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EXTENDED VERTEX EDGE EQUITABLE LABELING (EVEEL) A NEW METHOD OF GRAPH LABELING

MUKUND BAPAT*

Hindale, Devgad, Sindhudurg, Maharashtra–India, 416630.

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ABSTRACT

We define a new type of labeling called as Extended Vertex Edge Equitable Labeling (eveel) of graphs. We allow the vertices to take values from {0,1,-1} and obtain the edge label as the absolute difference between the labels of the incident vertices. We show that $K_{1,n}$, Cn is eveel except for n≡3 (mod 6) and non isomorphic structures of one point union of Flag of C_3 are evee graphs.

***Key words**: vertex labeling, vertex, extended vertex, equitable labeling, edge labeling, graph, edge.*

***Subject Classification:** 5C78.*

1. INTRODUCTION

The graphs considered here are finite, simple and connected. I.Cahit first proposed equitable labeling in 1990 by distributing vertices and edges evenly. We attempt to distribute vertices among 0, 1, -1 and edges among 0, 1, 2. Define f: V(G)→{0, -1, 1} such that the edge e = (uv) ϵ E(G) then f(e) = |f(u)-f(v)| (mod 3). Further the condition to be satisfied is that $|e_f(i) - e_f(j)| \leq 1$, i, j ϵ{0, 1, 2} and $|v_f(a) - v_f(b)| \leq 1$, a ≠ b, a, b ϵ{0, 1, -1}. This condition is called as pairity condition. $e_f(0)$ and $e_f(1)$, $e_f(2)$ stands for number of edges labeled with 0, 1 and 2 respectively. $v_f(a)$ is the number of vertices with label a, aϵ{0, 1, -1}. As usual $e_f(0, 1, 2)$ = (a, b, c) will stand for number of edges labeled with 0 are a and that with 1 are b and with 2 are c in numbere. $v_f(0, 1, -1)$ = (a, b, c) indicates the the number of vertices labeled with 0 are a, that with 1 are b and with -1 are c in number. Without using the digit 2 in labeling of vertices we are getting edges with label 2 and satisfies pairity condition. The graph that admits evee labeling is called evee graph, the function f as evee function.

2. RESULTS PROVED

Result 2.1: $K_{1,n}$ is eveel graph.

Proof: The pendent vertices be $u_1, u_2,...,u_n$ and the central vertex be u.f :V(G)→{-1, 0, 1} is given by f(u) = 1.

Further,
Case-i: n ≡ 0 (mod 3). Let n = 3x. then label x vertices with 0, other x vertices with 1 and remaining x vertices with -1. We have number distribution given by $e_f(0,1,-1)$ = (x, x, x) and $v_f(0, 1, -1)$ = (x, x+1, x).

Case-ii: n ≡ 1(mod 3). Let n = 3x+1. For 3x pendent vertices follow the function as in case 1. The remaining vertex can be labeled as 0. We have number distribution given by $e_f(0, 1, -1)$ = (x, x+1, x) and $v_f(0, 1, -1)$ = (x+1, x+1, x).

Case-iii: n ≡2(mod 3). Let n = 2+3x. For 3x pendent vertices follow the function as in case 1. The two vertices must be labeled with 0 and -1. We have number distribution given by $e_f(0, 1, -1)$ = (x, x+1, x+1) and $v_f(0, 1, -1)$ = (x+1, x+1, x+1).

*Corresponding Author: Mukund Bapat**
Hindale, Devgad, Sindhudurg, Maharashtra–India, 416630.

Result 2.2: One point union of k copies of C_3 i.e. $(C_3)^{(k)}$ is eveel graph iff k is a even number.

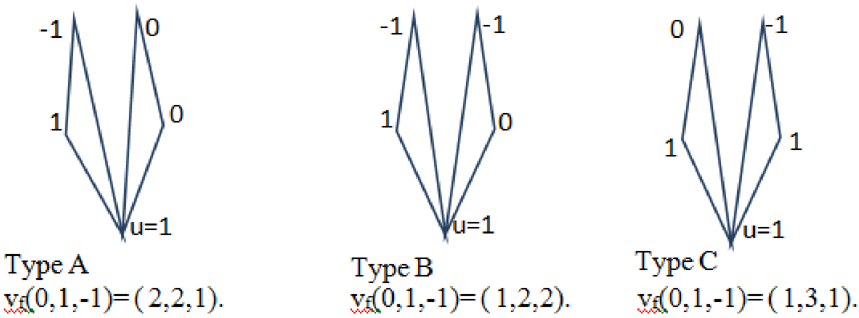

Figure-2.1: Labeled copies, Type C is not evel. In each case. $e_f(0, 1, -1) = (2, 2, 2)$

k	number of times particular labeling is used			$v_f(0,1,-1)$	Remarks
	Type A	Type B	Type C		
2	1	0	0	2,2,1	It is evee labeling
4	1	1	0	3,3,3,	
6	1	1	1	4,5,4	
2+6t	t+1	t	t	4t+2,4t+2,4t+1	
4+6t	t+1	t+1	t	4t+3,4t+3,4t+3	
6t	t	t	t	4t+3,4t+4,4t+3	

