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EASYMOVE GAME REPRESENTED IN GRAPH DOMINATION

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ABSTRACT

The domination game played on a graph G consists of two players, Dominator and Staller who alternate taking turns choosing a vertex from G such that whenever a vertex is chosen by either player, at least one additional vertex is dominated. Dominator wishes to dominate the graph in as few steps as possible and Staller wishes to delay the process as much as possible. The game domination number $\gamma_g(G)$ is the number of vertices chosen when Dominator starts the game and the Staller-star game domination number $\gamma'_g(G)$ when Staller starts the game. An imagination strategy is developed as a general tool for proving results on the domination game.Domination by pawns on a square beehive

Keywords: *Domination, Bipartite Graph, Game Domination number.*

1. INTRODUCTION

Then a game theory problem is one of the potential applications for various problems of theoretical computer science. Domination problems associated with the placement of various EasyMove pawns on EasyMove game board. On the one hand for their intrinsic interest but on other hand because they can also be formulated as domination problem in graph theory [1, 3]. The history of EasyMove (Sarasu Mane in local Kannada language) two-player 4×4 board game using wooden pegs originated in Karnataka state India in 19th century played in many parts of India. This is an alignment game played by 2 players, each player gets 4 pawns and some of its origins are in the simulation of war. However, the playing boards that are used in war games often have regular tetragon (quadrilateral) cells. We introduce a chessboard-like game board consisting of n^2 tetragon cells a 'square hive', and a board piece called a 'pawn' which can execute moves on lines through any of the four sides of a cell on which pawn is placed. Because of the similarity with the structure of honeycombs we call the board a beehive or hive. We study various parameters of easymove on such hives.

EasyMove (Sarasu Mane) Game is the Indian equivalent of tag. As the name suggests, there are the pawns and then there is the place. The players in this game divide up into two teams. The player has the task of move the place and then the place either horizontal/vertical. It is one of the most popular games played by children all over the world with different names and a slight change in rules, but the overall objective remains the same. Organization of the template.

EasyMove (Sarasu Mane Aata) Game

Figure-1: Placing 8 pawns on board B_{11}.

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The game of EasyMove (Sarasu Mane Aata) Game is played on a board consisting of two concentric squares connected by lines from the middle of each of the inner square's sides to the middle of the corresponding outer square's side.

Pieces are played on the points where two or more lines meet or intersect, so there are 16 playable points. This is a two-player game and each player gets 4 pawns of different colors.

1.1 How to play:

1. To begin with the 8 pawns placing diagonally on board.
2. Pawns have to be moved only on intersections of lines (shown by in Fig. 2). During a turn only one coin has to be played.
3. Players toss a coin to decide who plays first and has a slight advantage as a result.
4. To begin with, players alternately place one of their pawns on any unoccupied point on the board.
5. A player has to place a pawn such that pawn can make a 'Mill' or blocking the opponent from making a Mill.
6. A Mill is a formation of four pawns of a player in a line either horizontally or vertically (Fig.2)

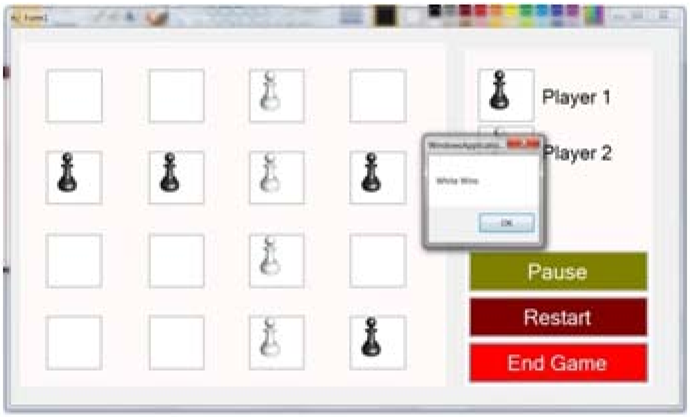

Figure-2: Player 2 winning status.

2. IMPLEMENTATION

If you design this game in graph theory, if you consider numbers of places are vertices and there is relation between the vertices is called edge. Then the above game looking like as shown below.

