International Journal of Mathematical Archive-9(3), 2018, 163-167
IJMA Available online Through www.ijma.info ISSN 2229 – 5046

CYCLIC PATH COVERS IN ZERO DIVISOR GRAPH

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(Received On: 05-01-18, Revised & Accepted On: 14-02-18)

ABSTRACT

Let R be a commutative ring and let $\Gamma(z_n)$ be the zero divisor graph of a commutative ring R, whose vertices are non-zero zero divisors of z_n, and such that the two vertices u,v are adjacent if n divides uv. In this paper, we have analyzed the maximum number of zero divisor graph $\Gamma(z_n)$, where n is a positive integer, by studying its properties and structure and thereby decomposing it into a finite number of paths and cycles, whose sum of the vertices in the cycle are equal where decomposition is a set of subgraphs $H_1,\ldots H_k$ that partition of the edges of G. That is for all i and j is $_{1\le i\le k} H_i = G$ and $E\ H_i \ \cap E\ H_j \ = \emptyset$.

***Keywords**: Commutative ring, Zero divisor graph, Decomposition, Cycles.*

1. INTRODUCTION

Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph Γ(R). to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of non-zero zero divisors of R and for distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent if and only if $xy = 0$ [1, 2, 5, 6]. Throughout this paper, Consider the commutative ring R as z_n. and zero divisor graph Γ (R) as $\Gamma(z_n)$. In this paper we are about to decompose [3, 4, 7, 8]. the zero divisor graph into paths and cycles whose sum of the vertices in the cycle are equal . Problems of this type are not only interesting in their own right, but also have potential applications in communication and switching networks. Sometimes it is desirable to decompose a communication or switching network into parts of certain specified types.

Definition 1.1: A graph G is decomposable into $H_1; H_2; : : : ;H_k$ if G has subgraphs $H_1; H_2; : : : ;H_k$ such that
1. each edge of G belongs to one of the H_i's for some i = 1; 2; : : : ; k; and
2. If $i \neq j$, then H_i and H_j have no edges in common.

Definition 1.2: Let G and H be two graphs. A graph G is decomposable into H's if each of the H_i's in the definition above is isomorphic to H.

2. DECOMPOSITION OF ZERO DIVISOR GRAPH

Theorem 2.1: For any $p > 3$, $\Gamma(z_{3p})$ can be decomposed into $(p-1)/2c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 5$,
The vertex set of $\Gamma(z_{15}) = \{3,5,6,9,10,12\}$. That is $|V(\Gamma(z_{15}))| = 6$. Let $u = 3$ and $v = 6$ then 15 does not divide uv. Then there is no path between u and v. Similarly, any two vertices which are multiple of 3 are non adjacent.

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Let $x = 5$ and $y = 15$ then 15 divides both ux and ux. Clearly, ux and uy are adjacent. Then $V(\Gamma(z_{15}))$ can be partitioned into two parts $v_1 = \{5,10\}$. and $v_2 = \{3,6,9,12\}$. Clearly, $\Gamma(z_{15})$ can be decomposed into two c_4. That is, {3, 5, 12, 10, 3} and {9, 5, 6, 10, 9}. Since, 5, 10 are the common points in both the cycles. This cycle is chosen in such a way that the total value of each cycle is $= 3 + 5 + 12 + 10 = 30 = 2(15) = 2(3p)$. Hence the number of $c_4 = 2 = (5-1)/2 = (p-1)/2$.

Case-(ii): When $p = 7$,
The vertex set of $\Gamma(z_{21}) = \{3,6,7,9,12,14,15,18\}$. That is $|V(\Gamma(z_{21}))| = 8$. Let $u = 3$ and $v = 12$ then 21 does not divide v . Clearly, uv are non adjacent. Let $u = 18$, $x = 7$ and $y = 14$, then 21 divides both ux and uy. That is 7, 14 are adjacent to all the remaining vertices. Then $V(\Gamma(z_{21}))$ can be partitioned into two parts $v_1 = \{7,14\}$ an $v_2 = \{3,6,9,12,15,18\}$. Clearly, $\Gamma(z_{21})$ can be decomposed into three c_4. That is cycles are {3, 7, 18, 14, 3}, {6, 7, 15, 14, 6} and {9, 7, 12, 14, 9}. Since, 7, 14 are the common points in all the cycles and these cycles are chosen in such a way that the total value of each cycle is 3 + 7 + 18 + 14 = 42 = 2(21) = 2(3p). The number of $c_4 = 3 = (7-1)/2 = (p-1)/2$. Hence, $\Gamma(z_{21})$ can be decomposed into $(p-1)/2c_4$, where $p = 7$.

