International Journal of Mathematical Archive-9(3), 2018, 142-147
JMA Available online through www.ijma.info ISSN 2229 – 5046

CONSTRUCTION OF SHEAVES OF ALGEBRAS VIA TOLERANCES

P. VAMSI SAGAR*
Research Scholar, Department of Mathematics,
Rayalaseema University, Kurnool, Andhra Pradesh, India.

M. P. K. KISHORE
Professor, G. V. P. College of Engineering (Autonomous),
Madhurawada, Visakhapatnam, Andhra Pradesh, India.

(Received On: 02-02-18; Revised & Accepted On: 05-03-18)

ABSTRACT

In this paper, a method to construct sheaf of Ω-algebras via compatible tolerance relations is presented. A necessary and sufficient condition for the existence of sheaf of Ω-algebras via tolerances is also observed. Further, using the method a sheaf is constructed over a bounded distributive lattice L such that L is isomorphic with the collection of all global sections.

Keywords: *Compatible tolerances, Sheaf of algebras, Bounded distributive lattice.*

Mathematics Subject Classification 2010:*16G99, 14F05, 06D99.*

I. INTRODUCTION

The theory of sheaves dates back to 1950's when Jean Leray first introduced the concepts. Later Grothendieck [10] gave a category theoretic approach to sheaves. Subsequently the sheaf concepts are developed by several mathematicians in different fields like Complex analysis and Partial Differential Equations. Recently applications of sheaf theory are also reported in the domain of Philosophy. Grant Malcolm [9] applied sheaves concepts in theoretical computer science. However, the Algebraic, Topological approach is developed by the works of researchers like Comer [22], Keimel [16], Hofmann [15], Swamy [23].

Swamy [23] and Wolf [1] independently developed a construction mechanism for sheaves of Universal algebras based on Chinese reminder theorem. The construction of global sheaves of algebras over Boolean spaces is done via congruences. The elements of the stalks are the congruence classes in the sheaves of algebras as per the construction given by Swamy [23], which in turn are generated from equivalence classes. However, in real life on a collection of objects establishing equivalence relations restricts the information and hence there is a need to relax them to tolerance relations as they inherits a great amount of information of the objects as well as easier to establish in several ways.

However, the tolerances are not that friendly to establish the algebraic structures when compared with equivalences. The algebraic theory of tolerances and compatible tolerances were studied by Chajda [11], Pogonowski [13] and Zelinka [5]. Recently the construction of sheaves of sets via tolerance relations is studied by M.P.K.Kishore [19] *et al.*, in which a construction mechanism based on tolerances is presented on the lines of construction given by Swamy [23] and established linkages with graphs.

In the present work, the construction of sheaves of Ω-algebras over compatible tolerances is studied and established a necessary and sufficient conditions for existence of global sheaves of algebras over compatible tolerances. The construction is applied on a Distributive lattice in which tolerances are constructed through Prime ideals.

2. PRELIMINARIES

To make the paper almost self-contained we recall some basic definitions that will be useful for out presentation. We start with the following.

Corresponding Author: P. Vamsi Sagar*,
Research Scholar, Department of Mathematics,
Rayalaseema University, Kurnool, Andhra Pradesh, India.

Definition 2.1: ***Lattice:*** An algebraic structure $(L, \vee, \wedge)$, consisting of a set L and two binary operations $\vee$ and $\wedge$ on L is a lattice if the following axiomatic identities hold for all elements a, b, c of L.

(i) Commutative laws: $a \vee b = b \vee a$, $a \wedge b = b \wedge a$

(ii) Associative laws: $a \vee (b \vee c) = (a \vee b) \vee c$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

(iii) Absorption laws: $a \vee (a \wedge b) = a$, $a \wedge (a \vee b) = a$

(iv) Idempotent laws: $a \vee a = a$, $a \wedge a = a$

Definition 2.2: *Bounded lattice:* A bounded lattice is an algebraic structure of the form $(L, \vee, \wedge, 0, 1)$ such that $(L, \vee, \wedge)$ is a lattice, 0 (the least element) is the identity element for the join operation $\vee$, and 1 (the greatest) is the identity element for the meet operation $\wedge$.