Table-2.1: For even value of k the graph $(C_3)^{(k)}$ is eadc.At any stage $e_f(0,1,-1) = (k, k, k)$

Result 2.3: Cn is eveel.except for n≡3, i≡1(mod6).

Proof: We have considered cases n ≡ 0, 4, 5 (mod 6) and since C_3 is not eadel and there is no cycle on 1 or 2 vertices we consider n ≡ 7, 8(mod 6). We first obtain th elabeling of Cn for

Case: n≡4 (mod 6). Taken = 4 + 6x, x=0, 1, 2....The ordinary labels of C_{4+6x} is given by $v_1, v_2, v_3, v_4, u_1, u_2, ...u_6, u_{6+1}.....u_{6x}$. Define a function f: V(G)→{0, 1, -1} as follows: $f(v_1) = 0$, $f(v_2) = -1$, $f(v_3) = 1$, $f(v_4) = 0$. Further $f(u_i) = 0$ for i≡1(mod6), $f(u_i) = -1$ for i ≡2(mod 6), $f(u_i) = 1$ for i≡3(mod 6), $f(u_i) = 1$ for i≡4(mod 6), $f(u_i) = -1$ for i ≡5(mod 6), $f(u_i) = 0$ for i ≡ 0(mod 6). The number distribution is $e_f(0, 1, 2) = (2x+1, 2x+2, 2x+1)$, $v_f(0, 1, -1) = (2x+2, 2x+1, 2x+1)$

Case: n≡5 (mod 4). Taken = 5+ 6x, x=0, 1, 2....The ordinary labels of C_{5+6x} is given by $v_1, v_2, v_3, v_4, v_5, u_1, u_2, ...u_6, u_{6+1}.....u_{6x}$. Define a function f: V(G)→{0, 1, -1} as follows: $f(v_1) = 0$, $f(v_2) = -1$, $f(v_3) = 1$, $f(v_4) = 1$, $f(v_5) = 0$. Further $f(u_i) = 0$ for i≡1(mod6), $f(u_i) = -1$ for i≡2(mod 6), $f(u_i)=1$ for i≡3(mod 6), $f(u_i) = 1$ for i≡4(mod 6), $f(u_i) = -1$ for i ≡5(mod 6), $f(u_i) = 0$ for i ≡ 0(mod 6). The number distribution is $e_f(0, 1, 2) = (2x+2, 2x+2, 2x+1)$, $v_f(0, 1, -1) = (2x+2, 2x+2, 2x+1)$

Case: n≡0 (mod 6).Taken = 6x, x=1, 2....The ordinary labels of C_{6x} is given by $u_1, u_2, ...u_6, u_{6+1}.....u_{6x}$. Define a function f:V(G)→{0,1,-1} as follows: $f(u_i) = 0$ for i≡1(mod6), $f(u_i)=-1$for i≡2(mod 6), $f(u_i)=1$ for i≡3(mod 6), $f(u_i) = 1$ for i≡4(mod 6),$f(u_i) = -1$ for i ≡5(mod 6), $f(u_i) = 0$ for i ≡ 0 (mod 6). The number distribution is $e_f(0, 1, 2) = (2x, 2x, 2x)$, $v_f(0, 1, -1) = (2x, 2x, 2x)$.

Case: n≡7 (mod 4). Taken = 7+ 6x, x = 0, 1, 2....The ordinary labels of C_{7+6x} is given by $v_1, v_2, v_3, v_4, v_5, v_6, v_7, u_1, u_2, ...u_6, u_{6+1}....u_{6x}$. Define a function f:V(G)→{0, 1, -1} as follows: $f(v_1) = 0$, $f(v_2) = -1$, $f(v_3) = 1$, $f(v_4) =1$, $f(v_5) = 1$, $f(v_6) = -1$, $f(v_7) = 0$. Further $f(u_i) = 0$ for i≡1(mod6), $f(u_i) = -1$ for i≡ 2(mod 6), $f(u_i) =1$ for i≡3(mod 6),$f(u_i) =1$ for i≡4(mod 6), $f(u_i) = -1$ for i ≡5(mod 6), $f(u_i) = 0$ for i ≡ 0(mod 6). The number distribution is $e_f(0, 1, 2) = (2x+3, 2x+2, 2x+3)$, $v_f(0, 1, -1) = (2x+2, 2x+3, 2x+2)$.

