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ON GENERALIZED SEMI REGULAR CLOSED SETS IN INTUITIONISTIC TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define a new class called the intuitionistic generalized semi regular closed set. Also intuitionistic generalized semi regular connectedness is dealt with and their properties are discussed.

***Key words**: intuitionistic topological space, intuitionistic sets, intuitionistic generalized semi regular closed sets.*

1. INTRODUCTION

The notion of closed set is very much fundamental in the study of topological spaces. The introduction of generalized closed sets was put forward by Levine [6] in the year 1970. This gave rise to a study of the separation axioms, connectedness and continuity of generalized closed sets. The intuitionistic fuzzy set introduced by Atanassov [1] was later developed by Coker [3], who came up with a new concept of intuitionistic sets and intuitionistic points which was helpful to introduce vagueness in Mathematics. Coker [2, 8] dealt with the study of continuity and compactness followed by connectedness. There are at present many research works undertaken in this field and many others [9, 10] have contributed their works. Gnanambal Ilango [4, 5] and Selvanayaki studied in detail about the generalized pre regular closed sets in intuitionistic topological spaces, its properties, connectedness, continuity, compactness. A study on generalized semi regular closed sets in soft topology was done by Mohana K *et.al* [7] where in they studied soft gsr-closed sets, soft gsr-open sets and soft gsr-$T_{1/2}$ spaces. Herewith, in this paper we have studied the concepts of intuitionistic generalized semi regular closed sets in intuitionistic topological spaces.

2. PRELIMINARIES

In this present study, a space X means an intuitionistic topological space (X,τ).

Definition 2.1[5]: Let X be a non empty set. An intuitionistic set (IS) A is an object having the form $A = < X, A_1, A_2>$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2.2[5]: Let X be a non empty set and let A, B are intuitionistic sets in the form $A = < X, A_1, A_2>$, $B = < X, B_1, B_2>$ respectively. Then

(a) $A \subseteq B$ if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
(b) $A = B$ if $A \subseteq B$ and $B \supseteq A$
(c) $\bar{A} = < X, A_2, A_1>$
(d) $[\ ]\,A = < X, A_1, (A_1)^c>$
(e) $A - B = A \cap \bar{B}$.
(f) (f)$\underset{\sim}{\varphi} = < X, \varphi, X >$, $\underset{\sim}{X} = < X, X, \varphi >$
(g) $A \cup B = < X, A_1 \cup B_1, A_2 \cap B_2 >$.
(h) $A \cap B = < X, A_1 \cap B_1, A_2 \cup B_2 >$.

Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = < X, A_i\,(1), A_i\,(2) >$. Then

(i) $\cap A_i = < X, \cap A_i^{(1)}, \cup A_i^{(2)} >$.
(j) $\cup A_i = < X, \cup A_i^{(1)}, \cap A_i^{(2)} >$.

Remark 2.3[5]: Any topological space (X,τ) is obviously an ITS of the form $\tau = \{A' : A \in \tau\}$ where $A' = < X, A, A^c >$.

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Definition 2.4[5]: An intuitionistic topology (IT for short) on a non empty set X is a family of IS's in X containing, and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS for short). Any intuitionistic set in τ is known as an intuitionistic open set (IOS for short) in X and the complement of IOS is called intuitionistic closed set (ICS for short).

Definition 2.5[5]: Let (X,τ) be an ITS and $A = < X, A_1, A_2 >$ be an IS in X. Then the interior and closure of A are defined as
$Icl(A) = \cap\{K : K \text{ is an ICS in X and } A \subseteq K\}$
$Iint(A) = \cup\{G : G \text{ is an IOS in X and } G \supseteq A\}$.

It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A) = A and is an IOS in X iff Iint(A) = A.

Definition 2.6[5]: Let (X,τ) be an ITS. An intuitionistic set A of X is said to be
(i) Intuitionistic semiopen if $A \subseteq Icl(Iint(A))$.
(ii) intuitionistic preopen if $A \subseteq Iint(Icl(A))$.
(iii) intuitionistic regular open (intuitionistic regular closed) if A = Iint(Icl(A)) (A = Icl(Iint(A))).
(iv) intuitionistic α-open if $A \subseteq Iint(Icl(Iint(A)))$.