Identity laws: a $\vee$ 0 = 0 $\vee$ a = a, a $\wedge$ 1 = 1 $\wedge$ a = a.

Definition 2.3: *Distributive lattice:* A lattice L is distributive if for all elements a, b, c of L.

i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

Definition 2.4: *Tolerance relation:* A tolerance relation is a reflexive and symmetric relation on a set.

Definition 2.5: *Compatible tolerance:* Let (A, Ω) be an algebra. Let η be a tolerance on A. Then η is said to be compatible tolerance if for every $\sigma \in \Omega_n$, $a_{1,}a_{2,} \dots, a_{n,}b_{1,}b_{2,} \dots, b_n \in A$ such that $(a_i, b_i)_{1\leq i\leq n} \in \eta$ implies $(\sigma(a_1, a_2, \dots, a_n),\ \sigma(b_1, b_2, \dots, b_n)) \in \eta$.

Definition 2.6: *Bijective tolerance:* A tolerance η on A is said to be bijective tolerance if $A \cong A|\eta$.

Example 2.7: Let $A = \{1,2,3,4\}$ and $\eta = \Delta \cup \{(1,2),(2,1),(3,4),(4,3),(2,3),(3,2)\}$ where $\Delta = \{(1,1),(2,2),(3,3),(4,4)\}$ then the tolerances η on A are $\eta[1] = \{1,2\}$, $\eta[2] = \{1,2,3\}$, $\eta[3] = \{2,3,4\}$, $\eta[4] = \{3,4\}$ and the tolerance classes are

$$A|\eta = \{\eta[1], \eta[2], \eta[3], \eta[4]\}$$

Definition 2.8: *Sheaf:* A sheaf (of sets) is a triple (S, π, X) satisfying the following

(i) S, X are topological spaces.

(ii) π is a local homeomorphism of S onto X,

that is, $\pi: S \to X$ is surjection such that for any $s \in S$, there exists open sets G, U in S, X respectively such that $s \in G$, $\pi(s) \in U$ and $\pi|_G: G \to U$ is a homeomorphism. S is called the sheaf space, X is called the base space and π is called the projection. Often we say that (S, π, X) is a sheaf over X. For any $p \in X, \pi^{-1}(p)$ is a non-empty set and is called the stalk at p, denote it by S_p. Note that S is a disjoint union of all $S_p{}'s$.

Definition 2.9: *Sheaf Ω-of algebras:* By a sheaf of Ω-algebras we mean a sheaf (S, π, X) satisfying the following.

(i) For each $p \in X$, the stalk S_p is an Ω-algebra.

(ii) The Ω operations are all continuous, that is, if σ is an n-ary operation, then the map $(s_{1,}s_{2,} \dots, s_n) \to \sigma(s_{1,}s_{2,} \dots, s_n)$ is a continuous map of $S^{(n)} = \{(s_{1,}s_{2,} \dots, s_n) \in S^n | \pi(s_1) = \pi(s_2) = \cdots = \pi(s_n)\}$ into S. Here, we regard $S^{(n)}$ as a subspace of the product space S^n.

Theorem 2.1: *Let (A, Ω) be an algebra. Let η be a compatible tolerance on A. For each $a_{1,}a_{2,} \dots, a_n \in A$, $\sigma \in \Omega_n$ define $(\sigma(\eta(a_1),\ \eta(a_2), \dots, \eta(a_n)) = \eta(\sigma(a_1, a_2, \dots, a_n))$ then $A|\eta$ is also an Ω-algebra.*

Proof: Let $\eta(a_1)$, $\eta\,(a_2), \dots, \eta(a_n) \in A/\eta$. Observe that $\sigma[\eta(a_1),\ \eta(a_2), \dots, \eta(a_n)] = \eta[\sigma(a_1, a_2, \dots, a_n)]$. Since A is an Ω-algebra, $\sigma(a_1, a_2, \dots, a_n) \in A$ implies $\eta(\sigma(a_1, a_2, \dots, a_n)) \in A|\eta$ implies $\sigma[\eta(a_1),\ \eta(a_2), \dots, \eta(a_n)] \in A|\eta$. Hence $A|\eta$ is an Ω-algebra.

