{G_1}(u_1)+d_{G_1}(v_1)\right) + d_{G_2}(u_2) + d_{G_2}(v_2)$

Proof:

a)
$$\begin{aligned}
d_{G_1\circ G_2}\big((u_1,u_2),(u_1,v_2)\big) &= \textstyle\sum_{w_2\neq v_2,u_2w_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(u_2,w_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(u_2,w_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,w_2v_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(w_2,v_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(w_2,v_2)\big)\}\\
&+ \textstyle\sum_{w_2=v_2,w_1u_1\in E_1}\{\min\big(\mu_2(w_1,u_1),\mu_1'(v_2)\big),\max\big(\gamma_2(w_1,u_1),\gamma_1'(v_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq v_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(w_2),\mu_1'(v_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(w_2),\gamma_1'(v_2)\big)\}\\
&= \textstyle\sum_{w_2\neq v_2,u_2w_2\in E_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\}+\sum_{w_2=u_2,u_1w_1\in E_1}\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}|V-\{u_2\}|\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}\\
&+ \textstyle\sum_{w_2\neq u_2,w_2v_2\in E_2}\{\mu_2'(w_2,v_2),\gamma_2'(w_2,v_2)\}+\sum_{w_2=v_2,w_1u_1\in E_1}\{\mu_2(w_1,u_1),\gamma_2(w_1,u_1)\}\\
&+ \textstyle\sum_{w_2\neq v_2,u_1w_1\in E_1}|V-\{v_2\}|\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}\\
&= \textstyle\sum_{w_2\neq v_2,u_2w_2\in E_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\} + \sum_{w_2\neq u_2,w_2v_2\in E_2}\{\mu_2'(w_2,v_2),\gamma_2'(w_2,v_2)\}\\
&+ d_{G_1}(u_1) + (p_2-1)d_{G_1}(u_1) + d_{G_1}(u_1) + (p_2-1)d_{G_1}(u_1)
\end{aligned}$$

$$d_{G_1\circ G_2}\big((u_1,u_2),(u_1,v_2)\big) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$$

b)
$$\begin{aligned}
d_{G_1\circ G_2}\big((u_1,u_2),(v_1,u_2)\big) &= \textstyle\sum_{u_1=w_1,w_2u_2\in E_2}\{\min\big(\mu_1(u_1),\mu_2'(u_2,w_2)\big),\max\big(\gamma_1(u_1),\gamma_2'(u_2,w_2)\big)\}\\
&+ \textstyle\sum_{w_2=u_2,w_1u_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2)\big)\}\\
&+ \textstyle\sum_{w_2\neq u_2,u_1w_1\in E_1}\{\min\big(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)\big),\max\big(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2)\big)\}\\
&+ \textstyle\sum_{u_1=w_1,w_2u_2\in E_2}\{\min\big(\mu_1(v_1),\mu_2'(w_2,u_2)\big),\max\big(\gamma_1(v_1),\gamma_2'(w_2,u_2)\big)\}
\end{aligned}$$

$+ \sum_{w_2=u_2, w_1v_1\in E_1}\{\min(\mu_2(w_1,v_1),\mu_1'(u_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(u_2))\}$
$+ \sum_{w_2\neq u_2, w_1v_1\in E_1}\{\min(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(u_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(u_2))\}$
$= \sum_{w_2\in V_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\}+\sum_{w_1\in V_1}\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}$
$+ |V-\{u_2\}|\sum_{w_1\in V_1}\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}$
$+ \sum_{w_2\in V_2}\{\mu_2'(w_2,u_2),\gamma_2'(w_2,u_2)\}+\sum_{w_2=v_2, w_1u_1\in E_1}\{\mu_2(w_1,v_1),\gamma_2(w_1,v_1)\}$
$+ |V-\{u_2\}|\sum_{w_1\in V_1}\{\mu_2(w_1,v_1),\gamma_2(w_1,v_1)\}$

$$d_{G_1\circ G_2}((u_1,u_2),(v_1,u_2)) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))$$