Definition 2.7: Let (X,τ) be a non empty intuitionistic topological space and let $A = < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be
(i) intuitionistic α −generalized closed (Iαg-closed) if $I\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic open in X.
(ii) intuitionistic generalized semiclosed (Igs-closed) if $Iscl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic open in X.

Definition 2.8[5]: An intuitionistic subset A of (X,τ) is said to be I-dense if Icl(A) = X.

Definition 2.9[5]: A space (X,τ) is called intuitionistic irreducible or I hyper connected if every intuitionistic open subset of X is I-dense.

3. INTUITIONISTIC GENERALIZED SEMI REGULAR CLOSED SETS:

Definition 3.1: Let (X,τ) be an intuitionistic topological space and let $A = < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be intuitionistic generalized semi regular closed (Igsr-closed) if $Iscl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic regular open in X.

Theorem 3.2:
a) Every I-closed set is Igsr-closed.
b) Every Irg-closed set is Igsr-closed.
c) Every Ig-closed is Igsr-closed
d) Every Iαg-closed set is Igsr-closed
e) Every Igs-closed set is Igsr-closed.

Converse of the above theorem is not true and is shown in the following examples:

Example 3.3: Let X = {a, b} and $\tau = \{\varphi, X, < X, \{a\}, \varphi >, < X, \{a\}, \{b\} >, < X, \varphi, \{b\} >\}$. Then the intuitionistic subsets $< X, \{a\}, \varphi >$, $< X, \varphi, \{b\} >$, $< X, \{a\}, \{b\} >$ are Igsr-closed, but not I- closed.

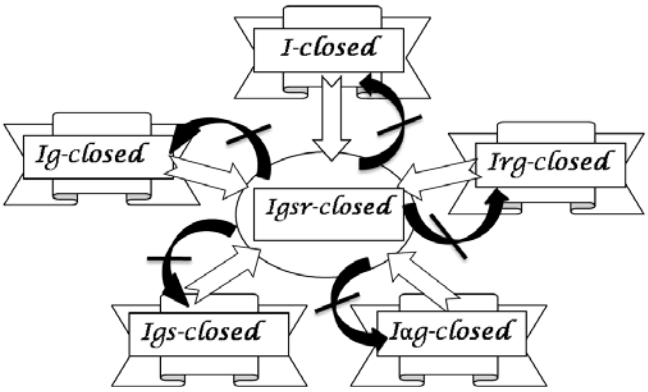
Example 3.4: Let X = {a, b} and $\tau = \{\varphi, X, < X, \{a\}, \varphi >, < X, \{a\}, \{b\} >, < X, \varphi, \{b\} >\}$. Then the intuitionistic subset $< X, \{a\}, \varphi >$ is Igsr-closed, but not Irg-closed.

Example 3.5: Consider X = {a, b} and $\tau = \{\varphi, X, A, B, C\}$ where $A = < X, \{a\}, \varphi >$, $B = < X, \{a\}, \{b\} >$, $C = < X, \varphi, \{b\} >\}$. Then the intuitionistic subsets $< X, \{a\}, \varphi >$, $< X, \varphi, \{b\} >\}$ are Igsr-closed, but not Ig-closed.

Example 3.6: Let X = {a, b} and $\tau = \{\varphi, X, A, B, C\}$ where $A = < X, \{a\}, \varphi >$, $B = < X, \{a\}, \{b\} >$, $C = < X, \varphi, \{b\} >\}$. Then the intuitionistic subsets $< X, \{a\}, \varphi >$, $< X, \varphi, \{a\} >$, $< X, \varphi, \{b\} >$ are Igsr-closed, but not Iαg-closed.

Example 3.7: Let X = {a, b} and $\tau = \{\varphi, X, A, B, C\}$ where $A = < X, \{a\}, \varphi >$, $B = < X, \{a\}, \{b\} >$, $C = < X, \varphi, \{b\} >\}$. Then the intuitionistic subsets $< X, \{a\}, \{b\} >$ is I-open, but not I-regular open.

The diagrammatic representation of the above theorem is shown below:



Theorem 3.8: If A is I-regular open and Igsr-closed, then A is I-semi closed.