Case: n≡8 (mod 4). Taken = 8 + 6x, x = 0, 1, 2....The ordinary labels of C_{8+6x} is given by $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, u_1, u_2, ...u_6, u_{6+1}.....u_{6x}$. Define a function f:V(G)→{0, 1, -1} as follows: $f(v_1) = 0$, $f(v_2) = 0$, $f(v_3) = 1$, $f(v_4) = -1$, $f(v_5) =-1$, $f(v_6) =1$, $f(v_7) = 1$, $f(v_8) = -1$, Further $f(u_i) = 0$ for i≡1(mod6),$f(u_i) = -1$for i ≡2(mod 6), $f(u_i) = 1$ for i≡3(mod 6), $f(u_i) =1$ for i≡4(mod 6), $f(u_i) = -1$for i ≡5(mod 6), $f(u_i) = 0$ for i ≡ 0(mod 6).The number distribution is $e_f(0, 1, 2) = (2x+2, 2x+3, 2x+3)$, $v_f(0, 1, -1) = (2x+2, 2x+3, 2x+3)$

Result 2.4: The non isomorphic structures of one point union of Flag of C_3 are evee graphs.

Proof: There are three ways we can take non isomorphic structures of one point union on $FL(C_3)$. The fig 4.2 shows that we can take one point union at point x or point y or point u.

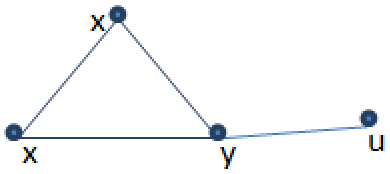

Figure-2.2: FL(C_3): Pair wise non-isomorphic One point union can be taken at x, y, u

