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VERTEX EQUITABLE LABELING OF CERTAIN GRAPHS

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ABSTRACT

Let G be a graph with p vertices and q edges and $A = \{0,1,2,\ldots,\left\lceil\frac{q}{2}\right\rceil\}$*. A vertex labeling* $f: V(G) \rightarrow A$ *induces an edge labeling* f^* *defined by* $f^*(uv) = f(u) + f(v)$ *for all edges uv. For* $a \in A$*, let* $v_f(a)$ *be the number of vertices v with* $f(v) = a$*. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A,* $\left|v_f(a) - v_f(b)\right| \leq 1$ *and the induced labels are 1, 2,…, q. In this paper we examine the vertex equitable labeling of Jewel graph, Quadrilateral snake and Alternate quadrilateral snake.*

Key Words: *Labeling, Vertex equitable labeling, Jewel graph, Quadrilateral snake, Alternate quadrilateral snake.*

1. INTRODUCTION

Graph labeling was first introduced in the 1960's. In many labeled graphs, the labels are used for identification only. The kind of labeling, we are interested in conserve dual purpose; it means a labeling can be used to identify vertices and edges & also to signify some additional properties, depending on the particular labeling. Graph labeling is one of the fascinating areas of graph theory with wide range of applications. An enormous body of literature has grown around graph labeling in the last four decades.

Labeled graphs provide mathematical models for a broad range of applications. Most graph labeling methods trace their origin to one introduced by Rosa in 1967, or one given by Graham and Sloane in 1980. In the intervening 50 years nearly 200 graph labeling techniques have been studied in over 2000 papers. The area of graph labeling basically deals with theoretical study. The topic of graph labeling has been the subject of research for a long time in the applied fields also. Rosa identified three types of labeling, which we called α- labeling, β- labeling and ρ- labeling. Rosa introduced graceful labeling in 1966 and later graceful labeling was called as β -labeling.

The term graceful was introduced by Golomb in 1972. Rosa showed that if every tree is graceful, then Ringel's conjecture holds. Since then, researchers have been trying to prove Ringel's conjecture through the graceful tree conjecture, which claims that every tree is graceful. The extension of graceful labeling to directed graphs arose in the characterization of finite neofields by Hsu and Keedwell.

The quantitative labeling of graph elements have been used in diverse field such as conflict resolutions in social psychology, energy crises etc. Quantitative labeling of graph elements are also used in missile guidance codes, radar location codes, coding theory, X-ray crystallography, astronomy , circuit design, communication network etc.

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P. Jeyanthi and A. Maheswari [4] proved that $T\widehat{O}P_n$, $T\widehat{O}2P_n$, $T\widehat{O}C_n$, $T\widetilde{O}C_n$ are vertex equitable graphs. P. Jeyanthi and A. Maheswari [6] proved that jewel graph J_n, jelly fish graph $(JF)_n$, balanced lobster graph $BL(n,2,m)$, $L_n \odot K_m$ (mean value) and $L_n \ddot{O} K_{1,m}$ are vertex equitable graphs. A. Maheswari and K. Thanalakshmi [10] proved that $T\widehat{O}nC_4$ and zig – zag triangle are vertex equitable graphs. P.Jeyanthi, A.Maheswari and M.Vijayalakshmi [9] have proved $KY(m,n), P(2,QS_n), P(m,QS_n), C(n,QS_n), NQ(m)$ and $K_{1,n} \times P_2$ are also vertex equitable graphs. For more results on graph labeling one may refer to Gallian survey [1]. In our study we have considered jewel graph, quadrilateral snake, alternate quadrilateral snake and proved that they are vertex equitable.

Notation: In the course of our proof we use the notation J_n for jewel graph, QS_n for quadrilateral snake and $A(QS_n)$ for alternate quadrilateral snake.

2. JEWEL GRAPH

Definition: The *Jewel graph* J_n is a graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \le i \le n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \le i \le n\}$. See Figure 1.

Theorem 2.1: Jewel graph J_n, $n \ge 2$ is a vertex equitable graph.