c) $d_{G_1\circ G_2}((u_1,u_2),(v_1,v_2)) = \sum_{u_1=w_1, w_2u_2\in E_2}\{\min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}$
$+ \sum_{w_2=u_2, w_1u_1\in E_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2))\}$
$+ \sum_{w_2\neq u_2, u_1w_1\in E_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2))\}$
$+ \sum_{u_1=w_1, w_2v_2\in E_2}\{\min(\mu_1(v_1),\mu_2'(w_2,v_2)), \max(\gamma_1(v_1),\gamma_2'(w_2,v_2))\}$
$+ \sum_{w_2=v_2, w_1v_1\in E_1}\{\min(\mu_2(w_1,v_1),\mu_1'(v_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(v_2))\}$
$+ \sum_{w_2\neq v_2, w_1v_1\in E_1}\{\min(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(v_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(v_2))\}$
$= \sum_{w_2\in V_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\}+\sum_{w_1\in V_1}\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}$
$+ \sum_{w_2\neq u_2, u_1w_1\in E_1}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)), \max(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2))\}$
$- (\mu_2(u_1,v_1),\gamma_2(u_1,v_1))$
$+ \sum_{w_2\in V_2}\{\mu_2'(w_2,v_2),\gamma_2'(w_2,v_2)\}+\sum_{w_2=v_2, w_1u_1\in E_1}\{\mu_2(w_1,v_1),\gamma_2(w_1,v_1)\}$
$+ \sum_{w_2\neq v_2, w_1v_1\in E_1}\{\min(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(v_2)), \max(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(v_2))\}$
$-(\mu_2(u_1,v_1),\gamma_2(u_1,v_1))$

$$d_{G_1\circ G_2}((u_1,u_2),(v_1,v_2)) = d_{G_1}(u_1,v_1) + (p_2-1)\left(d_{G_1}(u_1)+d_{G_1}(v_1)\right) + d_{G_2}(u_2) + d_{G_2}(v_2)$$

Remark 3.2: If G_1 and G_2 are two edge regular intuitionistic fuzzy graph, then $G_1\circ G_2$ need not be edge regular intuitionistic fuzzy graph.

Remark 3.3: If $G_1\circ G_2$ is an edge regular intuitionistic fuzzy graph, then G_1 and G_2 need not be edge regular intuitionistic fuzzy graph

Theorem 3.4: Let $G_1:(A_1,B_1)$ and $G_2:(A_2,B_2)$ be two regular intuitionistic fuzzy graph of same degree on $G_1^*:(V,E)$ and $G_2^*:(V,E)$ such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$, Then G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree if and only if $G_1\circ G_2$ is an edge regular intuitionistic fuzzy graph.

Proof: Let $d_{G_1}(u_1) = d_{G_2}(u_2) = (k_1, k_2)$, for all $u_1 \in V_1$, $u_2 \in V_2$ and (k_1, k_2) is a constant.

Assume that G_1 and G_2 are (c_1, c_2)- edge regular intuitionistic fuzzy graph. Then,
$d_{G_1}(u_1,v_1) = d_{G_2}(u_2,v_2) = (c_1, c_2)$, for all $(u_1, v_1) \in E_1$, $(u_2, v_2) \in E_2$.

When $u_1 = v_1$, $(u_2, v_2) \in E_2$ we have
$d_{G_1\circ G_2}((u_1,u_2),(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2,v_2) = 2p_2(k_1, k_2)+(c_1, c_2)$.

When $u_2 = v_2$, $(u_1, v_1) \in E_1$ we have,
$d_{G_1\circ G_2}((u_1,u_2),(v_1,u_2)) = 2d_{G_2}(u_2) + d_{G_1}(u_1,v_1) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))$
$= (c_1, c_2) + 2(k_1, k_2) + (p_2-1)((k_1, k_2) + (k_1, k_2))$
$= (c_1, c_2) + 2p_2(k_1, k_2)$.

When $(u_1, v_1) \in E_1$, $(u_2, v_2) \notin E_2$ we have
$d_{G_1\circ G_2}((u_1,u_2),(v_1,v_2)) = d_{G_1}(u_1,v_1) + (p_2-1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2)$
$= (c_1, c_2) + 2p_2(k_1, k_2)$.

Hence $G_1 \circ G_2$ is an edge regular intuitionistic fuzzy graph.

Conversely assume that $G_1 \circ G_2$ is an edge regular intuitionistic fuzzy graph.

Let $(u_1, v_1), (w_1, x_1) \in E_1$ be two edges of G_1 and let $u \in V_2$. Then $((u_1, u), (v_1, u)), ((w_1, u), (x_1, u)) \in E$.

