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PRIME LABELING TO UDUKKAI GRAPHS

A. EDWARD SAMUEL*[1] AND S. KALAIVANI[2]

[1]Ramanujan Research Centre,
P.G. and Research Department of Mathematics,
Government Arts College (Autonomous), Kumbakonam – 612 001, Tamil Nadu, India.

[2]Bharatidasan University, Tiruchirapalli, Tamil Nadu, India.

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ABSTRACT

In this paper, we investigate prime labeling for some graphs related to udukkai graph. We discuss prime labeling in the context of some graph operations namely duplication, fusion, switching in udukkai graph A_n. We prove that the duplication of any vertex of udukkai graph are prime graph. We prove that the identifying any vertex of udukkai graph are prime graph. We prove that the switching of an apex vertex and other vertex of udukkai graph are prime graph and also apply coloring of the graph operations of the udukkai graph is satisfying coloring condition.

Keywords: *Prime Labeling; Prime Graph; Udukkai Graph; Duplication; Fusion; Switching; coloring.*

1. INTRODUCTION

we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy [1]. Many researchers have studied prime graph for example in Fu. H [8] have proved that the path P_n on *n* vertices is a prime graph. In Dretsky .T[3] have proved that the cycle C_n on *n* vertices is a prime graph In [10] S. Meena and K. Vaithilingam have proved the prime labeling for some Fan related graphs. In [4] A.Edward Samuel and S. Kalaivani have proved that the prime labeling for some octopus related graphs. In [5] A.Edward Samuel and S. Kalaivani have proved that the prime labeling for some planter related graphs. In [6] A.Edward Samuel and S. Kalaivani have proved that the prime labeling for some Vanessa related graphs. In [7] A.Edward Samuel and S. Kalaivani have proved that the square sum labeling for some lilly related graphs. For latest survey on graph labeling we refer to [9] (Gallian. J. A., 2009).

2. PRELIMINARY DEFINITIONS

Definition [10]: Let $G = (V(G), E(G))$ be a graph with *p* vertices. A bijection $f : V(G) \to \{1, 2, \dots, p\}$ is called a *prime labeling* if for each edge $e = uv$, $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*.

Definition [10]: *Duplication* of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words a vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' also.

Definition [10]: Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying (fusing)* two vertices *u* and *v* by a single vertex *x* is such that every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.

Definition [10]: A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.

Definition [2]: A coloring is *proper* if adjacent vertices have different colors. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

Corresponding Author: A. Edward Samuel[1],*
[1]Ramanujan Research Centre, P.G. and Research Department of Mathematics,
Government Arts College (Autonomous), Kumbakonam – 612 001, Tamil Nadu, India.

3. PRIME LABELING TO UDUKKAI RELATED GRAPHS

3.1. Udukkai graph
An *Udukkai graph* $A_n, n \geq 2$ can be constructed by two fan graphs $2F_n, n \geq 2$ joining two path graphs $2P_n, n \geq 2$ with sharing a common vertex. i.e., $A_n = 2F_n + 2P_n$.

Example 3.2:

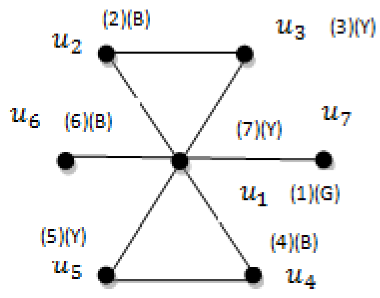

Figure-3.1: Udukkai graph A_2.

Theorem 3.3: An udukkai graph $A_n, n \geq 2$ admits prime graph, where *n* is any positive integer.

Proof: Let A_n be the udukkai graph with vertices $\{u_1, u_2, \dots, u_{4n-1}\}$. Here $|V(A_n)| = 4n - 1$. Define a labeling $f : V(A_n) \to \{1, 2, \dots, 4n - 1\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \leq i \leq 4n - 1$$

Clearly vertex labels are distinct. Then for any edge $e = u_1u_i \in A_n$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$ and for any edge $e = u_iu_{i+1} \in A_n$, $\gcd(f(u_i), f(u_{i+1})) = 1$ for $2 \leq i \leq 4n - 1$. Since it is consecutive positive integers. Then *f* admits prime labeling. Thus A_n is a prime graph.

