International Journal of Mathematical Archive-9(3), 2018, 96-100
Available online through www.ijma.info ISSN 2229 – 5046

GROUP ACTION ON FUZZY LEFT N-SUBGROUP OF A NEAR RING

G. BALASUBRAMANIAN*[2] AND P. VANI[3]

[1]Head & Associate Professor of Mathematics,
Government Arts College for Men, Krishnagiri, Tamil Nadu, India.

[2]Guest Lecturer in Mathematics,
Government Arts College for Women, Krishnagiri – 635 002, India.

(Received On: 02-02-18; Revised & Accepted On: 01-03-18)

ABSTRACT

Zadeh [1965] introduced fuzzy set. Mukherjee and Bhattacharya [1986] studied on a fuzzy groups, and some group theoretic results. Wu [1981] obtained few properties on normal fuzzy subgroup. Das [1981] found some new results on fuzzy group and level subgroups. It is also motivated to find N-fuzzy group in hemi-ring. Mukherjee and Bhattacharya [1984] analyzed on fuzzy normal subgroups and fuzzy cosets, and investigates on properties on normality. Nagarajan and Manermaran [2013] gave few algebraic properties on M-fuzzy factor group, and obtained some its power fuzzy group, homomorphic image and preimage, union and intersection.

In this paper, the notion of Q-fuzzification of left N-subgroups is introduced in a near ring and investigated some related properties. Characterization of Q-fuzzy left N subgroup with respect to a triangular norm is given.

Keywords: *Set action on a fuzzy set; group action on fuzzy left N-subgroup in a near-ring.*

SECTION-1: INTRODUCTION

Ray [1992] initiated isomorphic fuzzy group. Makamba [1992] discussed direct product and isomorphism of fuzzy subgroups. Ajmal [1994] studied on homomorphism on factor fuzzy group. They gave an idea to introduce M-fuzzy group, & its normality in a ring. Fang [1994] studied on fuzzy homomorphism & fuzzy isomorphism, and it leads to N-fuzzy subgroup in near-ring. Kim [1997], Kim & Kim [1994], Mukherjee and Bhattacharya [1984, 1986], and analyzed on some characterizations of fuzzy subgroups, and it give an idea to verify also few characterizations on N-fuzzy group in a near-ring. Kumar *et.al* [1992] obtained some new structures on fuzzy normal subgroup and fuzzy quotient group which initiate to make an attempt on N-fuzzy normal subgroup in a near-ring. Kumbhoikar and Bapal [1991] found correspondence theorem for fuzzy ideals in near-ring. Liu *et.al* [2001] and Morsi & Yehia [1994] got few properties on quotient fuzzy group and quotient fuzzy ring induced from fuzzy ideals in near-ring, and same argument will argue quotient N-fuzzy subgroup in near-ring. Further in this paper, we introduce the notion of Q-fuzzification of left N-subgroups in a near ring and discuss usual algebraic properties like as union, intersection, and level cut-set of Q-fuzzification of left N-subgroups in a near ring a. Characterization of Q fuzzy left N-subgroups are given.

SECTION-2: BASIC DEFINITIONS AND PRELIMINARIES

Definition 2.1: A near ring is a non – empty set R with two binary operations + and . satisfying the following axioms: (1). (R, +) is a group; (2). (R, .) is a semigroup; (3). x. (y + z) = x. y + x. z for all x, y, z, \ in R. Then it is called a left near – ring by (3). In this paper, it will use the word near- ring. Here xy denotes x.y; (2). x.0 = 0, and x(-y) = -(xy) for x, y in R.

Definition 2.2: A two sided R – subgroup of a near – ring R is a subset H of R such that (1). (H, +) is a subgroup of (R, +); (2). $RH \subset H$; (3). $HR \subset H$. If H satisfies (1) and (2), then it is a left R-subgroup of R, while if H satisfies (1) and (3), then it is a right R-subgroup of R.

Definition 2.3: A fuzzy set μ in a set R is a function $\mu: R \to [0, 1]$.