So, $d_{G_1 \circ G_2}\big((u_1,u),(v_1,u)\big) = d_{G_1 \circ G_2}\big((w_1,u),(x_1,u)\big)$
$\Rightarrow d_{G_1}(u_1,v_1) + 2d_{G_2}(u) + (p_2-1)(d_{G_1}(u_1) + d_{G_1}(v_1)) = d_{G_1}(w_1,x_1) + 2d_{G_2}(u) + (p_2-1)(d_{G_1}(w_1) + d_{G_1}(x_1))$
$\Rightarrow d_{G_1}(u_1,v_1) + (p_2-1)(2(k_1,k_2)) = d_{G_1}(w_1,x_1) + (p_2-1)(2(k_1,k_2))$
$\Rightarrow d_{G_1}(u_1,v_1) = d_{G_1}(w_1,x_1)$

Hence G_1 is an edge regular intuitionistic fuzzy graph.

Similarly G_2 is an edge regular intuitionistic fuzzy graph.

Now, suppose that G_1 is (s_1, s_2)-edge regular intuitionistic fuzzy graph and G_2 is (s_3, s_4)-edge regular intuitionistic fuzzy graph with $(s_1, s_2) \neq (s_3, s_4)$

Then $d_{G_1 \circ G_2}\big((u_1,u_2),(u_1,v_2)\big) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$
$= 2p_2(k_1, k_2) + (s_3, s_4)$.

Also, $d_{G_1 \circ G_2}\big((u_1,u_2),(v_1,u_2)\big) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2) + (p_2-1)(d_{G_1}(u_1) + d_{G_1}(v_1))$
$= (s_1, s_2) + 2(k_1, k_2) + (p_2-1)((k_1, k_2) + (k_1, k_2))$
$= (s_1, s_2) + 2p_2(k_1, k_2)$

So, $d_{G_1 \circ G_2}\big((u_1,u_2),(u_1,v_2)\big) \neq d_{G_1 \circ G_2}\big((u_1,u_2),(v_1,u_2)\big)$

which is a contradiction to $G_1 \circ G_2$ is an edge regular intuitionistic fuzzy graph. Hence G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree.

4. EDGE REGULAR PROPERTY OF ALPHA PRODUCT OF INTUITIONISTIC FUZZY GRAPHS

In this section, we see about the edge regular property of alpha product of two intuitionistic fuzzy graph.

Definition 4.1: Let $G_1 : (A_1,B_1)$ and $G_2 : (A_2,B_2)$ where $A_1 = (\mu_1,\gamma_1)$, $B_1 = (\mu_2,\gamma_2)$, $A_2 = (\mu_1', \gamma_1')$, $B_2 = (\mu_2', \gamma_2')$ be two intuitionistic fuzzy graph on $G^*(V,E)$ Where $V = V_1 \times V_2$ and $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1, u_2v_2 \in E_2$ or $u_2 = v_2, u_1v_1 \in E_1$ or $u_1v_1 \in E_1, u_2v_2 \notin E_2$ or $u_1v_1 \notin E_1, u_2v_2 \in E_2\}$. Then the alpha product of G_1 and G_2 is defined as

$G_1 \times_\alpha G_2 = \{((\mu_1 \times_\alpha \mu_1')(,\gamma_1 \times_\alpha \gamma_1'), (\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2'))\}$

where

$((\mu_1 \times_\alpha \mu_1')(,\gamma_1 \times_\alpha \gamma_1'))(u_1, u_2) = \{\min(\mu_1(u_1), \mu_1'(u_2)), \max(\gamma_1(u_1), \gamma_1'(u_2))\}$

$((\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2'))((u_1, u_2)(v_1, v_2))$

$$= \begin{cases} \{min(\mu_1(u_1),\mu_2'(u_2,v_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,v_2))\} & u_1 = v_1, u_2v_2 \in E_2 \\ \{(\min(\mu_1'(u_2),\mu_2(u_1,v_1)), max(\gamma_1'(u_2),\gamma_2(u_1,v_1))\} & u_2 = v_2, u_1v_1 \in E_1 \\ \{\min\big(\mu_2(u_1,v_1),\mu_1'(u_2),\mu_1'(v_2)\big), max(\gamma_2(u_1,v_1),\gamma_1'(u_2),\gamma_1'(v_2))\} & u_1v_1 \in E_1, u_2v_2 \notin E_2 \\ \{min(\mu_2'(u_2,v_2),\mu_1(u_1),\mu_1(v_1)) max(\gamma_2'(u_2,v_2), \gamma_1(u_1),\gamma_1(v_1))\} & u_1v_1 \notin E_1, u_2v_2 \in E_2 \end{cases}$$