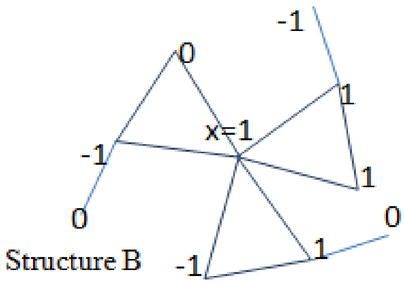

Figure-2.4: $v_f(0,1,-1) = (3,4,3,)$; $e_f(0,1,2) = (4,4,4)$

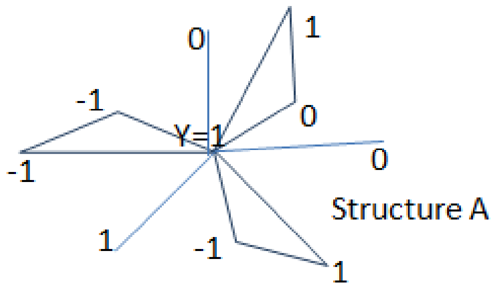

Figure-2.5: $v_f(0,1,-1) = (3,4,3,)$; $e_f(0,1,2) = (4,4,4)$

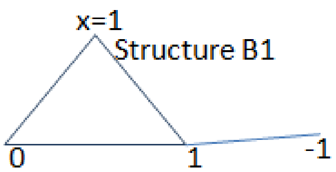

Figure-2.6: $v_f(0,1,-1) = (1,2,1)$;e $_f(0,1,2) = (1,2,1)$

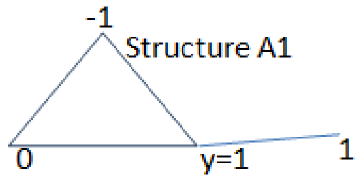

Figure-2.7: $v_f(0,1,-1) = (1,2,1)$; $e_f(0,1,2) = (1,2,1)$

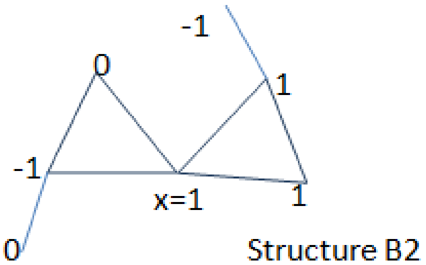

Figure-2.8: $v_f(0,1,-1) = (2,3,2)$; $e_f(0,1,2) = (3,3,2)$

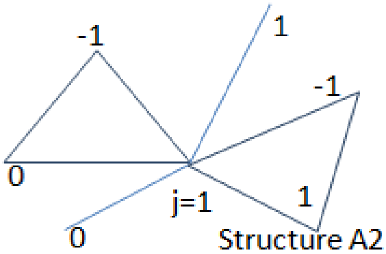

Figure-2.9: $v_f(0,1,-1) = (3,2,2)$; $e_f(0,1,2) = (2,3,3)$

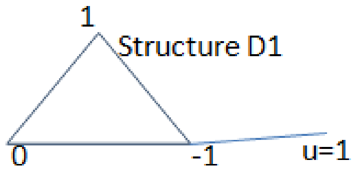

Figure-2.10: $v_f(0,1,-1) = (1,2,1)$; $e_f(0,1,2) = (1,2,1)$

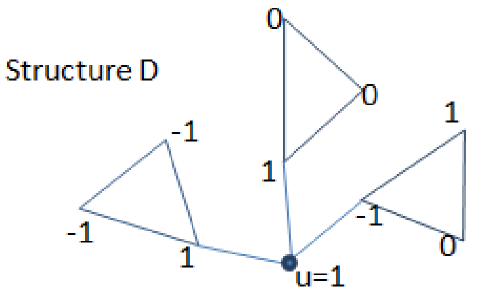

Figure-2.11: $v_f(0,1,-1) = (3,4,3,)$; $e_f(0,1,2) = (4,4,4)$

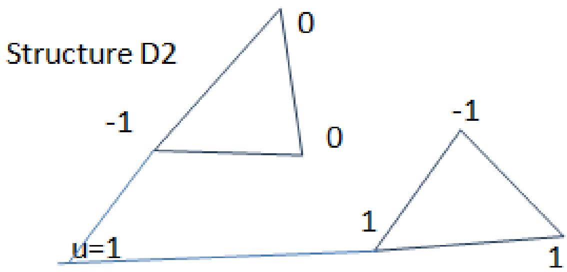

Figure-2.12: $v_f(0,1,-1) = (2,3,2)$; $e_f(0,1,2) = (3,2,3)$

The table below gives us a method to obtain three types of one point union of flag of C_3, yet all are eveel.

Structure Of type A .Point common to all copies is point y with label 1						
k	Number of copies used			Resultant vertex distribution	Resultant edge distribution	Remarks
	A	A1	A2	$v_f(0,1,-1)$	$e_f(0,1,2)$	
1	-	1	-	(1,2,1)	(1,2,1)	Graph is eveel
2	-	-	1	(3,2,2)	(2,3,3)	
3x	x	-	-	(3x,3x+1,3x)	(4x,4x,4x)	
3x+1	1+x	x	x	(3x+1,3x+2,3x+1)	(4x+1,4x+2,4x+1)	
3x+2	x	x	x+1	(3x+3,3x+2,3x+2)	(4x+2,4x+3,4x+3)	

Table-2.2: Shows How To Obtain Labeled Copy Of $(C_3)^{(k)}$ Structure Type A

Structure Of type B .Point common to all copies is point x with label 1						
k	Number of copies used			Resultant vertex distribution	Resultant edge distribution	Remarks
	B	B1	B2	$v_f(0,1,-1)$	$e_f(0,1,2)$	
1	-	1	-	(1,2,1)	(1,2,1)	Graph is eveel
2	-	-	1	(2,3,2)	(3,3,2)	
3x	x	-	-	(3x,3x+1,3x)	(4x,4x,4x)	
3x+1	1+x	x	x	(3x+1,3x+2,3x+1)	(4x+1,4x+2,4x+1)	
3x+2	x	x	x+1	(3x+2,3x+3,3x+2)	(4x+3,4x+3,4x+2)	

Table-2.3: Shows How To Obtain Labeled Copy Of $(C_3)^{(k)}$ Structure Type B

Structure Of type D .Point common to all copies is point u with label 1						
k	Number of copies used			Resultant vertex distribution	Resultant edge distribution	Remarks
	D	D1	D2	$v_f(0,1,-1)$	$e_f(0,1,2)$	
1	-	1	-	(1,2,1)	(1,2,1)	Graph is eveel
2	-	-	1	(2,3,2)	(3,2,3)	
3x	x	-	-	(3x,3x+1,3x)	(4x,4x,4x)	
3x+1	1+x	x	x	(3x+1,3x+2,3x+1)	(4x+1,4x+2,4x+1)	
3x+2	x	x	x+1	(3x+2,3x+3,3x+2)	(4x+3,4x+2,4x+3)	

Table-2.4: Shows How To Obtain Labeled Copy Of $(C_3)^{(k)}$ Structure Type D

CONCLUSIONS

The new labeling helps us understand how the three non isomorphic structures on $(C_3)^{(k)}$ are evee graphs. It is important to study what happens to non-isomorphic structures of $(C_n)^{(k)}$#

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