Case-(iii): when $p > 7$,
In general, $\Gamma(z_{3p}) = \{3,6,\ldots,3\ p-1\ ,p,2p\}$. that is $|V(\Gamma((z_{3p}))| = p+1$. Using the above two cases $\Gamma(z_{3p})$ can be decomposed into $(p-1)/2c_4$. then the number of c_4. in $k_{2,p-1}$ is $(p-1)/2c_4$.

Theorem 2.2: For any $p > 5$, then $\Gamma(z_{5p})$ can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 7$,
The vertex set of $\Gamma\ z_{35}\ =\ 5,7,10,14,15,20,21,25,28,30$. That is $V\ \Gamma\ z_{35}\ = 10$. Let $u = 5$ and $v = 20$ then 35 does not divide 100. Clearly u and v are non - adjacent. Let $u = 10$ and $X =\ 7,14,21,28$ then 35 divides ux for all $x \in X$. Clearly ux are adjacent. Then $V\ \Gamma\ z_{35}$ can be partitioned into two parts $V_1 =\ 7,14,21,28$ and $V_2 = \{5,10,15,20,25,30\}$. Clearly, $\Gamma\ z_{35}$ is $k_{4,6}$ and this $k_{4,6}$ can be decomposed into six c_4. That is, {5, 7, 30, 28, 5},{15, 7, 20, 28, 15}, {25, 7, 10, 28, 25}, {5, 14, 30, 21, 5}, {15, 14, 20, 21, 15}, {25, 14, 10, 21, 25}. Clearly, 7, 28 are the common points in three cycles and 17, 21 are the common points in the remaining three cycles. These cycles are chosen in such a way that the total value of each cycle is $= 5 + 7 + 30 + 28 = 70 = 2\ 35 = 2\ 7p$. Then number of $c_4 = 6 = \frac{7-1}{2} = \frac{p-1}{2}$. Similarly it can be decomposed into $\{5,7,30,28,5,14,30,21,5\}$, $\{15,7,20,28,15,14,20,21,15\}$, $\{25,7,10,28,25,14,10,21,25\}$ with the total value of each cycle is $= 5 + 7 + 30 + 28 + 5 + 14 + 30 + 21 = 140 = 4(35) = 4(5p)$. Then the number of C_8 is $3 = (7-1)/2 = (p-1)/2$. Therefore, Γ (Z35) can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Case-(ii): When $p = 11$.
The vertex set of $\Gamma(z_{55}) = \{5,10,15..,50,11,..,44\}$. That is $|V(\Gamma(z_{35}))| = 14$. Let $u = 5$ and $v = 30$ then 55 does not divide 150. That is, u and v are non-adjacent. Let $u = 30$ and $X = \{11,22,33,44\}$ then 55 divides ux for all $x \in X$. Clearly ux are adjacent. Then $V(\Gamma(z_{55}))$ can be partitioned into two parts $V_1 = \{11, 22, 33, 44\}$ and $V_2 = \{5,10,15,20,25,30,35,40,45,50\}$. Clearly, $\Gamma(Z_{55})$ is $k_{4,10}$ and this $k_{4,10}$ can be decomposed into ten c_4. That is, {5, 11, 50, 44, 5}, {10, 11, 45, 44, 10}, {15, 11, 40, 44, 15}, {20, 11, 35, 44, 20}, {25, 11, 30, 44, 25}, {5, 22, 50, 33, 5}, {10, 22, 45, 33, 10}, {15, 22, 40, 33, 15}, {20, 22, 35, 33, 20},{25, 22, 30, 33, 25}. Clearly, 11, 44 are the common points in five cycles and 22, 33 are the common points in the remaining five cycles. These cycles are choosen in such a way that the total value of each cycle is $5 + 11 + 50 + 44 = 110 = 2(55) = 2(5p)$.Then number of $c_4 = 10 = (11-1)/2 = (p-1)/2$ or it can be decomposed into {5, 11, 50, 44, 5, 22, 50, 33, 5}, {10, 11, 45, 44, 10, 22, 45, 33, 10}, {15, 11, 40, 44, 15, 22, 40, 33, 15}, {25, 11, 30, 44, 25, 22, 30, 33, 25}, {20, 11, 35, 44, 20, 22, 35, 33, 20} with the total value of each cycle is $= 5 + 11 + 50 + 44 + 5 + 22 + 50 + 33 = 220 = 4(55) = 4(5p)$. Then the number of C_8 is $5 = (11-1)/2 = (p-1)/2$ Therefore, $\Gamma(Z_{55})$ can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Case-(iii): When $p > 11$
In general, $\Gamma(z_{5p}) = \{5,10,\ldots,5(p-1),p,2p,3p,4p\}$. That is $|V(\Gamma(z_{5p}))| = p+3$. Using the above two cases the $V(\Gamma(z_{5p}))$can be partitioned into two parts $V_1 = \{p,2p,3p,4p\}$ and $V_2 = \{5,10,\ldots,5(p-1)\}$. Clearly, $\Gamma(z_{5p})$ is $k_{4,p-1}$ with the common points $\{p,2p,3p,4p\}$ which is adjacent to all the remaing vertices. Then the number of cycles in $k_{4,p-1}$ is $(p-1)/2\ c_8$ or $(p-1)c_4$.