Degree of an edge

$$\begin{aligned} d_{G_1\times_\alpha G_2}((u_1,u_2),(v_1,v_2)) &= \textstyle\sum_{((u_1,u_2),(w_1,w_2))\in E}(\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2')((u_1,u_2),(w_1,w_2)) \\ &+ \textstyle\sum_{(w_1,w_2),(v_1,v_2))\in E} (\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2')(w_1,w_2),(v_1,v_2)) \\ &- 2(\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2')\big((u_1,u_2),(v_1,v_2)\big) \\ &= \textstyle\sum_{u_1=w_1,u_2w_2\in E_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\} \\ &+ \textstyle\sum_{u_2=w_2,u_1w_1\in E_1}\{\min(\mu_1'(u_2),\mu_2(u_1,w_1)), max(\gamma_1'(u_2),\gamma_2(u_1,w_1))\} \\ &+ \textstyle\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)), max(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2))\} \\ &+ \textstyle\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\{\min\{\mu_2'(u_2,w_2),\mu_1(u_1),\mu_1(w_1))\max(\gamma_2'(u_2,w_2),\gamma_1(u_1),\gamma_1(w_1))\} \\ &+ \textstyle\sum_{u_1=w_1,v_2w_2\in E_2}\{min(\mu_1(v_1),\mu_2'(v_2,w_2)), \max(\gamma_1(v_1),\gamma_2'(v_2,w_2))\} \\ &+ \textstyle\sum_{v_2=w_2,v_1w_1\in E_1}\{\min(\mu_1'(v_2),\mu_2(v_1,w_1)), max(\gamma_1'(v_2),\gamma_2(v_1,w_1))\} \\ &+ \textstyle\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\{\min(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(v_2)), max(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(v_2))\} \\ &+ \textstyle\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\{\min\{\mu_2'(w_2,v_2),\mu_1(w_1),\mu_1(v_1))\max(\gamma_2'(w_2,v_2),\gamma_1(w_1),\gamma_1(v_1))\} \\ &- 2(\mu_2 \times_\alpha \mu_2')(\gamma_2 \times_\alpha \gamma_2')\big((u_1,u_2),(v_1,v_2)\big) \end{aligned}$$

Remark 4.2: If G_1 and G_2 are two edge regular intuitionistic fuzzy graphs, then $G_1 \times_\alpha G_2$ need not be edge regular intuitionistic fuzzy graph.

Remark 4.3: If $G_1 \times_\alpha G_2$ is an edge regular intuitionistic fuzzy graph, then G_1 and G_2 need not be edge regular intuitionistic fuzzy graph

Theorem 4.4: Let G_1:(A_1,B_1) and G_2 :(A_2,B_2) be two intuitionistic fuzzy graphs such that G_1^*: (V,E) and G_2^*: (V,E) are complete graphs. Suppose $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$, Then for any $(u_1, u_2), (v_1, v_2) \in E$

1. When $u_1 = v_1$, $u_2v_2 \in E_2$, $d_{G_1 \times_\alpha G_2}\big((u_1,u_2),(v_1,v_2)\big) = d_{G_2}(u_2,v_2)+2\, d_{G_1}(u_1)$
2. When $u_2 = v_2$, $u_1v_1 \in E_1$, $d_{G_1 \times_\alpha G_2}\big((u_1,u_2),(v_1,v_2)\big) = d_{G_1}(u_1,v_1)+2\, d_{G_2}(u_2)$