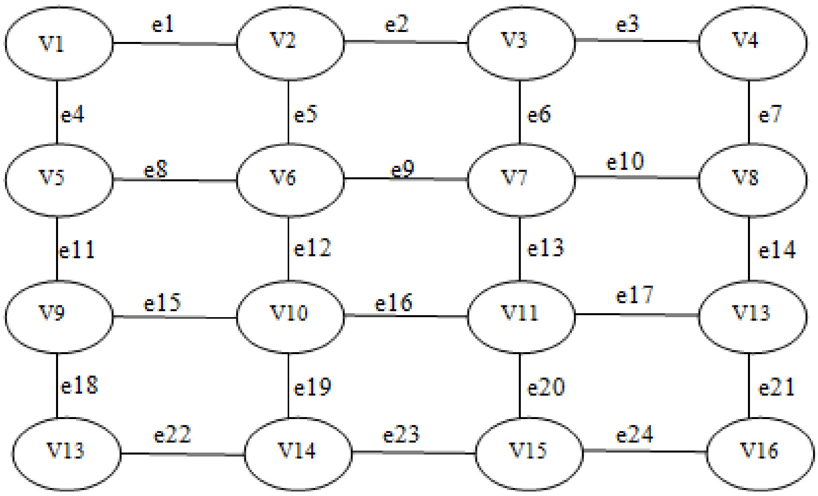

Figure-3: B_{11} Board Representing Graph

2.1 Graph: A graph G (V, E) consists of a finite nonempty set V of objects called vertices (the singular is vertex) and a set E of 2-elements subsets of V called edges [1].

2.2 Starting with the Game these vertices

Game start with the placing each player 4 pawns on the diagonal and anti-diagonal of the vertices:

Player A $=\{v_1, v_6, v_{11}, v_{16}\}$

Player B $=\{v_4, v_7, v_{10}, v_{13}\}$

2.3 Solution Matrix: A Mill is a formation of four pawns of a player in a line either horizontally or vertically, there are 8 possible solutions, Therefore the solution matrix of 4×4 as shown below

$$\mathrm{SM[G]} = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \\ v_5 & v_6 & v_7 & v_8 \\ v_9 & v_{10} & v_{11} & v_{12} \\ v_{13} & v_{14} & v_{15} & v_{16} \end{bmatrix}_{4\times4}$$

3. DEFINITIONS OF THE ELEMENTS OF A HIVE

For any positive integer n, a square beehive B_n of order n has n columns, each consisting of n tetragon cells which are considered to run from any direction can also numbered as follows. The in the top left corner is numbered (1, 1). If a cells has the number (i, j), then its neighbors (where they exist) on its boundaries are numbered (i, j+1), (i+1, j+1), (i+1, j), respectively.

In Fig. 1 we show B_{11}. Eight pawns are placed on the hive as indicated by the pawns.

Let S_i denotes the diagonal consisting of cells (r, c) of B_n such that r+c = i. such a diagonal is called a sum diagonal. Similarly, d_j denote the diagonal consisting of all cells (r, c) of B_n such that r-c = j. such diagonal is called a difference diagonal. It could happen that a diagonal consists of only one cell. Only one pawn may be placed on a cell.

A column is empty if there is no pawn in that particular column. An empty column is covered if each cell in the column is covered by at least one diagonal from pawns in other columns.

A particular cell is a private neighbor of pawn q if that cell is covered only by a line from q. A line from a pawn is essential if it covers at least one private neighbor. A line from a pawn is superfluous if it contain no private neighbor. The cell which contains a pawn may be its own private neighbor. A pawn is essential for a covering if it has at least one private neighbor (which may be itself).

3.1 Dominating and Independence sets

Board games provide ready illustrations of domination and of independence. For example 4×4 EasyMove board can be represented by a graph at the squares corresponding to u and v would challenge one another. Any vertex adjacent to the vertex v is said to be dominated by v whilst any other vertex is independent of v.

For any graph a subset of its vertices is an independent set if no two vertices in the subset are adjacent. An independent set is maximal if any vertex not in the set is dominated by at least one vertex in it. The independent number, I(G), of a graph G is the cardinality of the largest independent set[26].