Example 3.4:

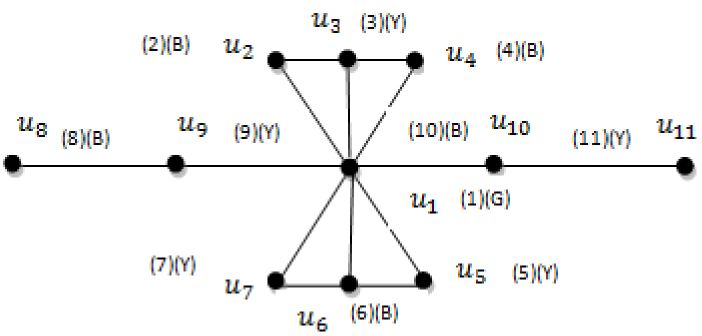

Figure-3.2: Prime labeling for A_3.

Theorem 3.5: The graph obtained by duplication of any vertex u_k to u_k' of an udukkai graph $A_n, n \geq 2$ is a prime graph, where *n* is any positive integer.

Proof: Let G be the graph of an udukkai graph $A_n, n \geq 2$. Let u_k be the vertex of an udukkai graph $A_n, n \geq 2$ and u_k' be its duplicated vertex and G_k be the graph resulted due to duplication of the vertex u_k in $A_n, n \geq 2$, where *n* is any positive integer. Let $V(A_n) = \{u_1, u_2, \dots, u_{4n-1}\}$. Here $|V(G_k)| = 4n$.

We define a labeling $f : V(G_k) \to \{1, 2, \dots, 4n\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \leq i \leq n + 6$$
$$f(u_k') = n + 7$$
$$f(u_i) = i + 1 \quad \text{for } n + 7 \leq i \leq 4n - 1$$

Clearly vertex labels are distinct. Then *f* admits prime labeling. Thus G_k is a prime graph.

Example 3.6:

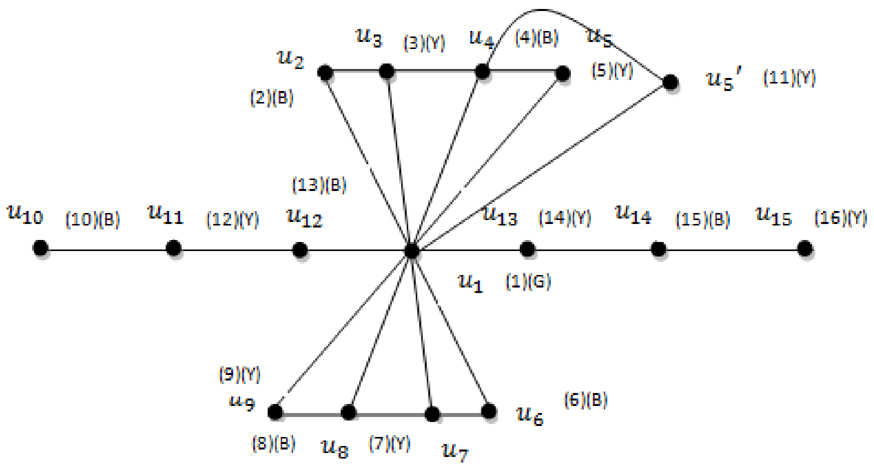

Figure-3.3: Duplication of u_5 in A_4.

Theorem 3.7: The graph obtained by identifying any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of an udukkai graph $A_n, n \geq 2$ is a prime graph, where n is any positive integer.

Proof: Let $A_n, n \geq 2$ be an udukkai graph with vertices $\{u_1, u_2, \dots, u_{4n-1}\}$ and the vertex u_i be the fused with u_k. Denote the resultant graph as G_k. Here we note that $|V(G_k)| = 4n - 2$.
Define a labeling $f: V(G_k) \to \{1,2, \dots, 4n-2\}$ as follows

$f(u_i) = i$ for $1 \leq i \leq n-1$
$f(u_5 = u_6) = n$
$f(u_i) = i - 1$ for $n+2 \leq i \leq 4n-1$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd\big(f(u_i), f(u_k)\big) = 1$.Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of an udukkai graph $A_n, n \geq 2$ is a prime graph.