Corresponding Author: G. Balasubramanian[2],*
[1]Head & Associate Professor of Mathematics,
Government Arts College for Men, Krishnagiri, Tamil Nadu, India.

Definition 2.4: Let G be any group. A mapping μ: G→ [0, 1] is a fuzzy group if (FG1). $\mu(xy) \geq \min\{\mu(x), \ \mu(y)\}$ and (FG2). $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

Definition 2.5: Let (S, +) be a group, and G be a non-empty set. Then G acts on S if there exists a function $* : G \times S \to S$ (denoted * (g, s) = gs for all g ∈ G, and s ∈ S) such that es = s and (g + h) ∗ s = g ∗ (h ∗ s) for all s in S, and for all g, h in G.

Definition 2.6: A map $f : R \to S$ is called homomorphism if $f(x + y) = f(x) + f(y)$ for all x, y in S.

Definition 2.7: (T-norm) A triangular norm is a function $T: [0,1] \times [0,1] \to [0,1]$ that satisfies the following conditions for all x, y, z in [0, 1].
(T1): T(x, 1) = x;
(T2). T(x, y) = T(y, x);
(T3): T(x, T(y, z)) = T(T(x, y), z);
(T4). T(x, y) ≤ T(x, z) if y ≤ z.

Definition 2.8: Let .there exist a map (namely multiplication) from N × R → R. A group (G, Δ) with identity 0 acts on a N-fuzzy group A of a near-ring (R, +, .) if (GAFS1) the group G acts on R [there exists a function ∗ : G × R → R with the conditions g ∗ (h∗ s) = (g + h) ∗ s and e ∗ s = s or all s in S, and for all g, h in G];

(GAFG2) $A(x \ * n(s - \ t) \geq T\{\ A(x * s), \ A(x * t)\}$;

(GAFG3) $A(x \ * (ns)) \geq T\ (A(x * s))$; for all n in N; and for all x, y ∈ G and s, t ∈ S.

Definition 2.9: Let θ be a mapping from X to Y. (i) Let (G′, Δ′) a group acting on R′-fuzzy group B under a near-ring (R′, +′, .′). Then the inverse image of B under θ denoted by $\theta^{-1}(B)$ is fuzzy set in (G, Δ) defined by $\theta^{-1}(B) = \mu_\theta^{-1}(B)$ where $\mu_\theta{}^{-1}{}_{(B)}(x) = \mu_B(\theta(x))$; (ii) Let (G, Δ) be a group acting on R-fuzzy group A under a near-ring (R, +, .).

Definition 2.10: Then the image of A under θ denoted by θ(A), where $\mu_{\theta(A)}(y) = \{\text{Sup } \mu_{A(x)} : x \in \theta^{-1}(y)$ if $\theta^{-1}(y) \neq 0$; 0, otherwise. Then $\mu_\theta{}^{-1}{}_{(B)}(x * s) = \mu_B(\theta(x) * s)$ for all s in S. Also $\mu_{\theta(A)}(y * s) = \{\text{Sup } \mu_A(x * s) : (x * s) \in (\theta^{-1}(y) *^1 s)$ for all s in S if $\theta^{-1}(y) \neq 0$; = 0, otherwise.

SECTION-3: GROUP ACTION ON FUZZY N-GROUP OF A NEAR-RING

Theorem 3.1: Let 'T' be a t-norm. Then every imaginable fuzzy left N-subgroup μ of a near ring R acted by a group (G, Δ) is a fuzzy left N-subgroup of R acted by G.

Proof: Assume μ is an imaginable fuzzy left N-subgroup of R. Then it gets that

μ (x ∗ n(s-t)) ≥ T{μ (x ∗ s), μ (x ∗ t)} and μ (x ∗ (ns)) ≥ μ (x ∗ s). for all s, t in R and x in G.