Theorem 2.3: For any $p > 7$, $\Gamma(z_{7p})$ can be decomposed into $(3(p-1)/2)c_4$or $(p-1)c_6$

Proof: The proof is based on induction over p.

Case-(i): When $p = 11$,
The vertex set of Γ(z_77)= $\{ 7, 14, 21, .., 70, 11, .., 66\}$. That is $|V(\Gamma \ z_{77})| = 16$. Let $u = 7$ and $v = 35$ then 77 does not divide 245. Clearly u and v are nonadjacent. Let $u = 21$ and $X = 11, 22, 33, 44, 55, 66$ then 77 divides ux for all x ϵ X. Clearly ux are adjacent. Then V Γ z_{77} can be partitioned into two parts $V_1 = 11, 22, 33, 44, 55, 66$ and $V_2 = 7, 14, 21, 28, 35, 42, 49, 56, 63, 70$. Clearly, Γ z_{77} is $k_{4,10}$ and this $k_{4,10}$ can be decomposed into fifteen c_4. That is,{7, 11, 70, 66, 7}, {14, 11, 63, 66, 14}, {21, 11, 56, 66, 21}, {28, 11, 49, 66, 28}, {35, 11, 42, 66, 35}, {7, 22, 70, 55, 7},{14, 22, 63, 55, 14}, {21, 22, 56, 55, 21},{28, 22, 49, 55, 28} ,{35, 22, 42, 55, 35}, {7, 33, 70, 44, 7}, {14, 33, 63, 44, 14}, {21,33, 56, 44, 21} {7, 33,70, 44,7} {14, 33, 63, 44, 14} {21, 33, 56, 44, 21}{28, 33, 49, 44, 28} and $\{35, 33, 42, 44, 35\}$. Clearly, V_1 are the common vertices in the cycles and these cycles are chosen in such a way that the total value of each cycle is $7 + 11 + 70 + 66 = 154 = 2(77) = 2(7p)$. Then number of $c_4 = 15 = 3(11 - 1)/2 = 3(p - 1)/2$ or $k_{4,10}$ can be decomposed into $10c_6$, That is, {7, 11, 70, 66, 7, 22, 70, 55, 7}, {14, 11, 63, 66, 14, 22, 63, 55, 14}, {21, 11, 56, 66, 21, 22, 56, 55, 21},{28, 11, 49, 66, 28,22, 49, 55, 28}{35, 11, 42, 66, 35,22, 42,55, 35}, {7, 11, 70, 66, 7, 33, 70, 44, 7}, {14, 11, 63, 66, 14, 33, 63, 44, 14}, {21, 11, 56, 66, 21, 22, 56, 55, 21}{28, 11, 49, 66, 28,33, 49, 44, 28} ,{35, 11, 42, 66, 35, 33, 42, 44, 35},.Then the number of c_6 is 10 = 11 − 1 = p – 1 Therefore, $\Gamma(z_{77})$ can be decomposed into $(3(p - 1)/2)c_4$or $(p - 1)c_6$.