Proof: Let A = < X, A_1, A_2 > be I-regular open and Igsr-closed. By definition, we have Iscl(A)⊆A (because U is I-ropen). We know that A ⊆ Iscl(A). Therefore Iscl(A) = A, ⇒ A is I-semi closed.

Remark 3.9: In (X,τ) an intuitionistic topological space, if Iscl(A) = A, then A is Igsr-closed.

Proposition 3.10: Let (X,τ) be an ITS and ISC(X) (IRO(X)) be the family of all intuitionistic semi closed (intuitionistic regular open) sets of X. If ISC(X) = IRO(X) then every intuitionistic subset of X is Igsr-closed in X.

Proof: Let A be an intuitionistic subset of X such that A ⊆ U where U is intuitionistic regular open in X. Given that ISC(X) = IRO(X), then by hypothesis, U is intuitionistic semi closed in X. ⇒Iscl(U) = U. Iscl(A) ⊆ Iscl(U) = U ⇒ A is Igsr-closed in X.

Definition 3.11: Let (X,τ) be an ITS and let A be a subset of X.
Then Igsr-cl(A) = ⋂ {K; K is Igsr-closed in X and A ⊆ K} and
Igsr-int(A) = ⋃ {G; G is Igsr-open in X and G ⊆ A}

Theorem 3.12: Let A be an intuitionistic subset of an intuitionistic topological space (X,τ). Then A is Igsr-open ⟺ U ⊆ Isint(A) whenever U is intuitionistic regular closed and U ⊆ A.

Proof:
Necessity: Let A be Igsr-open in X. U is intuitionistic regular closed in X such that U ⊆ A.
U^c is intuitionistic regular open in X such that $A^c \subseteq U^c$. We know that A^c is Igsr-closed,
Therefore, Iscl(A^c) ⊆ U^c. But Iscl(A^c) = $(\text{Isint}(A))^c \subseteq U^c$.⇒ U ⊆ Isint(A).

Sufficiency: Let B be an intuitionistic regular open set ∋ $A^c \subseteq K$. Then B^c is intuitionistic regular closed in X and $K^c \subseteq$ A. To prove that A^c is Igsr-closed. By hypothesis, $K^c \subseteq$ Isint (A) ⇒ Iscl(A^c) = $(\text{Isint}(A))^c \subseteq K$. Hence A^c is Igsr-closed and ∴ A is Igsr-open in X.

Lemma 3.13: Let A and B be subsets of ITS, then the following results are true;

i) Igsr-cl($\underset{\sim}{X}$) = $\underset{\sim}{X}$ and Igsr-cl($\underset{\sim}{\phi}$) = $\underset{\sim}{\phi}$
ii) If A ⊆ B then Igsr-cl(A) ⊆ Igsr-cl(B)
iii) A ⊆ Igsr-cl(A)
iv) Igsr-cl(A) = Igsr-cl(Igsr-cl(A))

Lemma 3.14: Let A and B be two subsets of the ITS(X,τ), then Igsr-cl(A⋂B) ⊂ Igsr-cl(A) ⋂ Igsr-cl(B).

Proof: A ⋂ B ⊂ A. We have Igsr-cl(A⋂B) ⊆ Igsr-cl(A) and Igsr-cl(A⋂B) ⊆ Igsr-cl(B).
Thus Igsr-cl(A⋂B) ⊂ Igsr-cl(A) ⋂ Igsr-cl(B).

Theorem 3.15: If ISC(X,τ) be closed under finite unions, then IGSRC(X,τ) is closed under finite unions.

Proof: Let ISC(X,τ) be closed under finite unions, let A,B ∈ IGSRC(X,τ) and let A ⋃ B ⊂ U, U is I-ropen in X. A ⊆ U, B ⊆ U. Hence Iscl(A) ⊆ U, Iscl(B) ⊆ U ⇒ Iscl(A) ⋃ Iscl(B) ⊂ U.

By hypothesis, Iscl(A) ⊆ U. So A⋃B ∈ IGSRC(X,τ).

Theorem 3.16: If ISO(X,τ) be open under finite unions, then IGSRO(X,τ) is open under finite unions.