A subset of the vertices of a graph is a dominating set if every vertex not in the subset is adjacent to at least one vertex in the subset. A minimal dominating set(Example 2) containing no proper subset that is also a dominating set. The dominating set, D(G), of graph G is the cardinality of the smallest dominating set[26].

Consider again the graphical representation of a EasyMove game board. The problem of placing four pawns on the board either horizontal/vertical so that no one pawn sat another line, is precisely the problem of finding a maximal independent set for the graph which contain the edge(u, v) where u and v are vertices corresponding to squares in the row or the column. There are, in fact, 8 such maximal independent sets and of course I(G) = 8. Another problem asks what is the minimum number of steps that can take to place all 4 pawns on a standard EasyMove board such that each square is dominated by at least one pawn. This problem is equivalent to finding D(G) for the graph of the first problem. Example 2 shows a minimal dominating set of smallest cardinality and so D(G) = 4.

3.2 Neighbor: A vertex v of a graph G, recall that a neighbor of v is a vertex adjacent to v in G.

3.3 Open neighborhood: The neighborhood N(v) of v is the set of neighborhood of v.

3.4 Closed neighborhood: The closed neighborhood N[v] is defined as N[v] =N (v) $\cup\{v\}$.

3.5 Minimum Dominating Set: A Minimum Dominating Set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$ [2].

Among the variations of domination, the K-tuple domination was introduced in [7]; also see [8, p.189]. For a fixed positive integer k, a k-tuple dominating set of G=(V,E) is a subset D of V such that every vertex in V is dominating by at least k vertices of D. the k-tuple dominating number $\gamma \times$ k(G) is the minimum cardinality of a k-tuple dominating set of G.

3.6 Coverings: In a graph G, a set g of edges is said to cover G if every vertex in G is incident on at least one edge in G. A set of edges that covers a graph G is said to be an edge cover, a covering sub graph, or simply a covering of G. for example, a graph G is trivially its own covering. A Hamiltonian circuit (if it exists) in a graph is also a covering (Lemma 2) [3].

3.7 Bipartite Graph: A Graph G is a bipartite graph if V(G) can be partitioned into two subsets U and W, called partite sets, such that every edge of G joins a vertex of U and a vertex of W[1].

3.8 Coloring: A coloring of a graph G, we mean an assignment of colors (elements of some set) to the vertices of G, one color to each vertex, such that adjacent vertices are colored differently. The smallest number of colors in any coloring of a graph G is called *Chromatic number* of G and is denoted by χ(G)[2].

4. GAME STRATEGIES

4.1 Both are playing to win the game
In this strategy both are trying to win the game so both Dominator and Staller follows the right strategy to win the game. Here no need of analysis.

One trying to win the game, another one trying to delay the game much as possible, in this situation staller needs to follow some strategy as follows.

4.2 To avoid opponent to win
In EasyMove game Dominator wishes to dominate the graph in as few steps as possible and Staller wishes to delay the process as much as possible. If staller wishes to delay the game, He should play most of the time either Minimum dominating vertices or Minimal dominating vertices. Then he can take game away from dominator. For that we need to model EasyMove game as a domination game as below.

The above EasyMove game there are total eight possible solutions, Player A uses a strategy to end the game in as few moves as possible; Player B uses a strategy that will require the most moves before the game ends. To run the game most of the moves, player B need to play most of the time Dominating vertices, such that player A not possible to win. So try to find domination vertices in a graph.

This situation can be modeled by the graph G on Fig 4. The graph G actually the Cartesian product $P_4 \times P_4$ which is bipartite graphs. The game intersections points are the vertices of G and two vertices are adjacent if the vertices represent intersections point on the same point at opposite ends of game block. Looking for the opponent not possible to win game of fig 4 is the same problem seeking dominating number of the graph G. The vertices $\{v_2, v_8, v_9, v_{15}\}$ and $\{v_3, v_5, v_{12}, v_{14}\}$ corresponds to the placement the pawns in Fig 4.