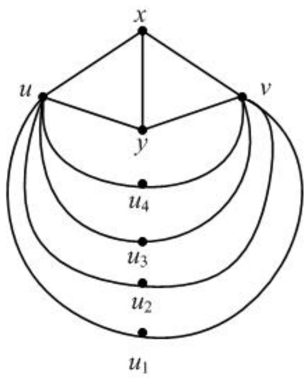
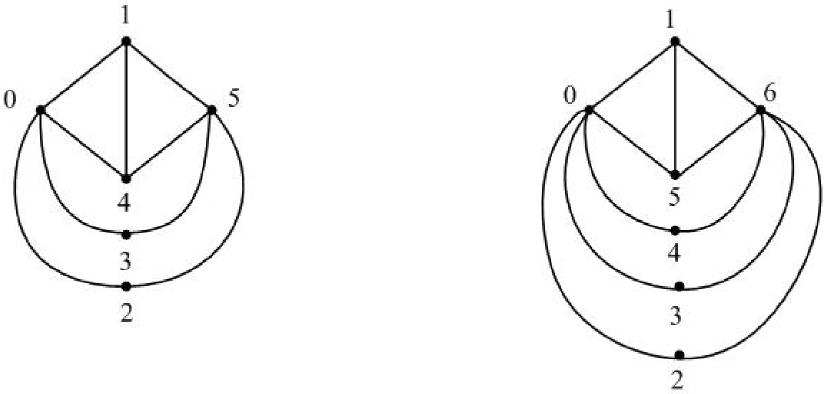


Figure-1



(a) J_2 (b) J_3

Figure-2

Proof: Let $G = J_n$ be any jewel graph on n+4 vertices and 2n+5 edges. The vertices and edges of J_n are defined as shown in Figure 1.

Define $f : V(G) \longrightarrow \{1, 2, \ldots, \left\lceil \frac{2n+5}{2} \right\rceil\}$ as follows:

$f(u) = 0,\ f(x) = 1,\ f(v) = \left\lceil \frac{q}{2} \right\rceil,\ f(y) = \left\lceil \frac{q}{2} \right\rceil - 1,\ f(u_i) = i + 1,$ for $1 \le i \le n$

From the above definition of f we can see that the edge induced labels are 1, 2,…, q as in definition of vertex equitable labeling. Hence J_n admits vertex equitable labeling. Vertex equitable labeling of J_2 and J_3 are shown in Figure 2.

3. QUADRILATERAL SNAKE

Definition: Let $Q(n)$ be the *quadrilateral snake* obtained from the path $v_1, v_2, v_3, \ldots, v_n$ by joining v_i and v_{i+1} to new vertices u_i and w_i .That is, every edge of a path is replaced by a cycle C_4.

Theorem 3.1: Quadrilateral snake QS_n, $n \geq 2$ is a vertex equitable graph.

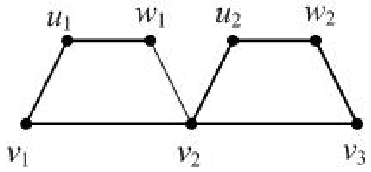
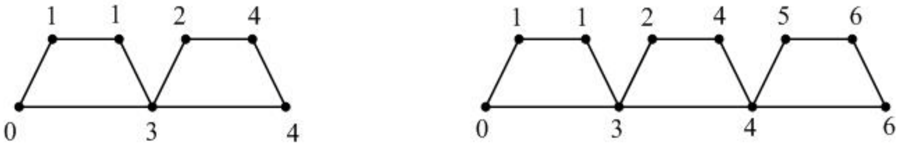


Figure-3



(a) QS_2 (b) QS_3

Figure-4

Proof: Let $G = QS_n$ be any quadrilateral snake graph on $3n+1$ vertices and $4n$ edges. Let $v_1, v_2, v_3, \ldots, v_{n+1}$ be the vertices on the path P_{n+1}. Let $u_1, u_2, \ldots, u_n$ and $w_1, w_2, \ldots, w_n$ be n vertices which lies above the path. QS_n is obtained by joining v_i and v_{i+1} to new vertices u_i and w_i for $1 \leq i \leq n$. See Figure 3. Now define $f: V(G) \longrightarrow \{1, 2, \ldots, \left\lceil \frac{4n}{2} \right\rceil\}$ as follows:

$f(v_1) = 0, f(v_2) = 3, f(v_i) = v_{i-1} + 2$, for $i \geq 4$,

$f(u_i) = i$, for $i = 1,2$,

$f(u_i) = 2i - 1$, for $i > 2$,

$f(w_1) = 1, f(w_i) = 2i$, for $i > 1$.

From the above definition of f we can clearly see that the edge induced labels are 1, 2,…, q as in definition of vertex equitable labeling. Hence QS_n admits vertex equitable labeling. Vertex equitable labeling of QS_2 and QS_3 are shown in Figure 4.

4. ALTERNATE QUADRILATERAL SNAKE

Definition: An *alternate quadrilateral snake* $A(QS_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i. That is, every alternate edge of path is replaced by C_4.

Theorem 4.1: Alternate quadrilateral snake $A(QS_n)$, $n \geq 2$ is a vertex equitable graph.

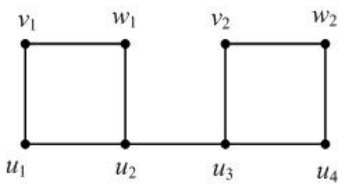
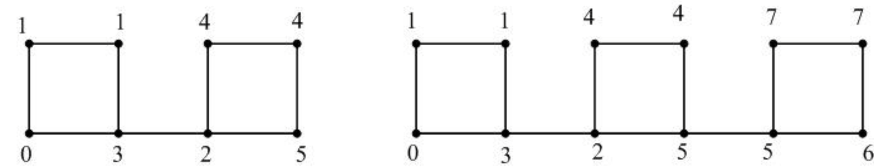


Figure-5



(a) $A(QS_2)$ (b) $A(QS_3)$

Figure-6

Proof: Let $G = A(QS_n)$ be any alternate quadrilateral snake graph on $4n$ vertices and $5n$-1 edges. Label the vertices u_i, v_i and w_i of $A(QS_2)$ as shown in Figure 6(a). Now label $A(QS_3)$ as follows:

(i) Label the vertices of the two blocks C_4 in $A(QS_3)$ as in $A(QS_2)$.

(ii) The remaining vertices of $A(QS_3)$ are labeled as follows:

Define $f: V(G) \longrightarrow \{1, 2, \ldots, \left\lceil \frac{5n-1}{2} \right\rceil\}$ as follows:

(a) First label the vertices v_i, w_i which lie above the path

When n is odd and for $n \geq 3$ define

$f(v_3) = 7$ and $f(v_n) = f(v_{n-2}) + 5$

When n is even and for $n \geq 4$ define

$f(v_4) = 8$ and $f(v_n) = f(v_{n-2}) + 5$

w_i's are also labeled in a similar way.

(b) Now label the path vertices

When n is odd and for $n \geq 5$ define

$f(u_5) = 5$

$f(u_7) = f(u_{n-2}) + 4$

$f(u_9) = f(u_{n-2}) + 1$

When n is even and for $n \geq 6$ define

$f(u_6) = 5$

$f(u_8) = f(u_{n-2}) + 4$

$f(u_{10}) = f(u_{n-2}) + 1$

The above procedure is followed for the rest of the vertices. From the above definition of f we can clearly see that the edge induced labels are 1, 2,…, q as in definition of vertex equitable labeling. Hence $A(QS_n)$ admits vertex equitable labeling. Vertex equitable labeling of $A(QS_2)$ and $A(QS_3)$ are shown in Figure 6.

CONCLUSION

In this paper we have verified vertex equitable labeling for graphs like jewel graph, quadrilateral snake and alternate quadrilateral snake. The study of triangular series graph and interconnection networks like honeycomb network and hexagonal network are under investigation.

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