3. EXISTENCE OF SHEAVES OF Ω-ALGEBRAS

Theorem 3.1: *Let (A, Ω) be an Ω-algebra. Let X be a topological space. Let T(A) be the set of all compatible tolerances on A. Suppose there exists a continuous map $p \mapsto \eta_p$ from X to $T(A)$ where η_p is a compatible tolerance on A. Then there exists a sheaf of Ω-algebras on A if and only if for any $a, b \in A$, $X(a, b) = \{p \in X | \hat{a}(p) = \hat{b}(p)\}$ is open in X.*

Proof: *First we observe the construction of sheaf of algebras via compatible tolerances.*

Sheaf Construction: Let X be a topological space and (A,Ω) *be an* Ω*-algebra*. Let $T(A)$be the set of all compatible tolerance relations on A. Let $p \mapsto \eta_p$ be a map of X into $T(A)$. LetS_p be the quotient space $A|\eta_p$ for any $p \in X$. Since η_p is a compatible tolerance, by the above theorem (2.1) each $S_p = A|\eta_p$ is an Ω-algebra. Define $S = \bigcup_{p\in X}^{+} S_p$,be the disjoint union of $S_p{'}s$. Define $\hat{a}: X \to S$ by $\hat{a}(p) = \eta_p(a)$ for $a \in A$. Consider a largest topology on S such that $\hat{a}$ is open and continuous for each $a \in A$ and define $\pi: S \to X$ by $\pi(s) = p$ for all $s \in S_p$. (S, π, X) forms a triple.

Now we prove the necessary and sufficient condition for *(S, π, X)* to be a global sheaf of algebras.

Let (S,π,X) be a global sheaf. First we prove that for $a \in A, \hat{a}$ is a global section. Continuity of $\hat{a}$ is clear from the definition. Also $\pi \circ \hat{a}(p) = \pi(\hat{a}(p)) = \pi\left(\eta_p(a)\right) = p$, for all $p \in X$. Therefore $\pi \circ \hat{a}$ is the identity and hence$\hat{a}$is a global section. Now we claim that $X(a,b)$ is open in X. Let $p \in X(a,b)$ that is, $p \in X$ and $\hat{a}(p) = \hat{b}(p) = s$ (say), $s \in S$. By the definition of sheaf there exists open sets G and U in S and X respectively such that $s \in G$ and $\pi|_G: G \to U$ is a homeomorphism. Observe that $\pi(s) = \pi(\hat{a}(p)) = p,\ p \in U$.

Now take $V = \hat{a}^{-1}(G) \cap \hat{b}^{-1}(G) \cap U$. Since $\hat{a}, \hat{b}$ are continuous and U is open, it follows that V is open in X and $p \in V$.

Now for any $q \in V$, $\hat{a}(q), \hat{b}(q) \in G$ and $\pi(\hat{a}(q)) = \pi\left(\hat{b}(q)\right)$. From the fact that $\pi|_G$ is a one-one map, it follows that, $\hat{a}(q) = \hat{b}(q)$. Therefore $q \in X(a,b)$ and hence $X(a,b)$ is open.

Conversely assume that $X(a,b)$is open in X. We now prove that(S,π,X) is a global sheaf. Let $s \in S$. Then there exists $p \in X,\ a \in A$ such that $s \in \eta_p(a)$. Now since $\eta_p(a) = \hat{a}(p),\ \hat{a}(p) \in \hat{a}(X)$ it follows that $s \in \hat{a}(X)$.

We now prove that $\pi|_{\hat{a}(X)}: \hat{a}(X) \to X$ is a homeomorphism.

Suppose $\pi|_{\hat{a}(X)}\left(\eta_p(a)\right) = \pi|_{\hat{a}(X)}\left(\eta_q(a)\right)$. By definition of π, it follows that $p = q$. Thus, $\eta_p(a) = \eta_q(a)$ and hence $\pi|_{\hat{a}(X)}$ is one-to-one.

Given $p \in X$, observe that $\pi|_{\hat{a}(X)}\left(\eta_p(a)\right) = p$ for $a \in A$, $\eta_p(a) \in \hat{a}(X)$. Therefore $\pi|_{\hat{a}(X)}$ is onto.