Case-(ii): When $p = 13$.
The vertex set of □(z_{91}) = $\{7, 14, .., 84, 13, 26, ..78\}$ That is $|V(\Gamma(z_{91}))| = 18$. Let $u = 7$ and $v = 35$ then 91 does not divide 245. Clearly u and v are non-adjacent. Let $u = 21$ and $X = \{ 13, 26, 39, 52, 65, 78 \}$ then 91 divides ux for all x ϵ X. Clearly ux are adjacent for all x ϵ X. Then V Γ (z_{91}) can be partitioned into two parts V_1 = {13, 26, 39, 52, 65, 78} and V_2 ={7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84}. Clearly, $\Gamma(z_{91})$ is $k_{4,12}$ and this $k_{4,12}$ can be decomposed into eighteen c_4.

That is, {7, 13, 84, 78, 7}, {14,13,77,78,7}, {21,13,70,78,21}, {28,13,63,78,28}, {35,13,56,78,35},{42, 13, 49, 78, 42}, {7, 26, 84, 65, 7}, {14,26,77,65,7}, {21,26,70,65,21}, {28,26,63,65,28}, {35,26,56,65,35},{42, 26, 49, 65, 42},{7, 39, 84, 52, 7}, {14,32,77,52,7}, {21,39,70,52,21},{28,39,63,52,28}, {35,39,56,52,35},{42, 39, 49, 52, 42}. Clearly, V_1 are the common vertices in the cycles and these cycles are chosen in such a way that the total value of each cycle is $= 7 + 13 + 84 + 78 = 185 = 2(91) = 2(7p)$. Then number of $c_4 = 18 = 3(13 - 1)/2 = 3(p - 1)/2$ or $k_{4,12}$ can be decomposed into $12c_6$, That is, $\{7, 13, 84, 78, 7, 26, 84, 65, 7\}$, {14,13,77,78,7,26,77,65,7}, {21,13,70,78,21,26,70, 65, 21}, {28,13,63,78,28,26,63,65,28}, {35,13,56,78,35,39,56,52,35}, {42,13,49,78,42,39,49,52} $\{7, 13, 84, 78, 7, 39, 84, 52, 7\}$, {14,13,77,78,7,39,77,52,7}, {21,13,70,78,21,39,70,52,21}, {28,13,63,78,28,39,63,52,28},{35,13,56,78,35,39, 56,52,35} ,{42,13,49,78,42,39,49,52,42}. Then the number of c_6 is $12 = 13 - 1 = p - 1$ Therefore, $\Gamma(z_{91})$ can be decomposed into $(3(p - 1)/2)c_4$or $(p - 1)c_6$

Case-(iii): When $p > 13$.
In general, $\Gamma(z_{7p}) = \{7, 14, \ldots, 7(p - 1), p, 2p, .., 6p\}$. That is $|V(\Gamma(z_{7p})| = p + 5$. Using the above two cases the $\Gamma(z_{7p})$ can be partitioned into two parts $V_1 = \{ p, 2p, 3p, 4p, 5p, 6p \}$ and $V_2 = \{77, 14, \ldots, 7(p - 1)\}$. Clearly, $\Gamma(z_{7p})$ is $k_{6,p-1}$ with the common points $\{p, 2p, 3p, 4p, 5p, 6p\}$ which is adjacent to all the remaining vertices. Then the number of cycles in $k_{6,p-1}$is $(3(p - 1)/2)c_4$or $(p - 1)c_6$

Theorem 2.4: FOR any distinct prime p and q, $\Gamma(z_{pq})$ can be decomposable into (q − 1) Cp−1, where $q > p$.

Proof: The vertex set of $\Gamma(z_{pq})$ is $\{p, 2p, 3p, ..(p - 1)p, q, 2q, ..(p - 1)q\}$. That is $|V(\Gamma(z_{pq}))| = p + q - 2$. Using the above theorem $\Gamma(z_{pq})$ is $k_{p-1,q-1}$ for $p > 3$, with the common points $\{q, 2q, ..(p - 1)q\}$ which is adjacent to all the remaining vertices. Therefore the number of cycles in $k_{p-1,q-1}$ is $(q - 1)c_{p-1}$.