Proof: 1. By definition for any $\big((u_1,u_2),(v_1,v_2)\big) \in E$

$$
\begin{aligned}
d_{G_1\times_\alpha G_2}((u_1,u_2),(v_1,v_2)) &= \textstyle\sum_{((u_1,u_2),(w_1,w_2))\in E}(\mu_2 \times_\propto \mu_2')(\gamma_2 \times_\propto \gamma_2')((u_1,u_2),(w_1,w_2))\\
&+ \textstyle\sum_{(w_1,w_2),(v_1,v_2))\in E} (\mu_2 \times_\propto \mu_2')(\gamma_2 \times_\propto \gamma_2')(w_1,w_2),(v_1,v_2))\\
&-2(\mu_2 \times_\propto \mu_2')(\gamma_2 \times_\propto \gamma_2')\big((u_1,u_2),(v_1,v_2)\big)\\
&= \textstyle\sum_{u_1=w_1,u_2w_2\in E_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}\\
&+ \textstyle\sum_{u_2=w_2,u_1w_1\in E_1}\{\min(\mu_1'(u_2),\mu_2(u_1,w_1)), max(\gamma_1'(u_2),\gamma_2(u_1,w_1))\}\\
&+ \textstyle\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\{\min(\mu_2(u_1,w_1),\mu_1'(u_2),\mu_1'(w_2)), max(\gamma_2(u_1,w_1),\gamma_1'(u_2),\gamma_1'(w_2))\}\\
&+ \textstyle\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\{\min\{\mu_2'(u_2,w_2),\mu_1(u_1),\mu_1(w_1))\max(\gamma_2'(u_2,w_2),\gamma_1(u_1),\gamma_1(w_1))\}\\
&+ \textstyle\sum_{u_1=w_1,v_2w_2\in E_2}\{min(\mu_1(v_1),\mu_2'(v_2,w_2)), \max(\gamma_1(v_1),\gamma_2'(v_2,w_2))\}\\
&+ \textstyle\sum_{v_2=w_2,v_1w_1\in E_1}\{\min(\mu_1'(v_2),\mu_2(v_1,w_1)), max(\gamma_1'(v_2),\gamma_2(v_1,w_1))\}\\
&+ \textstyle\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\{\min(\mu_2(w_1,v_1),\mu_1'(w_2),\mu_1'(v_2)), max(\gamma_2(w_1,v_1),\gamma_1'(w_2),\gamma_1'(v_2))\}\\
&+ \textstyle\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\{\min\{\mu_2'(w_2,v_2),\mu_1(w_1),\mu_1(v_1))\max(\gamma_2'(w_2,v_2),\gamma_1(w_1),\gamma_1(v_1))\}\\
&- 2(\mu_2 \times_\propto \mu_2')(\gamma_2 \times_\propto \gamma_2')\big((u_1,u_2),(v_1,v_2)\big)
\end{aligned}
$$

Given G_1^* and G_2^* are complete graphs. So the edge set is given by

E={ $(u_1,u_2),(v_1,v_2): u_1 = w_1, u_2v_2 \in E_2$ or $u_2 = w_2, u_1v_1 \in E_1$}

$$
\begin{aligned}
d_{G_1\times_\alpha G_2}((u_1,u_2),(u_1,v_2)) &= \textstyle\sum_{u_1=w_1,u_2w_2\in E_2}\{min(\mu_1(u_1),\mu_2'(u_2,w_2)), \max(\gamma_1(u_1),\gamma_2'(u_2,w_2))\}\\
&+ \textstyle\sum_{u_2=w_2,u_1w_1\in E_1}\{\min(\mu_1'(u_2),\mu_2(u_1,w_1)), max(\gamma_1'(u_2),\gamma_2(u_1,w_1))\}\\
&+ \textstyle\sum_{u_1=w_1,v_2w_2\in E_2}\{min(\mu_1(v_1),\mu_2'(w_2,v_2)), \max(\gamma_1(v_1),\gamma_2'(w_2,v_2))\}\\
&+ \textstyle\sum_{v_2=w_2,v_1w_1\in E_1}\{\min(\mu_1'(v_2),\mu_2(v_1,w_1)), max(\gamma_1'(v_2),\gamma_2(v_1,w_1))\}\\
&- 2(\mu_2 \times_\propto \mu_2')(\gamma_2 \times_\propto \gamma_2')\big((u_1,u_2),(v_1,v_2)\big)\\
&= \textstyle\sum_{u_2w_2\in E_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\}+\sum_{u_1w_1\in E_1}\{\mu_2(u_1,w_1),\gamma_2(u_1,w_1)\}\\
&+ \textstyle\sum_{w_2v_2\in E_2}\{\mu_2'(w_2,v_2),\gamma_2'(w_2,v_2)\}+\sum_{w_1v_1\in E_1}\{\mu_2(w_1,v_1),\gamma_2(w_1,v_1)\}\\
&-- 2(\mu_2(u_2,v_2),\gamma_2(u_2,v_2))\\
&= \textstyle\sum_{u_2w_2\in E_2}\{\mu_2'(u_2,w_2),\gamma_2'(u_2,w_2)\} + \sum_{w_2v_2\in E_2}\{\mu_2'(w_2,v_2),\gamma_2'(w_2,v_2)\}\\
&- 2(\mu_2(u_2,v_2),\gamma_2(u_2,v_2))+d_{G_1}(u_1)+ d_{G_1}(u_1)
\end{aligned}
$$