Let U be open in X and $s \in \left(\pi|_{\hat{a}(X)}\right)^{-1}(U)$. Then $\pi|_{\hat{a}(X)}(s) \in U$. Now since $s \in S_p$ for some p, there exist $a \in A$ such that $s = \eta_p(a)$ and hence $\pi|_{\hat{a}(X)}\left(\eta_p(a)\right) \in U$. Since $\pi|_{\hat{a}(X)}\left(\eta_p(a)\right) = p$, it follows that $p \in U$, clearly $\hat{a}(p) \in \hat{a}(U)$.

From the fact that $\hat{a}$ is an open map, it is clear that $\hat{a}(U)$ is open in S.

Let $s' \in \hat{a}(U)$. Then $s' = \hat{a}(q)\left(= \eta_q(a)\right)$ for some $q \in U$.

It can be observed that $\pi|_{\hat{a}(X)}\left(\eta_q(a)\right) \in U$ as $\pi\left(\eta_p(a)\right) = q$. Therefore $s' = \eta_q(a) \in (\pi|_{\hat{a}})^{-1}(U)$.

Thus $\hat{a}(U) \subseteq \pi|_{\hat{a}(X)}(U)$ and hence $\pi|_{\hat{a}(X)}$ is continuous.

Let H be an open set in $\hat{a}(X)$. By the subspace topology induced by S, there exists an open set G in S such that $H = \hat{a}(X) \cap G$. Let $s \in H$; then there exists $q \in X$ such that $s = \hat{a}(q)\left(= \eta_q(a)\right), s \in G$. Since $q \in \hat{a}^{-1}(G)$, consider $W = \hat{a}^{-1}(G) \cap X$. Clearly $q \in W$ and W is open in X.

Now let $p \in W$, that is, $p \in \hat{a}^{-1}(G) \cap X$. Then $\hat{a}(p) \in G$ and since $\hat{a}(p) \in \hat{a}(X)$, it follows that
$\hat{a}(p) \in \hat{a}(X) \cap G = H. p = \pi|_{\hat{a}(X)}(\hat{a}(p)) \in \pi|_{\hat{a}(X)}(H)$. Thus $\pi|_{\hat{a}(X)}$ is an open map.

Now it is enough to show that every n-ary operation $\sigma \in \Omega_n$ is continuous from $S^{(n)} \to S$.

Let $\sigma \in \Omega_n$. To show that $\sigma: S^{(n)} \to S$ is continuous, let H be an open set in S. We have to show that $\sigma^{-1}(H)$ is open in $S^{(n)}$. Let $\sigma(s_1, s_2, \dots, s_n) \in H$, where $s_1, s_2, \dots, s_n \in S^{(n)}$ and there exists $a_1, a_2, a_3, \dots, a_n \in A$ such that $S_i = \eta_p(a_i)$ for some p (Since (S,π,X) is global) and each S_p being an algebra. Now $\sigma(\eta_p(a_1), \eta_p(a_2), \eta_p(a_3), \dots \eta_p(a_n)) \in H$ implies $\eta_p(\sigma(a_1, a_2, \dots, a_n)) \in H$ which implies $\widehat{\sigma(a_1, a_2, \dots, a_n)}(p) \in H$. Since $\widehat{\sigma(a_1, a_2, \dots, a_n)}$being continuous there exists an open set U in X containing p such that $\widehat{\sigma(a_1, a_2, \dots, a_n)}(U) \subseteq H$. Now consider $W = \left(\widehat{a_1}(U) \times \widehat{a_2}(U) \times \dots \times \widehat{a_n}(U)\right) \cap S^{(n)}$. Clearly W is an open set in $S^{(n)}$ containing$\left(s_1, s_2, \dots, s_n\right)$. Let $t = \left(\widehat{a_1}(q), \widehat{a_2}(q), \dots, \widehat{a_n}(q)\right) \in W$ for some $q \in U$.

Then $\sigma(t) = \sigma(\widehat{a_1}(q), \widehat{a_2}(q), \dots, \widehat{a_n}(q)) = \sigma(\eta_q(a_1), \eta_q(a_2), \eta_q(a_3), \dots, \eta_q(a_n)) = \eta_q\left(\sigma(a_1, a_2, \dots, a_n)\right) = \sigma(a_1, \widehat{a_2, \dots, a_n})(q) \in H$, since $\sigma(a_1, \widehat{a_2, \dots}, a_n)(U) \subseteq H$. Which implies $\sigma(W) \subseteq H$ and hence σ is continuous. Thus (S, π, X) is a sheaf of Ω-algebras.