Theorem 2.5: For any prime $p > 4$, $\Gamma(z_{4p})$ can be decomposable into $k_{1,2(p-1)}$ and $k_{2,p-1}$ or $(p - 1)/2c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 5$,
The vertex set of $\Gamma(z_{20})$ = {2, 4, 6, ..18, 5, 10, 15}. That is $|V(\Gamma(z_{20}))| = 11$. Let u = 4 and v = 10 then 20 divides uv. Clearly u and v are adjacent. Let $X = \{5, 15\}$ and $v = 10$ then 20 divides vxfor all x ϵ X. Clearly ux are non-adjacent. Then first $\Gamma(z_{20})$is decomposed into $k_{1,8}$. Consider $u = 4$ and $v = 5$ then 20 divides uv. That is u and v are adjacent. Then the remaining vertices can be partitioned into two parts V_1 = {4, 8, 12, 16} and V2 = {5, 15} which are adjacent to each other. Secondly, $\Gamma(z_{20})$is further decomposed into $k_{2,4}$ Futher it can be decomposed into two cycles of c_4, that is {4, 5, 16, 15, 4} and {8, 5, 12, 15, 8}. These cycles are chosen in such a way that the total value of each cycle is $40 = 2(20) = 2(5p)$. Finally, this implies $\Gamma(z_{20})$can be completely decomposed into $k_{1,8}$ and $k_{2,4}$ or $2c_4$.

Case-(ii): When p = 7.
The vertex set of $\Gamma(z_{28}) = \{2, 4, 6, ..26, 7, 14, 21\}$. That is $|V(\Gamma(z_{28})| = 15$ Let $u = 6$ and $v = 14$ then 28 divides uv. Clearly u and v are adjacent. Let $X = \{7, 21\}$ and $v = 14$ then 24 divides vx. Clearly ux are non-adjacent for all $x \in X$. Then first $\Gamma(z_{28})$is decomposed into $k_{1,12}$. Consider $u = 8$ and $v = 7$ then 24 divides uv. That is u and v are adjacent. Then the remaining vertices can be partitioned into two parts $V_1 = \{4, 8, 12, 16, 20, 24\}$ and $V_2 = \{7, 21\}$ which are adjacent to each other. Secondly, $\Gamma(z_{28})$ is further decomposed into $k_{2,6}$ which is further decomposed into three cycles of c_4, that is {4, 7, 24, 14, 4} ,{8, 7, 20, 14, 8} and {12, 7, 16, 14, 12}. These cycles are chosen in such a way that the total value of each cycle is $56 = 2(28) = 2(4p)$. Finally, this implies $\Gamma\ z_{28}$ can be completely decomposed into $k_{1,12}$ and $k_{2,6}$ or $3c_4$.

Case-(iii): When $p > 7$.
In general, $V((z_{4p})) = \{2, 4, 6\ldots 2(p-1), p, 2p, 3p\}$. That is$|V(\Gamma(z_{4p})| = 2p+1$. Using the above cases $\Gamma(z_{4p})$can be decomposed into $k_{1,2(p-1)}$ and $k_{2,p-1}$ which can be further decomposed into $(p-1)/2$ cycles of length 4 that is $(p-1)/2\ c_4$.