Example 3.8:

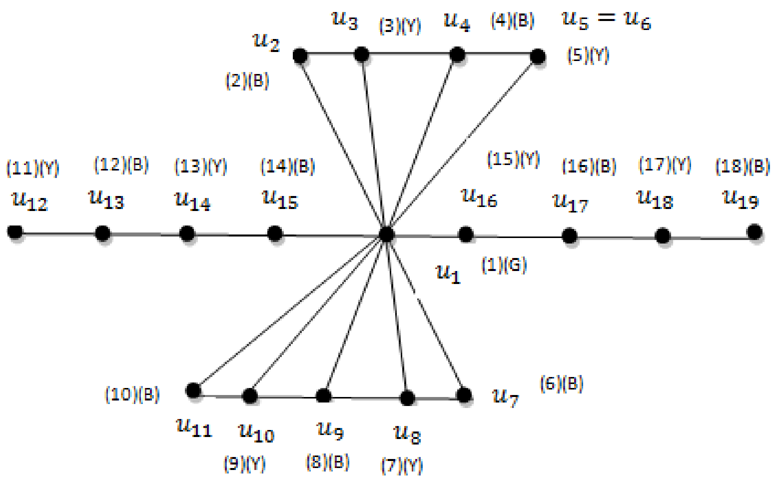

Figure-3.4: Fusion of u_5 and u_6 in A_5.

Theorem 3.9: The switching of any vertex u_k in an udukkai graph $A_n, n \geq 2$ produces a Prime graph, where n is any positive integer.

Proof: Let $G = A_n$ and $\{u_1, u_2, \dots, u_{4n-1}\}$ be the successive vertices of an udukkai graph $A_n, n \geq 2$ and G_u denotes the graph obtained by a vertex switching of G with respect to the vertex u. It is obvious that $|V(G_u)| = 4n - 1$.

Define a labeling $f: V(G_u) \to \{1,2, \dots, 4n-1\}$ as follows

$f(u_1) = 1$
$f(u_2) = 4n - 1$
$f(u_i) = i - 1$ for $3 \leq i \leq 4n-1$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$. Clearly vertex labels are distinct. Then f is a prime labeling and consequently G_u is a prime graph. That is, the switching of any vertex u_k in an udukkai graph $A_n, n \geq 2$ produces a Prime graph, where n is any positive integer.

Example 3.10:

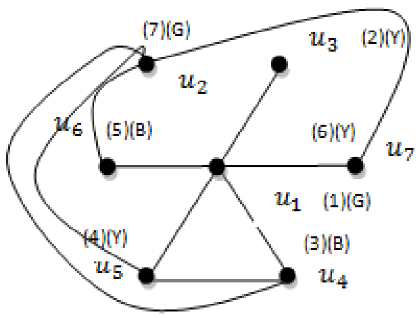

Figure-3.5: Switching the vertex u_2 in A_2.

Theorem 3.11: The switching of an apex vertex u_1 in an udukkai graph $A_n, n \geq 2$ produces a Prime graph, where n is any positive integer.

Proof: Let $G = A_n$ and $\{u_1, u_2, \ldots, u_{4n-1}\}$ be the successive vertices of an udukkai graph $A_n, n \geq 2$ and G_u denotes the graph obtained by an apex vertex switching of G with respect to the vertex u_1. It is obvious that $|V(G_u)| = 4n - 1$. Without loss of generality, we initiate the labeling from u_1 and proceed in the clock – wise direction.

Define a labeling $f: V(G_u) \to \{1, 2, \ldots, 4n - 1\}$ as follows

$$f(u_i) = i \quad \text{for} \quad 1 \leq i \leq 4n - 1$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$. Clearly vertex labels are distinct. Then f is a prime labeling and consequently G_u is a prime graph. That is, the switching of an apex vertex u_1 in an udukkai graph $A_n, n \geq 2$ produces a Prime graph and it is a disconnected graph.

Example 3.12:

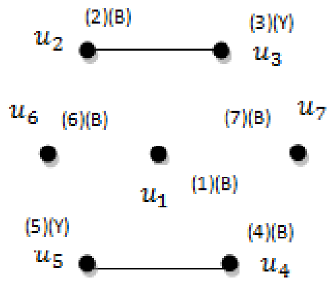

Figure-3.6: Switching an apex vertex u_1 in A_2.

APPLICATIONS

Several practical problems in real life situations have motivated the study of labeling of graphs which are required to obey a variety of conditions depending on the structure of graphs. Prime labeling are applied in the additive number theory, coding theory problems, communication network design, telecommunication.

CONCLUSION

In this paper we proved that an Udukkai graph A_n, duplication of an Udukkai graph A_n, fusing of an Udukkai graph A_n, switching of an Udukkai graph A_n are prime graphs. Coloring conditions satisfied to the Udukkai graph.

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