Theorem 3.2: *Let L be a bounded distributive Lattice $|L| > 2$. Let $X = Spec(L)$ be the spectrum of L. For each $P \in X$, define $\eta_P = \{(x, y) \in L \times L | (x = y) \vee (x \in P) \vee (y \in P)\}$. Then (1) η_P is a tolerance on L. (2) η_P is lower semi compatible.*

Proof: (1) Claim: η_P is a tolerance on L. For any $x \in L$, $(x, x) \in \eta_P$ therefore η_P is reflexive. Also for any $x, y \in L$, $(x, y) \in \eta_P$. we have to show that $(y, x) \in \eta_P$

Case (a): Let $x = y \Rightarrow y = x \Rightarrow (y, x) \in \eta_P$

Case (b): Let $x \in P \Rightarrow (y, x) \in \eta_P$

Case (c): Let $y \in P \Rightarrow (y, x) \in \eta_P$. Hence in all the cases for $(x, y) \in \eta_P \Rightarrow (y, x) \in \eta_P$. Therefore η_P is symmetric. Thus η_P is a tolerance on L.

(2) Claim: η_P is lower semi compatible.

Let $(x_1, y_1), (x_2, y_2) \in \eta_P$. We have to show that $(x_1 \wedge x_2,\ y_1 \wedge y_2) \in \eta_P$.

Case (i): Let $x_1 = y_1$ and $x_2 = y_2 \Rightarrow x_1 \wedge x_2 = y_1 \wedge y_2 \Rightarrow (x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$.

Case (ii): Let $x_1 = y_1$ and $x_2 \in P$, since $x_1 \wedge x_2 \leq x_2 \in P$ implies $x_1 \wedge x_2 \in P$ as a result $(x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$.

Case (iii): Also let $x_1 = y_1$ and $y_2 \in P$, since $y_1 \wedge y_2 \leq y_2 \in P$ implies $y_1 \wedge y_2 \in P$, as a result $(x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$.

Case (iv): Let $x_1 \in P$ and $x_2 = y_2$, since $x_1 \wedge x_2 \leq x_1 \in P$ implies $x_1 \wedge x_2 \in P$, and hence $(x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$.

Case (v): Let $y_1 \in P$ and $x_2 = y_2$, since $y_1 \wedge y_2 \leq y_1 \in P \Rightarrow y_1 \wedge y_2 \in P$ also $y_1 \in P, x_2 \in P$ and since $x_1 \wedge x_2 \leq x_2 \in P$ implies $x_1 \wedge x_2 \in P$. Also $y_1 \in P, y_2 \in P \Rightarrow y_1 \wedge y_2 \in P$ hence $(x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$.

Case (vi): Also $x_1 \in P, y_2 \in P$ and since $y_1 \wedge y_2 \leq y_2 \in P$, hence $(x_1 \wedge x_2, y_1 \wedge y_2) \in \eta_P$. Hence η_P is lower semi compatible.

By interchanging the roles of x_1, y_2 the other cases follows.

Theorem 3.3: *Let L be a bounded distributive Lattice. Let $X = Spec(L)$ be the spectrum of L. Let $T(L)$ be the tolerances on L. For each $P \in X$, define $\eta_P = \{(x, y) \in L \times L | (x = y) \vee (x \in P) \vee (y \in P)\}$ Topoligize $T(L)$ with the largest topology for which $\{< a, b > | a, b \in L\}$ forms a sub base where $< a, b >= \{\eta_P \in T(L) | \eta_P(a) = \eta_P(b)\}$ then the map $P \mapsto \eta_P : X \to T(L)$ is continuous.*

Proof: Let H be an open set in $T(L)$. To show that $f^{-1}(H)$ is open in X. Let $H =< a_1, b_1 > \cap < a_2, b_2 > \cap \dots \cap < a_n, b_n >$ for some $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n \in L$. Let $q \in f^{-1}(H)$. Then $f(q) \in H$ which implies $f(q) \in < a_1, b_1 > \cap < a_2, b_2 > \cap \dots \cap < a_n, b_n >$. Then there exists $W_1, W_2, \dots, W_n$ open in $T(L)$ such that $f(q) \in W_i \subseteq < a_i, b_i >_{(1 \leq i \leq n)}$. Thus $f(q) \in \bigcap_{i=1}^{n} W_i \subseteq \bigcap_{i=1}^{n} < a_i, b_i >= H$ which implies $q \in f^{-1}(\bigcap_{i=1}^{n} W_i) \subseteq f^{-1}(H)$. Therefore $f : P \mapsto \eta_P : X \to T(L)$ is continuous.