$d_{G_1\times_\alpha G_2}((u_1,u_2),(u_1,v_2)) = d_{G_2}(u_2,v_2)+2\, d_{G_1}(u_1)$.

Proof of (2) is similar to (1).

Theorem 4.5: Let G_1:(A_1,B_1) and G_2 :(A_2, B_2) be two regular intuitionistic fuzzy graph of same degree on G_1^*: (V, E) and G_2^*: (V, E) such that $\mu_1 \geq \mu_2'$, $\gamma_1 \leq \gamma_2'$, $\mu_1' \geq \mu_2$, $\gamma_1' \leq \gamma_2$, Then G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree if and only if $G_1 \times_\alpha G_2$ is an edge regular intuitionistic fuzzy graph.

Proof: Let $d_{G_1}(u_1)$ = $d_{G_2}(u_2)$ = (m_1, m_2), for all $u_1 \in V_1$, $u_2 \in V_2$ and (k_1, k_2) is a constant.

Assume that G_1 and G_2 are (k_1, k_2)- edge regular intuitionistic fuzzy graph. Then,

$d_{G_1}(u_1,v_1) = d_{G_2}(u_2,v_2)$ = (k_1, k_2), for all (u_1, v_1) $\in E_1$, (u_2, v_2) $\in E_2$.

When $u_1 = v_1$, (u_2, v_2) $\in E_2$ we have

$d_{G_1\times_\alpha G_2}\big((u_1,u_2),(u_1,v_2)\big) = d_{G_2}(u_2,v_2) + 2d_{G_1}(u_1)$= ($k_1$, k_2)+ 2 (m_1, m_2).

When $u_2 = v_2$, (u_1, v_1) $\in E_1$ we have,

$d_{G_1\times_\alpha G_2}\big((u_1,u_2),(v_1,u_2)\big) = 2d_{G_2}(u_2) + d_{G_1}(u_1,v_1)$=($k_1$, k_2)+ 2 (m_1, m_2).

Hence $G_1 \times_\alpha G_2$ is an edge regular intuitionistic fuzzy graph.

Conversely assume that $G_1 \times_\alpha G_2$ is an edge regular intuitionistic fuzzy graph.

Let (u_1, v_1), (w_1, x_1) $\in E_1$ be two edges of G_1 and let u $\in V_2$. Then ((u_1, u), (v_1, u)), ((w_1, u), (x_1, u)) $\in$ E.

So, $d_{G_1\times_\alpha G_2}((u_1,u),(v_1,u)) = d_{G_1\times_\alpha G_2}((w_1,u),(x_1,u))$

$\Rightarrow d_{G_1}(u_1,v_1) + 2d_{G_2}(u) = d_{G_1}(w_1,x_1) + 2d_{G_2}(u)$

$\Rightarrow d_{G_1}(u_1,v_1) + 2(m_1, m_2) = d_{G_1}(w_1,x_1) + 2(m_1, m_2)$

$\Rightarrow d_{G_1}(u_1,v_1) = d_{G_1}(w_1,x_1)$

Hence G_1 is an edge regular intuitionistic fuzzy graph.

Similarly G_2 is an edge regular intuitionistic fuzzy graph.

Now, suppose that G_1 is (k_1, k_2)-edge regular intuitionistic fuzzy graph and G_2 is (k_3, k_4)-edge regular intuitionistic fuzzy graph with $(k_1, k_2) \neq (k_3, k_4)$

Then $d_{G_1\times_\alpha G_2}((u_1,u_2),(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2,v_2)$
$= 2p_2(k_1, k_2) + (s_3, s_4)$.