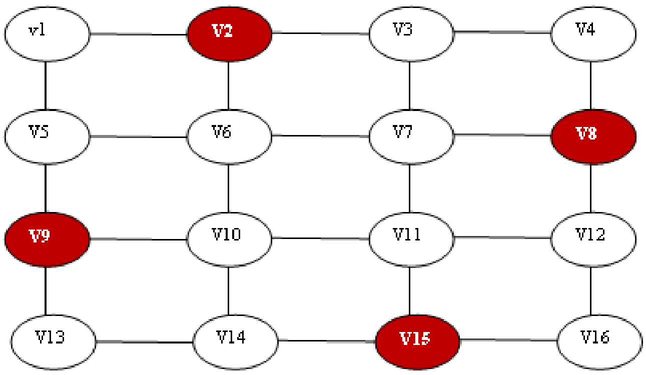

Figure-4: Dominating Vertices

Example 1: Minimum Dominating Sets of graph G
Minimum C= $\{v_2, v_8, v_9, v_{15}\}$ or
Minimum C= $\{v_3, v_5, v_{12}, v_{14}\}$

Example 2: Minimal Dominating Sets of graph G
Minimal C = $\{v_1, v_6, v_{11}, v_{16}, v_4, v_{13}\}$ or
Minimal C = $\{v_4, v_7, v_{10}, v_{13}, v_1, v_{16}\}$

Lemma 1: For the Graph G (Fig 3) Find
a. What are the possible degrees for a vertex in $G_{4,4}$?
b. How many edges are there in $G_{4,4}$?

Solution: The graph G has 16 vertices, four of which have degree 4, 8 vertices have degree 3 and the remaining four vertices have degree 2. Therefore using Handshaking Lemma.

$$\mathbf{E(G)} = \frac{1}{2}\sum_{v\in G(V)} d_G(G) = \frac{1}{2}\,(4\times 2 + 8\times 3 + 4\times 4) = 24$$

The total 24 edges in Graph G.

Lemma 2: Does Graph G (Fig 3) have Hamiltonian circuit.

A pawn that can move either one spaces horizontally and one space vertically or one space vertically and one spaces horizontally.

A Hamiltonian circuit in a connected graph is defined as a closed walk that traverse every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates.[3]

HC (G) = { $v_1 \to v_2 \to v_3 \to v_4 \to v_8 \to v_{13} \to v_{16} \to v_{15} \to v_{11} \to v_7 \to v_6 \to v_{10} \to v_{14} \to v_{13} \to v_9 \to v_5 \to v_1$ **}**

So Graph G (Fig 4) has a Hamiltonian cycle of length 16.

Lemma 2: shows that Graph G(Fig. 3) is an edge covering.

Theorem 1: For the graph G in Fig 4 $\gamma(G)$=4.

Proof: Since the Four solid vertices in Fig 4 form a dominating set of G, it follows that $\gamma(G) \leq 4$, to verify that $\gamma(G) \geq 4$, it is necessary to show that there is no dominating set with three in G.

By Lemma 1 graph G has 16 vertices, four of which have degree 4 and 8 vertices have degree 3, the remaining vertices have degree 2. Therefore there are 4 vertices that dominate four vertices each if open neighborhood or else five if closed neighborhood. Conceivably, then there is same set of three vertices that together dominate all 16 vertices of graph, However we have already noticed that G is bipartite and so its vertices can be colored with two colors say Red (R) and Green (G) without loss of generality, we can assume that the vertices of G are colored as in Fig 5 notice that the neighbors of each vertex have a color that is different from the color assigned to this vertex.