Now the general method described above is applied on an algebraic structure of a bounded distributive lattice.

Theorem 3.4: *Let L be a bounded distributive lattice, then there exists a sheaf of algebras (S, π, X) such that L is isomorphic with*

$\Gamma(X, S) = \{f - f \text{ is a continuous map from } X \text{ to } S \text{ such that } \pi \circ f = id_X\}$ and each stalk is a lower semi lattice.

Proof: Consider $X = Spec(L)$ be the spectrum of L, that is, $X = \{P | P \text{ is a prime ideal of } L\}$ equipped with the well-known hull-kernel topology. For each $P \in X$ define $S_p = L/\eta_P$ where η_P is defined as $\eta_p = \{(a, b) \in L | (a = b) \vee (a \in P) \vee (b \in P)\} = \Delta \cup (P \times L) \cup (L \times P)$ and $\eta_p(a) = \{b \in L | (a, b) \in \eta_p\}$

Case (i): If $a \in P$ then $\eta_P(a) = L$

Case (ii): If $a \notin P$ then $\eta_P(a) = \{b \in L | b \in P\} \cup \{a\}$, hence $\eta_P(a) = \begin{cases} L & if\ a \in P \\ P \cup \{a\} & if\ a \notin P \end{cases}$

Structure of $L/\eta_P = \{\eta_P(a) | a \in L\}$ $(=\{\hat{a}(p)/a \in L\})$

Define $\leq_p$ on $L|\eta_P$ by $\eta_P(a) \leq_p \eta_P(b)$ if and only if $a \leq b$.
Then for a, b in L if $a \leq b$,

Case (i): If $a, b \in P$ then $\eta_P(a) = L = \eta_P(b)$

Case (ii): If $a \in P, b \notin P$ then $\eta_P(a) = L, \eta_P(b) = P \cup \{b\}$

Case (iii): If $a \notin P$, $b \in P$ this case does not arise since P is an (prime) ideal.

Case (iv): If $a \notin P, b \notin P$ then $\eta_P(a) = P \cup \{a\} \leq_P P \cup \{b\} = \eta_P(b)$.

Hence $L \leq_{p,} P \cup \{b\}\ \forall b \notin P, P \cup \{a\} \leq_p P \cup \{b\} \leq_p P \cup \{1\}$ where 1 is the greatest element of L.

Thus L/η_P is a bounded lower semi-lattice with L as the least element and $P \cup \{1\}$ as the greatest element.

Let $S = \bigcup^{+}_{P \in X} S_P$, the disjoint union of $S_P{}'s$. For each $a \in L$ define $\hat{a}(P) = \eta_P(a)$.

Topologise S with the largest topology with respect to each $\hat{a}$ is continuous. Then by theorem 3.1 (S, π, X) is a global sheaf, since $P \mapsto \eta_P$ being continuous. It is enough to show that $L \cong \Gamma(X, S)$.

Let $\hat{a}(P) = \hat{b}(P)$ for all P. Then, $(a, b) \in \eta_P, \forall P$ and since P = {0} being the smallest prime ideal and $\eta_{\{0\}} = \Delta$, it follows that $(a, b) \in \Delta$ and thus $a = b$.

Let $f: X \to S$ be a global section, then there exists $a \in A$ such that $f(P) = \eta_P(a) = \hat{a}(P)\ \forall P$ implies $f = \hat{a}$. Hence $\Gamma(X, S)$ is onto. Therefore $L \cong \Gamma(X, S)$.