Also, $d_{G_1\times_\alpha G_2}((u_1,u_2),(v_1,u_2)) = d_{G_1}(u_1,v_1) + 2d_{G_2}(u_2) = (k_1, k_2) + 2(m_1, m_2)$

$d_{G_1\times_\alpha G_2}((u_1,u_2),(u_1,v_2)) = d_{G_2}(u_2,v_2) + 2d_{G_1}(u_1) = (k_3, k_4) + 2(m_1, m_2)$

So, $d_{G_1\times_\alpha G_2}((u_1,u_2),(u_1,v_2)) \neq d_{G_1\times_\alpha G_2}((u_1,u_2),(v_1,u_2))$

which is a contradiction to $G_1 \times_\alpha G_2$ is an edge regular intuitionistic fuzzy graph. Hence G_1 and G_2 are edge regular intuitionistic fuzzy graph of same degree.

REFERENCES

1. Akram. M., Dudek. W, *Regular intuitionistic fuzzy graphs*, Neural Computing and Application, 1007/s00521-011-0772.
2. Atanassov. K.T., *Intuitionistic fuzzy sets: Theory and applications*, Studies in fuzziness and soft computing, Heidelberg, New York, Physica-Verl., 1999.
3. Ismail Mohaideen. S., Nagoor Gani. A., Fathima Kani. B., Yasmin. C., Properties of Operation on Regular Intuitionistic Fuzzy Graphs, International Journal of Engineering Science and Computing, Vol.6, No.4, (2016), 3779-3783.
4. John N. Mordeson and Peng. C.S., *Operations on Fuzzy Graphs*, Inform. Sci., 79:159-170, 1994.
5. Karunambigai. M.G., Sivasankar. S and Palanivel. K, *Edge Regular Intuitionistic Fuzzy graph*, Advances in Fuzzy sets and Systems, Vol.20, Issue No.1;(2015), 25-46.
6. Nagoor Gani. A., and Fathima Kani. B., *Alpha Product of Fuzzy Graphs*, Advances in Fuzzy Sets and Systems, Vol.17, No.1, (2014), 27-48.
7. Radha, K., and Kumaravel, N., *The degree of an edge in Cartesian Product and Composition of two Fuzzy Graphs*, International Journal of Applied Mathematics and Statistical Sciences, Vol.2, Issue.2, May (2013), 65 - 78.
8. Radha. K., and Kumaravel. N., *Some Properties of edge regular fuzzy graphs*, Jamal Academic Research Journal, Special issue, 2014, 121-127.
9. Radha. K., and Kumaravel. N., *The degree of an edge in alpha, beta and gamma product of two fuzzy graphs*, IOSR Journal of Mathematics, Vol.10, Issue.5, (2014), 1-19.
10. Parvathi. R., Karunambigai. M.G., and Atanassov .K.T., *Operation on Intuitionistic Fuzzy Graphs*, Proceedings of IEEE International Conference on Fuzzy Systems (2009), 1396-1401.
11. Parvathi. R., Karunambigai. M.G., and Palaniappan. N., *Operation on Intuitionistic Fuzzy Graph,* Proceedings of International Conference on Number Theory and Combinatorics, (2009), 135-150.
12. Ravi Narayanan. S., and Murugesan. S., *Edge Irregular Intuitionistic Fuzzy Graphs*, International Journal of Mathematics and Soft Computing, Vol.7, No.2 (2017), 15-29.
13. Ravi Narayanan. S., and Murugesan. S., *(2, (c_1, c_2)) – Pseudo regular intuitionistic fuzzy graphs*, International Journal of Fuzzy Mathematical Archieve, Vol.10, No.2, (2016), 131-137.
14. Thilagavathi. S., Parvathi. R., Karunambigai. M.G., *Operations on Intuitionistic Fuzzy Graphs-I*, Proceedings, UGC Sponsored National conference on Applications of Mathematics and Statistics with reference to physical and social science, (2007).
15. Thilagavathi. S., Parvathi. R., Karunambigai. M.G., *Operations on Intuitionistic Fuzzy Graphs-II*, International Journal of Computer Applications, (2009).

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