Theorem 2.6: For any prime $p > 6$, $\Gamma(z_{6p})$can be decomposed into $k_{1,3(p-1)}$, $k_{2,p-1}$ and $k_{2,2(p-1)}$ or $3(p-1)/2\ c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 7$.
The vertex set of $\Gamma(z_{42}) = \{2, 3, 4, ...40, 7, 14, 21, 28, 30\}$. That is $|V(\Gamma(z_{42})| = 29$ Let $u = 21$ and let X be the set consisting of multiple of 2 other than multiple of 3. Then ux is divided by the 42.That is ux is adjacent. Then, $\Gamma(z_{42})$ is first decomposed into $k_{1,20}$ From the remaining vertices consider the set $V_1 = \{14, 28\}$ and $V_2 = \{3,9,\ldots,39,6,12,\ldots 36\}$. Clearly any vertex of V_1 is adjacent to any vertex of V_2. Then, $\Gamma(z_{42})$ is decomposed into $k_{2,12}$. Further it can be decomposed into six cycles of c_4. That is {14,3,28,39,14}, {14,9,28,33,14}, {14,15,28,27,14}, {14,6,28,36,14}, {14,12,28,30,14},{14,18,28,24,14}. These cycles are chosen in such a way that the sum of the vertices is 98 .Finally the remaining vertices can be partitioned into $V_3 = \{7, 35\}$ and $V_4 = \{6,12,\ldots 36\}$. Clearly any vertex of V_3 is adjacent to any vertex of V_4. Then, $\Gamma(z_{42})$ is decomposed into $k_{2,6}$. Further it can be decomposed into three cycles of c_4. That is {7,6,35,36,7},{7,12,35,30,7},{7,18,35,24,7}. These vertices is chosen in such a way that the sum of the vertices is 91. Finally, this implies $\Gamma\ z_{42}$ can be completely decomposed into $k_{1,20}$, $6c_4$ and $3c_4$. That is $k_{1,20}$, $9c_4$.

Case-(ii): When $p = 11$.
The vertex set of $\Gamma(z_{77}) = \{2, 3, 4, ...60, 11,22,33,44,55\}$. That is $|V(\Gamma(z_{77})| = 45$. Let $u = 33$ and let X be the set consisting of multiple of 2 other than multiple of 3. Then ux is divided by the 77. That is ux is adjacent.Then , $\Gamma(z_{42})$ is first decomposed into $k_{1,32}$. From the remaining vertices consider the set $V_1=\{22, 44\}$ and $V_2=\{3, 9,\ldots,63,6,12,\ldots 60\}$. Clearly any vertex of V_1 is adjacent to any vertex of V_2. Then, $\Gamma(z_{42})$ is decomposed into $k_{2,20}$. Further it can be decomposed into ten cycles of c_4. That is {22,3,44,63,22}, {22,9,44,57,22}, {22,15,44,51,22}, {22,21,44,45,22}, {22,27,44,39,22},{22,6,44,60,22},{22,12,44,54,22},{22,18,44,48,22},{22,24,44,42,22},{22,30,44,46,22}.These cycles are chosen in such a way that the sum of the vertices is $154 = 2(77) = 2(6p)$.Finally the remaining vertices can be partitioned into $V_3 = \{11, 55\}$ and $V_4 = \{6,12,\ldots 60\}$. Clearly any vertex of V_3 is adjacent to any vertex of V_4. Then , $\Gamma(z_{42})$ is decomposed into $k_{2,6}$. Further it can be decomposed into three cycles of c_4. That is {11,6,55,60,11}, {11,12,55,54,11},{11,18,55,48,11},{11,24,55,42,11},{11,30,55,36,11}. These vertices are chosen in such a way that the sum of the vertices is 143. Finally, this implies $\Gamma\ z_{42}$ can be completely decomposed into $k_{1,32}$, $10c_4$ and $5c_4$. That is $k_{1,32}$, $15c_4$.

Case-(iii): When $p > 11$.
In general, $V((z_{6p})) = \{2, 4, 6\ldots 2(3p-1), 3, 6 \ldots .3(2p-1), p, 2p, 3p, 4p, 5p\}$. That is $|V(\Gamma(z_{4p})| = 4p+1$. Using the above cases $\Gamma(z_{6p})$can be decomposed into $k_{1,3p-1}$ $k_{2,2(p-1)}$ and $k_{2,p-1}$ which can be further decomposed into $(p-1)/2$ cycles and $(p-1)$ cycles of length 4 that is $\frac{p-1}{2} + p - 1$ of c_4. That is $3(p-1)/2\ c_4$.

Theorem 2.7: For any prime p, $\Gamma\ z_{p^2}$ is not decomposable into Hamilton cycles.