Theorem 3.5: Let L be a bounded distributive Lattice. For $P \in Spec(L)$,
$\eta_P = \{(x, y) \in L \times L | (x = y) \vee (x \in P) \vee (y \in P)\}$ define $\sim_P$on $L/\eta_P$by $\eta_P(x) \sim_P \eta_P(y)$ if and only if $x, y \in P \vee (x = y)$ then $\sim_P$ is an equivalence relation.

Proof: Reflexive: Let $x \in P$, clearly $(x, x) \in \eta_p$ and hence $\eta_P(x) \sim_P \eta_P(x)$. Thus $\sim_P$is reflexive.

Symmetric: Let $(\eta_P(x), \eta_P(y)) \in \sim_P$, implies $x, y \in P \vee (x = y), \eta_P(y) \sim_P \eta_P(x)$, therefore $\sim_p$is symmetric.

Transitive: Let $\eta_P(x) \sim_P \eta_P(y)$ which implies $x, y \in P \vee (x = y)$ and $\eta_P(y) \sim_P \eta_P(z)$ which implies $y, z \in P \vee (y = z)$.

Case (i): let $x = y$, $y = z$ implies $x = z$.

Case (ii): let $x = y$ *and* y, $z \in P$ which implies x, $z \in P$.

Case (iii): let $y = z$ *and* $x, y \in P$ which implies $x, z \in P$.

Case (iv): let $x, y \in P$ *and* y, $z \in P$ implies $x, z \in P$ in all the cases $\eta_P(x) \sim_P \eta_P(z)$ therefore $\sim_P$ is transitive. Hence $\sim_P$ is an equivalence relation.

Observe that in $(L/\eta_p)/\sim_P, [\eta_P(0)]_{\sim_P} = [\eta_P(x)]_{\sim_P}\ \forall x \in P$ and $[\eta_P(y)]_{\sim_P} = \{\eta_P(y)\}$. Let us denote $(L/\eta_p)/\sim_P$ as $\widehat{L_P}$

Theorem 3.6: Let L be a bounded distributive Lattice. For $P \in Spec(L)$,
$\eta_P = \{(x, y) \in L \times L | (x = y) \vee (x \in P) \vee (y \in P)\}$ define $\sim_P$on $L/\eta_P$by $\eta_P(x) \sim_P \eta_P(y)$ if and only if $x, y \in P \vee (x = y)$ then $\sim_P$is an equivalence relation. Let $(L/\eta_p)/\sim_P$ be denoted as $\widehat{L_P}$ then, $(\widehat{L_P}, \leq_P)$ is a lattice, where $\leq_P$defined on $\widehat{L_P}$ by $[\eta_P(y)]_{\sim_P} \leq_P [\eta_P(z)]_{\sim_P}$ if and only if $y \leq z$

Proof: First we observe that $\widehat{L_P} - [\eta_P(0)]_{\sim_P}$ is a sub lattice of L.
Let $[\eta_P(x)]_{\sim_P}, [\eta_P(y)]_{\sim_P} \in \widehat{L_P} - [\eta_P(0)]_{\sim_P}$

which implies

$[\eta_P(x)]_{\sim P}$, $[\eta_P(y)]_{\sim P} \neq [\eta_P(0)]_{\sim P}$ and hene $x, y \notin P$. Since P is prime ideal, $x \wedge y \notin P, x \vee y \notin P$ which implies

$[(\eta_p(x \wedge y)]{\sim_P}, [(\eta_p(x \vee y)]_{\sim P} \in \widehat{L_P} - [\eta_P(0)]_{\sim P}$.

Hence $\widehat{L_P} - [\eta_P(0)]_{\sim P}$is a lattice which is also distributive since L is distributive and

$$[(\eta_p(x) \vee \eta_p(y)) \wedge \eta_p(z)]_{\sim P} = [(\eta_p(x \vee y)) \wedge \eta_p(z)]_{\sim P} = [(\eta_p(x \vee y)) \wedge z)]_{\sim P} \text{and}$$
$$[(\eta_p(x) \vee (\eta_p(y) \wedge \eta_p(z))]_{\sim P} = [(\eta_p(x) \vee \eta_p(y \wedge z)]_{\sim P} = [\eta_p(x \vee (y \wedge z)]_{\sim P}$$

distributive lattice $[\eta_P(1)]_{\sim P}$ as the greatest element. Hence $(\widehat{L_P}, \leq_P)$ is a distributive lattice. Hence each L/η_pcan be regarded as a bounded distributive lattice.

