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A NEW APPROACH FOR RANKING FUZZY NUMBERS TO SOLVE TRANSPORTATION PROBLEM

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ABSTRACT

Many methods have been discussed earlier for solving fuzzy transportation problem, where the cost coefficients, supply and demand quantities are considered in the form of trapezoidal fuzzy numbers. In this paper we have introduced a method for ranking of the trapezoidal fuzzy numbers with new method to find an optimum solution for fuzzy transportation problem. First, we transform the fuzzy quantities into crisp quantities by using our new method, then obtain an initial basic feasible solution and finally find an optimum solution. This is illustrated with a numerical example. The method is very easy to apply and economical.

Keywords: *Fuzzy Transportation Problem, Trapezoidal fuzzy numbers, Ranking of fuzzy numbers, Optimum solution.*

1. INTRODUCTION

Transportation problem is used globally in solving certain concrete world problems. It plays a vital role in production industry and also in many other purposes. It is a special case of Linear programming problem, which permits us to regulate the optimum shipping patterns between origins and destinations. A transportation problem in which the cost of transportation, supply and demand quantities are uncertain i.e. fuzzy in nature, it is a fuzzy transportation problem; also when the demand in the market and supply are uncertain, it satisfies the condition of vagueness. Ranking fuzzy number is a necessary step in many mathematical models. The concepts of fuzzy sets were first introduced by Zadeh [11]. Ranking normal fuzzy number was first introduced by Jain [6] for decision making in fuzzy situations. Many authors presented various approaches for solving the fuzzy transportation problem [1], [2], [4], [8], [9], [10]. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani [3], Maliniand and Ananthanarayanan [7].

New method is presented for the ranking of generalized fuzzy trapezoidal numbers of a fuzzy transportation problem. To illustrate this proposed method, an example is discussed. As the proposed ranking method is direct and simple, it is therefore easy to understand and to find out the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations.

In this paper, we tried to find out a fuzzy optimal solution for a fuzzy transportation problem, where all the parameters are trapezoidal fuzzy numbers. In this method, first we transformed the fuzzy quantities into crisp quantities by using our new ranking method, then obtained an initial basic feasible solution by using least cost method and then finally found an optimum solution in a fuzzy number or a crisp number form. This method gives effective solution for a fuzzy transportation problem.

2. FUZZY SET

A class that admits the possibility of partial membership in it, is called a Fuzzy Set.

Let $X=\{x\}$ denote a space of objects. Then a fuzzy set A in X is a set of ordered pairs

$$A=\{x, \mu_A(x)\}, \quad x \in X \tag{1}$$

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where $\mu_A(x)$ is the grade of membership of x in set A

$$\text{i.e. } \mu_A(x) \in [0,\ 1]$$

where

$$\mu_A(X) = \begin{cases} 1, & x \text{ is totally in A} \\ 0, & x \notin A \\ (0,1) & \text{if } x \text{ is partially in A} \end{cases} \tag{2}$$

This set is always a continuum of possible choices. Data may be classified as crisp data and fuzzy data.

3.1. Definition: A real fuzzy number A=(a, b, c, d) is a fuzzy subset of real line R with membership function $\mu_A(x)$ such that

$\mu_A(x) \rightarrow [0, 1]$

$\mu_A(x) = 0$ for a' ≤ a

$\mu_A(x)$ is increasing in [a, b]

$\mu_A(x) = 1$ i.e. constant in [b, c]

$\mu_A(x)$ is decreasing in [c, d] &

$\mu_A(x) = 0$ for a' ≥ d

Membership function represents the grade of membership associates with particular groups or a set by a member of that set or group. Determination of membership function in terms of shape and boundary has a clear effect on the result of classification performed by Fuzzy Logic. Right shape and the boundaries for the membership function increase the accuracy of the Fuzzy Logic application.

3.2. Definition: The Trapezoidal Membership Function has a function name trapmf. Mathematical representation of this membership function of a trapezoidal fuzzy number A=(a, b, c, d) where a, b, c, d $\in \boldsymbol{R}$ is

$$\mu_f(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b < x < c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & x > d \end{cases} \tag{3}$$

It has a flat top and really is just a truncated triangle curve; graphical representation of 3 is shown in Figure 1.