Proof: The vertex set of $\Gamma\ z_{p^2}$ is $\{p, 2p, 3p, ..(p-1)p\}$. Clearly p is adjacent to all the vertices in $V(\Gamma(z_{p^2}))$. Also note that any two vertices in $\Gamma(z_{p^2})$ is adjacent and hence $\Gamma(z_{p^2})$is a complete graph. Clearly each vertices of $\Gamma(z_{p^2})$has degree $p - 2$. Let q be the number of edges of $\Gamma(z_{p^2})$ then $q = (p-1)(p-2)/2$. Suppose $\Gamma(z_{p^2})$is decomposable into Hamilton cycles C_{p-1}. Since each C_{p-1} has $p-1$ edges. So the number of such cycles in the decomposition must be $q(p-1) = (p-1)(p-2)/2(p-1) = (p-2)/2 = p/2 - 1$, which is not an integer for any prime p. Hence this is impossible so $\Gamma(z_{p^2})$is not decomposable into Hamilton cycles.

Theorem 2.8: For any prime $p > 2$, $\Gamma(z_{p^2})$ is decomposable into $(p-1)/2$ Hamilton paths P_{p-1}

Proof: Using the above theorem, $\Gamma(z_{p^2})$ is a complete graph, where p is any prime and $|V(\Gamma(z_{p^2})| = p - 1$. Since, for any $n \geq 1$, k_{2n+1} is decomposable into Hamilton cycles. Consider $\Gamma(z_{7^2})$, the vertices set of $\Gamma(z_{7^2})$, is {7, 14, 21, 28, 35, 42} that is $\{p, 2p, 3p, .. (p-1)p\}$ where $p = 7$. Label the vertices x_1, x_2, x_3, x_4, x_5, x_6 and form the Hamilton path as follows
Path 1: $x_1, x_5, x_3, x_6, x_4, x_2$ (P_6)
Path 2: $x_3, x_1, x_2, x_6, x_5, x_4$ (P_6)
Path 3: $x_6, x_1, x_4, x_3, x_2, x_5$ (P_6)

Hence $\Gamma(z_{p^2})$ is decomposable into $(p-1)/2$ Hamilton path P_{p-1}

For example in $\Gamma(z_{5^2})$, the number of vertices are $\{5, 10, 15, 20\}$.

For $p = 5$, $p - 1/2 = 5 - 1/2 = 4/2 = 2$ Hamilton path P_4

3. CONCLUSION

In the zero divisor graph $\Gamma\ z_n$ where n is any positive integer, the number of cycles of c_4 that has been decomposed for $n = 3p, 4p, 5p, 6p, 7p$ forms an inequality $o(D(\Gamma(z_{3p}))) \leq o(D(\Gamma(z_{4p}))) < o(D(\Gamma(z_{5p}))) < o(D(\Gamma(z_{6p}))) \leq o(D(\Gamma(z_{7p})))$ where $o(D(\Gamma(z_n)))$ is known as the number of cycles of c_4, while the complete graph $\Gamma(z_{p^2})$ is only decomposable into Hamilton path P_{p-1} but not has Hamilton cycles.

REFERENCES

1. D.F. Anderson and P.S. Livingston, The zero-divisor graph of a commutative ring, J. Algebra, 217, (1999), No-2, 434 - 447.
2. K. Ananthi, J. Ravi Sankar and N. Selvi, *Bipartite Universal Domination of Zero Divisor Graph, Applied Mathematical Science, Vol.* 9, 2015, NO. 51, 2525
3. D. Bryant and C. A. Rodger, Cycle decompositions, in The CRC Handbook of Combinatorial Designs, 2nd edition (Eds. C. J. Colbourn, J. H. Dinitz), CRC Press, Boca Raton (2007) 373–382.
4. B. W. Jackson, Some cycle decompositions of complete graphs, J. Combin. Inform. System Sci., 13 (1988) 20–32.
5. J. Ravi Sankar and S. Meena, *Connected Domination Number of a Commutative Ring, International Journal of Mathematical Research* 5, (2012), NO-1, 5 - 11.
6. J. Ravi Sankar and S. Meena, *Changing and Unchanging the Domination Number of a Commutative Ring, International Journal of Algebra*, 6,(2012), NO-27, 1343 - 1352.
7. A. Rosa, On cyclic decompositions of the complete graph into 4m + 2-gons, Mat.-Fyz. Cas., 16 (1966), 349–352.
8. B. Alspach and B. N. Varma, Decomposing complete graphs into cycles of length 2pe, Ann. Discrete Math., 9 (1980) 155–162.

Source of support: Nil, Conflict of interest: None Declared.