Since μ is imaginable, it follows that

min {μ (x ∗ s), μ (x ∗ t)} = T{ min {μ (x ∗ s), μ (x ∗ t), min {μ (x ∗ s), μ (x ∗ t)}
≤ T{ min {μ (x ∗ s), μ (x ∗ t)}
≤ min {μ (x ∗ s), μ (x ∗ t)}

and so

T{min {μ (x ∗ s), μ (x ∗ t)} ≤ min {μ (x ∗ s), μ (x ∗ t)} .

It also finds that μ(x ∗ n(s-t)) ≥ T{min {μ (x ∗ s), μ (x ∗ t)}
= min {min {μ (x ∗ s), μ (x ∗ t)} for all s, t in R, and x in G.

Hence G acts on μ 'μ' is a fuzzy left N-subgroup of R acted by G.

Theorem 3.2: If μ is fuzzy left N-subgroup of a near ring (R, +, .) acted by a group (G, Δ), and Q is a endomorphism of R, then $\mu_{(Q)}$ is a fuzzy left N-subgroup of R acted by G..

Proof: μ is fuzzy left N-subgroup of a near ring (R, +, .) acted by a group (G, Δ).

Define $\mu_{(Q)}$: R → R by $\mu_{(Q)}(x * s) = \mu(Q(x * s))$ for all x in G, and s in R. Clearly G acts also on fuzzy subgroup $\mu_{(Q)}$ of R.

For any s, t in R, and x in G, it gets that

(i). $\mu_{(Q)}(x * n(s\text{-}t)) = \mu(Q(x * ((n(s\text{-}t))))$
= μ (Q(x ∗ s), Q(x ∗ t))
≥ T {μ(Q(x ∗ s), Q(x ∗ t))
= T {$\mu_{(Q)}(x * s)$, $\mu_{(Q)}(x * t)$}

(ii). $\mu_{(Q)}(x * (ns)) = \mu(Q(x * (ns))$
$\geq \mu(Q(x * s)$
$\geq \mu_{(Q)}((x * s)$

Hence G acts on fuzzy N-subgroup $\mu_{(Q)}$ of R.

Theorem 3.3: Onto homomorphism pre-image of a fuzzy left N-subgroup of near ring R′ acted by a group (G, Δ), is fuzzy left N-subgroup of a near-ring R acted by G.

Proof: Let f: R → R′ be an onto homomorphism of near rings and A be a fuzzy left N-subgroup of R′ acted by G. Let B be the pre-image of A under f.

Then (i).$B((x * (s-t)) = A(f(x * (s-t)))$
$= A(f(x * s), f(x * t))$
$\geq T\{B(x * s), B(x * t)\}$
$= T(f(x * s), f(x * t))$

(ii). $B((x * (ns))) = A(f(x * (ns))$
$= A(f(x * s)$
$= B(x * s).$

Thus B is a fuzzy left N-subgroup of R acted by G.

Theorem 3.3: Onto homomorphism pre-image **under supreme property** of a fuzzy left N-subgroup of near ring R′ acted by a group (G, Δ), is fuzzy left N-subgroup of a near-ring R′ acted by G.

Proof: Let f: R → R′ be an onto homomorphism on near-rings. Let μ be fuzzy N-subgroup of R′.

By superme property, let $x^1, y^1 \in R'$ and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$, be such that

$$\mu(x * x_0) = \sup_{(x*h) \in f^{-1}(x^1)} \mu(x * h)$$
$$\mu(x * y_0) = \sup_{(x*h) \in f^{-1}(y^1)} \mu(x * h) \text{ respectively.}$$

Then we can deduce that

(1). $\mu^f(x * (n(x^1 - y^1))) = \sup_{(x*z) \in f^{-1}(x*(n(x^1-y^1)))} \mu(x * z)$
$\geq T\{\mu(x * x_0), \mu(x * y_0)\}$
$\geq T\left\{\sup_{(x*h) \in f(x)} (x * x_0), \sup_{(x*h) \in f(y)} (x * y_0)\right\}$
$= T\{\mu^t(x * x^1), \mu^t(x * y^1)\}$

(2). $\mu^f(x * nx^1) = \sup_{(x*z) \in f^{-1}(x*(nx^1))} \mu(x * z)$
$\geq \mu(x * x_0)$
$= \sup_{(x*x_0) \in f^{-1}(x*x^1)} \mu(x * x_0)$
$= \mu^t(x * x^1).$

Hence μ^f is a fuzzy left N-subset of R acted by G.