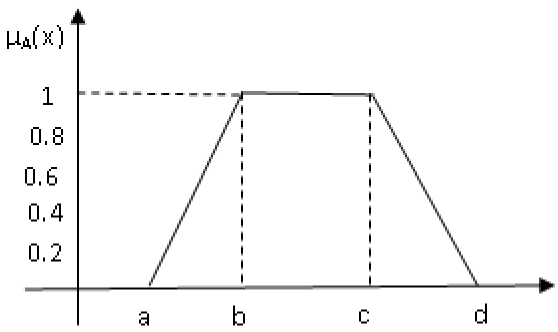

Figure 1: Trapezoidal Membership Function

3.3. Definition: Any set of fuzzy non negative allocations $x_{ij} = (-2\delta, -\delta,\ \delta,\ 2\delta)$ where δ small positive number, which satisfies the row and column sum is a fuzzy feasible solution.

3.4. Definition: If the number of non negative allocation is not more than (m + n-1) where m is the number of rows and n is the number of columns in transportation table then a feasible solution is called a fuzzy basic feasible solution.

3.5. Definition: If A= $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and B = $(\beta_1, \beta_2, \beta_3, \beta_4)$ two trapezoidal fuzzy numbers then the arithmetic operations on A and B are as follows:

Addition: $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) + (\beta_1, \beta_2, \beta_3, \beta_4) = (\alpha_1+\beta_1, \alpha_2+\beta_2, \alpha_3+\beta_3, \alpha_4+\beta_4)$

Subtraction: $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) - (\beta_1, \beta_2, \beta_3, \beta_4) = (\alpha_1-\beta_4, \alpha_2-\beta_3, \alpha_3-\beta_2, \alpha_4-\beta_1)$

Multiplication: $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \text{ X } (\beta_1, \beta_2, \beta_3, \beta_4) = (s_1, s_2, s_3, s_4)$

Where,
$s_1 = \text{minimum}(\alpha_1\beta_1, \alpha_1\beta_4, \alpha_4\beta_1, \alpha_4\beta_4)$
$s_2 = \text{minimum}(\alpha_2\beta_2, \alpha_2\beta_3, \alpha_3\beta_2, \alpha_3\beta_3)$
$s_3 = \text{maximum}(\alpha_2\beta_2, \alpha_2\beta_3, \alpha_3\beta_2, \alpha_3\beta_3)$
$s_4 = \text{maximum}(\alpha_1\beta_1, \alpha_1\beta_4, \alpha_4\beta_1, \alpha_4\beta_4)$

4. NEW APPROACH FOR RANKING OF FUZZY NUMBERS:

If A= $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is trapezoidal fuzzy number where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \boldsymbol{R}$ then the defuzzified value or the crisp value of A is given as

$$R(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \frac{2\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4}{8} \tag{4}$$

4. NUMERICAL EXAMPLE

Consider the following balanced fuzzy transportation problem where three sources S_1, S_2, S_3 and four destinations D_1, D_2, D_3, D_4, supply and demand are given as trapezoidal fuzzy numbers.

	D_1	D_2	D_3	D_4	Demand
S_1	(1,2,3,4)	(1,3,6,8)	(-1,0,1,2)	(3,5,6,8)	(0,2,4,6)
S_2	(4,8,12,16)	(6,7,11,12)	(2,4,6,8)	(1,3,5,7)	(2,5,9,13)
S_3	(1,5,9,13)	(0,4,8,12)	(0,6,8,14)	(4,7,9,12)	(2,4,6,7)
Supply	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Table-1: Transportation Problem with Trapezoidal Numbers

By using definition, Fuzzy Transportation Problem is balanced
i.e. Sum of supply = Sum of demand
(3, 11, 19, 27) = (4, 11, 19, 26)

By using our new approach given in equation (4) the fuzzy transportation problem is changed in to a crisp transportation problem as in table 2

	D_1	D_2	D_3	D_4	Demand
S_1	1.9	3.4	0.4	4.1	2.3
S_2	7.5	6.8	3.8	3	5.5
S_3	5.3	4.5	5.3	6	3.5
Supply	3	2.3	3	3	

Table-2: After ranking by new method

Using Least cost method the basic feasible solution of above crisp transportation problem is shown in table 3.