Proof: Let ISO(X,τ) be open under finite unions. Let A, B ∈ IGSRO(X,τ) and let A ⋃ B ⊂ U, U is I-ropen in X. A ⊆ U, B ⊆ U. Hence Iscl(A) ⊆ U, Iscl(B) ⊆ U ⇒ Iscl(A) ⋃ Iscl(B) ⊂ U.

By hypothesis, Iscl(A) ⊆ U. So A⋃B ∈ IGSRO(X,τ).

Corollary 3.17: If ISO(X,τ) be closed under finite unions, then IGSRO(X,τ) is closed under finite unions.

Theorem 3.18: Let A be an Igsr-closed set of an ITS(X,τ) and A ⊆ B ⊂ Iscl(A), then B is Igsr-closed in X.

Proof: Let A be an Igsr-closed set of an ITS(X,τ) and A ⊆ B ⊆ Iscl(A). Let U be I-ropen ∋ B ⊆ U. Since A is Igsr-closed, Iscl(A) ⊆ U whenever A ⊆ U. Therefore now B ⊆ Iscl(A), ⇒ Iscl(B) ⊆ Iscl(Iscl(A)) = Iscl(A) ⊆ U. Therefore, B is Igsr-closed in X.

Theorem 3.19: Let A be an intuitionistic generalized semiregular open set of ITS(X,τ) and Isint(A) ⊆ B ⊆ A, then B is Igsr-open.

Proof: Given Isint(A) ⊆ B ⊆ A. Since $(\mathrm{Isint}(A))^c = \mathrm{Iscl}(A^c)$. $A^c \subseteq B^c \subseteq \mathrm{Iscl}(A^c)$. A^c is Igsr-closed. By theorem 3.18, B^c is also Igsr-closed, then B is Igsr-open.

4. INTUITIONISTIC GENERALIZED SEMI REGULAR CONNECTEDNESS

Definition 4.1: Let (X,τ) be an intuitionistic topological space. Then X is called Igsr-connected if there does not exists an proper intuitionistic set ($\underset{\sim}{\varphi} \neq A \neq \underset{\sim}{X}$) of X which is both Igsr-open and Igsr-closed.

Proposition 4.2: Every Igsr-connected space is intuitionistic connected.

Proof: Let (X,τ) be an Igsr-connected space and assume it is not intuitionistic connected, then there exists a proper intuitionistic subset of X which is both intuitionistic open and intuitionistic closed. We know that every I-open set is Igsr-open, then X is not Igsr-connected which is a contradiction.

Proposition 4.3: Every Igsr-connected space is Ig-connected.

Proof: Let (X,τ) be an Igsr-connected space and assume (X,τ) is not Ig-connected, then there exists a proper intuitionistic subset of X which is both Ig-open and Ig-closed. We know that every Ig-open set is Igsr-open, then X is not Igsr-connected which is a contradiction.

Proposition 4.4: Every Igsr-connected space is Irg-connected.

Proof: Let (X,τ) be an Igsr-connected space and assume it is not Irg-connected, then there exists a proper intuitionistic subset of X which is both Irg-open and Irg-closed. We know that every Irg- open set is Igsr-open, then X is not Igsr-connected which is a contradiction.

Proposition 4.5: Every Igsr-connected space is Iαg-connected.

Proof: Let us consider an Igsr-connected space (X,τ). Suppose it is not Iαg-connected, then there exists a proper intuitionistic subset of X which is both Iαg-open and Iαg-closed. Since every Iαg-open set is Igsr-open, we get that X is not Igsr-connected. This is a contradiction.

Proposition 4.6: Every Igsr-connected space is Igs-connected.

Proof: Assume an Igsr-connected space and let it not be Igs-connected. There exists a proper intuitionistic subset of X which is both Igs-open and Igs-closed. As every Igs-open set is Igsr-open, we get X is not Igsr-connected which is a contradiction.

Theorem 4.7: For an ITS(X,τ), if X is I hyper connected then every intuitionistic subset of X is Igsr-closed.

Proof: If X is I hyper connected, then the only intuitionistic regular open subsets of X are < X, ϕ, X > and < X, X, ϕ >. So every intuitionistic subset of X is Igsr-closed.

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