4. CONCLUSION

In this paper, construction of sheaves of algebras via compatible tolerances is studied and the mechanism is implemented on a bounded distributive lattice. It is observed that, with the tolerance relation being semi compatible, the stacks are observed to be lower semi lattices isomorphic with the given lattice.

5. REFERENCES

1. A. Wolf, sheaf representations of arithmetical algebras, *Memoirs Amer.Math.Soc.* (148) (1974), pp. 87-93.
2. Arthur.Knoebel, *Sheaves of Algebras over Boolean Spaces*, Birkhauser, (2012).
3. Brian A. Davey, Sheaf spaces and sheaves of universal algebras, *Math. Z.,* (134) (1973), pp. 275-290.
4. BREDON, Glen E. *Sheaf theory*, 2nd ed. Springer, Berlin, 1977.
5. B. Zelinka, Tolerance in algebraic structures, *Czechoslovak Mathematical Journal,* Vol. (20) (1970), No.2, 179-183.
6. CORNISH, William H. The Chinese remainder theorem and sheaf representations. *Fund. Math.*(46) (1977), pp. 1059-1067.
7. Daniel Murfet, Special Sheaves of Algebras, October 5, 2006
8. GEORGESCU, George. Some sheaf constructions for distributive lattices. *Bull. Math. Soc. Sci. Math*. R.S. Roumanie (N.S) (32)(80) (1988), pp. 299-303.
9. Grant Malcolm, Sheaves, Objects and Distributed Systems, *Electronic Notes in Theoretical Computer Science*, Elsevier (225) (2009) 3-19.
10. Grothendieck, Alexander. Sur quelques points d'algebrehomologique, I. Tohoku Math. J. (2) (9) (1957), 119-221.
11. I.Chajda, Algebraic Theory of Tolerance Relations, Palacky University Olomouc, (1991).
12. J. Dauns and K. H. Hofmann, The representation of biregular rings by sheaves, Math. Z. (91) (1966), 103-123.
13. J.Pogonowski, Tolerance Spaces with Applications to Linguistics, Univ.Adama Mickiewicza, Poznan, (1981).
14. J. W. Gray, Fragments of the history of sheaf theory, in Applications of Sheaves, eds: M.P. Fourman, C.J.Mulvey, and D. S. Scott, Lecture Notes in Mathematics Vol 753 (Springer, 1979), pp.1 - 79.
15. K. H. Hofmann, Representation of algebras by continuous sections, Bull. Amer.math. Soc. (78), Number 3 (1972), 291-373.
16. K. Keimel, The representation of lattice-ordered groups and rings by sections in sheaves, in Lecture on the Applications of Sheaves to Ring Theory, Lecture Notes in Math. 248, Springer-Verlag, Berlin and New York, 1971, pages 1- 98.
17. Knoebel, Arthur. Sheaf representations of preprimal algebras.Abstracts Amer. Math.Soc. (24) (2003), p. 238
18. Martin Christian Olsson, Sheaves on Artin stacks. J. ReineAngew. Math. (603) (2007), 55-112.
19. M.P.K.Kishore, R.V.G.Ravi Kumar, P.VamsiSagar., Costruction of sheaves by tolerance relations, Asian-European Journal of Mathematics, Vol. 9, No. 2 (2016) 1650035 (8 pages), World Scientific Publishing Company, DOI: 10.1142/S1793557116500352
20. R. Balbes, and Dwinger, PH., Distributive lattices, U.Missouri Press, 1974
21. R. S. Pierce, Modules over commutative regular rings, Mem. Amer. Math. Soc. (70) (1967), 1-112.
22. S. D. Comer, Representation by algebras of sections over Boolean spaces, Pacific J. Math. (38) (1971), 29 -38.
23. U. M. Swamy, Representation of Universal algebras by sheaves, Proc. Amer. Math. Soc., (45) (1974), 55-58.
24. U.M. Swamy, and Manikyamba, P. Representation of certain classes of distributive lattices by sections of sheaves. Internat. J. Math. Math. Sci. (3) (1980), pp. 461-476.

Source of support: Nil, Conflict of interest: None Declared.