	D_1	D_2	D_3	D_4	Demand
S_1	1.9	3.4	**(2.3)** 0.4	4.1	2.3
S_2	**(1.8)** 7.5	6.8	**(0.7)** 3.8	**(3)** 3	5.5
S_3	**(1.2)** 5.3	**(2.3)** 4.5	5.3	6	3.5
Supply	3	2.3	3	3	

Table-3: Basic Feasible Solution by Least cost method

By using given fuzzy transportation problem in table 1 basic Feasible Solution in table 3 is converted into trapezoidal form as in table 4.

	D_1	D_2	D_3	D_4	Demand
S_1	(1,2,3,4)	(1,3,6,8)	**(0,2,4,6)** (-1,0,1,2)	(3,5,6,8)	(0,2,4,6)
S_2	**(-6,-1,5,11)** (4,8,12,16)	(6,7,11,12)	**(-5,-1,3,7)** (2,4,6,8)	**(1,3,5,7)** (1,3,5,7)	(2,5,9,13)
S_3	**(-4,0,4,7)** (1,5,9,13)	**(0,2,4,6)** (0,4,8,12)	(0,6,8,14)	(4,7,9,12)	(2,4,6,7)
Supply	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Table-4: Trapezoidal form of Transportation Problem for Least Cost

Here we get initial fuzzy basic feasible value (-193,-1,175,456) of the fuzzy transportation problem and we get the crisp value of the fuzzy transportation problem is 87.5

Optimal solution of this initial fuzzy basic feasible solution is obtain by Modi Method as shown in table 5 in trapezoidal form.

	D_1	D_2	D_3	D4	Demand
S_1	**(-6,-1,5,11)** (1,2,3,4)	(1,3,6,8)	**(-11,-3,5,12)** (-1,0,1,2)	(3,5,6,8)	(0,2,4,6)
S_2	(4,8,12,16)	(6,7,11,12)	**(-5,0,6,12)** (2,4,6,8)	**(1,3,5,7)** (1,3,5,7)	(2,5,9,13)
S_3	**(-4,0,4,7)** (1,5,9,13)	**(0,2,4,6)** (0,4,8,12)	(0,6,8,14)	(4,7,9,12)	(2,4,6,7)
Supply	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Table-5: Trapezoidal form of Transportation Problem by Modi Method

Transportation cost = (1, 2, 3, 4)* (-6,-1, 5, 11) + (-1, 0, 1, 2)* (-11,-3, 5, 12)
+ (2, 4, 6, 8)* (-5, 0, 6, 12) + (1, 3, 5,7)* (1,3,5,7)
+ (1, 5, 9, 13)* (-4,0,4,7) +(0,4,8,12)* (0,2,4,6)
= (-127, 11,149,376)

Here we get fuzzy optimal value (-127, 11,149,376) of the fuzzy transportation problem and we get the crisp value of the fuzzy transportation problem is 82.25.

6. RESULT & DISCUSSION

The basic feasible solution of the said fuzzy ranking transportation problem was derived by using least cost method. The basic feasible solution was further optimized by Modi method which is the least solution of the transportation problem. The optimal solution by our proposed method is 82.25, whereas the problem solved by the method [5] for optimization given optimal solution is 89.167. This proves that our proposed method is more economical and yields better solution.

7. CONCLUSION

In this paper new method is proposed to find the fuzzy optimal solution of fuzzy transportation problem in which the supply and demand costs are represented by trapezoidal fuzzy numbers. The optimal solution obtained by the proposed ranking method is least as compared to the proposed method of [5]. This new method is illustrated by solving numerical example. This method is very easy to understand and to apply for solving fuzzy transportation problems in real life situation.

This new concept of ranking method can be used to all types of transportation problems which would give effective solutions for any uncertain data.

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