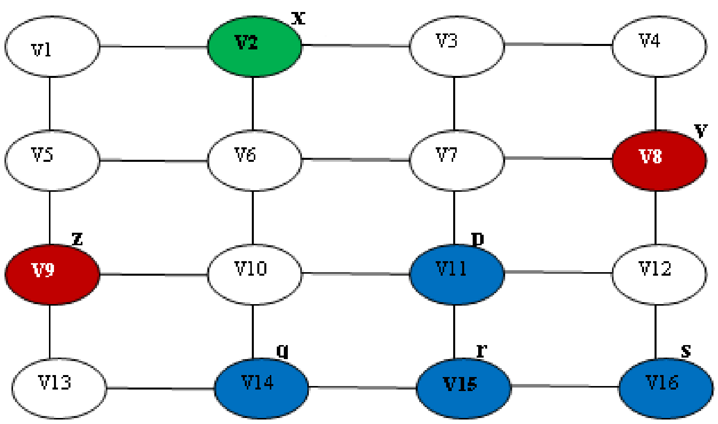

Figure-5: Colored Graph

Assume, to the contrary, that G has a dominating set S containing three vertices; at least two vertices S are colored the same. If all three vertices of S are colored the same, say Red (R) then only three of eight red vertices will be dominated. Therefore exactly two vertices of S are colored the same, say Red with the third vertex colored Green, if the Green vertex of S has degree at most three then it can dominate at most three Red vertices and S dominates at most five red vertices of G, which is impossible. Hence S must contain x (Fig 6) as its only Green vertex. Since y and z are only two red vertices not dominated by x, if follows that S={x, y, z}. However {p, q, r, s} are not dominated by any vertices of S, Which can't occur. Therefore $\gamma(G)$=4.

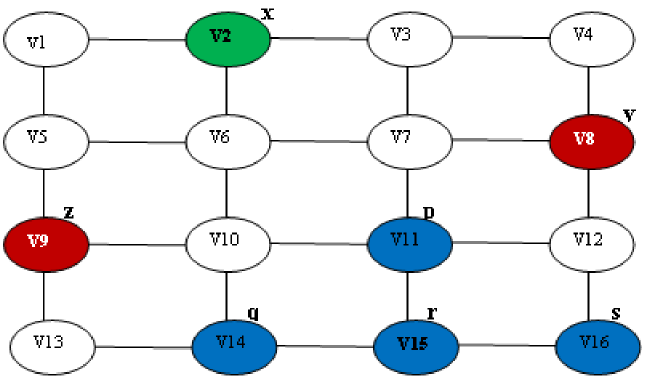

Figure-6: $P_4 \times P_4$

Theorem 2: An independence set is also a dominating set if and only if it is maximal. Thus I(G)≥D(G).

Proof: this follows directly from the definitions. Any vertex that is not in a maximal independent set is dominated by at least one vertex in the set, hence a maximal independent set is also a dominating set. Conversely, an independent set that is also a dominating set has to be because any vertex not in the set is dominated by at least one vertex in the set [26].

1. Complexity

The following decision problem for the domination number of a graph is known to be NP-Complete, even when restricted to bipartite graphs (see Dewdney [9]) or chordal graphs (see Booth [9] and Booth and Johnson [10]).

DOMINATION SET (DM)

INSTANCE: A graph G and a positive integer k $\leq |V(G)|$

QUESTION: Does G have a domination set of cardinality or less?

We will demonstrate a polynomial time reduction of this problem to our k-Game dominating problem,

k-Game Dominating Number(kGDF)

INSTANCE: A graph H and a positive integer j $\leq |V(H)|$

Theorem 3: kGDN is NP-Complete, even when restricted to bipartite or Chordal Graph.

Proof: It is obvious that kGDN is a member of NP since we can, in polynomial time, guess at a subset (H)→{ 0, 1, . . . k} and verify that H has weight at most j and is a kGDN. We next show how a polynomial time algorithm for kGDN could be used to solve DM in polynomial time. Given a graph G and positive integer construct the graph H by adding for each vertex V of G a star K1; k+1 joining v to an end vertex of this star and then subdividing the resulting edge once. Note that if k=1, then we have added a path of length 4 to v. it is easy to see that the construction of the graph H can be accomplished in polynomial time. Note that if G is a bipartite or chordal graph, then so too is H. [12]

2. Applications to Mathematics and other Areas

The EasyMove (Sarasu Mane) Game problem is really a puzzle but, surprisingly, there are some practical applications such as parallel memory storage schemes, traffic control, and deadlock prevention. The problem is also illustrative of more practical problems whose solutions are permutations, of which there are many. One such problem is the travelling salesman problem.

Problem: Find the shortest route on a map that visits each city exactly once and returns to the starting city. Here the permutation is the list of cities that are visited, except for the first.

Furthermore, the solution technique, backtracking is very important, and is best learnt on simple examples such as the EasyMove (Sarasu Mane) Game problem.

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