Theorem 3.5: Let T be a continuous t-norm and f be a homomorphism on a near ring R. If μ is fuzzy left N-subgroup of S acted by a group (G, Δ), then μ^f is a fuzzy left N-subgroup of f(R) acted by G.

Proof: It follows that μ^f is a fuzzy left N-subset f f(R) acted by G

Let $A_1 = f^{-1}(x * y_1)$; $A_2 = f^{-1}(x * y_2)$;

Let $A_{12} = f^{-1}(x * n(y_{1-} y_2))$ where $y_{1,} y_1$ in f(R), n in N, and x in G.

Consider the set $A_1 - A_2 = \{s \in R: (x * s) = (x * a_1) - (x * a_2)\}$ for some $(x * a_1) \in A_1$, and $(x * a_2) \in A_2$.

If $f((x * s) \in A_1 - A_2$, then $(x * s) = (x * a_1) - (x * a_2)$ for some $(x * a_1) \in A_1$, and $(x * a_2) \in A_2$.

It gets that $f(x * s) = f(x * a_1) - f(x * a_2) = y_1 - y_2$

Thus $(x * s) \in f^{-1}(x * y_1) - (x * y_2)) = f^{-1}(x * n(y_1 - y_2)) = A_{12}$.

Implies that $A_1 - A_2 \subset A_{12}$.

It follows that

$$\begin{aligned}\mu^t[x * n(y_1 - y_2)) &= Sup\ \{\ \mu(x * s) :\ (x * s) \in f^{-1}((x * n(y_1 - y_2))\\ &= Sup\ \{\ \mu(x * s) :\ (x * s)\ \in\ A_{12}\}\\ &\geq Sup\{\ \mu(x * s)\ /\ (x * s) \in A_1 - A_2\}\\ &\geq Sup\ \{\ \mu(x * x_1) - (x * x_2) : x * x_1 \in A_1 \text{ and } x * x_2 \in A_2.\end{aligned}$$

Since T is continuous, and every $\in > 0$, there exists $\delta > 0$ such that

$Sup\ \{\mu(x * x_1) : (x * x_1)\ \in\ A_1\ \} -\ (x * x_2) \in A_2\} \leq \delta$ and

$Sup\{\ \mu(x * x_{2)}) : x * x_1 \in\ A_2\} -\ (x * x_2)\ \in A_2\} \leq \delta$,, then we get

$T\ \{Sup\{\ \mu(x * x_1) : (x * x_1) \in A_1\ \}, Sup\ \{\ \mu(x * x_2) : x * x_2) \in A_2\ \} -\ T\ \{(x * x_1), (x * x_2)\} \leq \in.$

Choose $(x * a_1) \in A_1$, and $(x * a_2) \in A_2$ with $Sup\ \{\mu(x * x_1) : (x * x_1)\ \in A_1\} - \mu(x * a_1) \leq \delta\}$ then it finds that

$T\ \{\ Sup\ \{\ \mu(x * x_1) : (x * x_1) \in A_1\ \}, Sup\ \{\ \mu(x * x_2) : (x * x_2)\ \in\ A_2\} - T\{\mu(x * a_1) - \mu(x * a_2\ )\} \leq \in.$

It becomes now that

$$\begin{aligned}\mu^f(x * n(y_1 - y_2)) &\geq \sup\{T(\mu(x * x_1) \in A_1 and\ \mu(x * x_2) \in A_2) : (x * x_1) \in A_1\ ;\ (x * x_2) \in A_2\}\\ &\geq T\{sup\{\mu(x * x_1) : (x * x_1) \in A_1\}.\ \sup\ \{\mu(x * x_2) : (x * x_2) \in A_2)\}\}\\ &\geq T\{\mu^f(x * y_1), \mu^f(x * y_2))\}\end{aligned}$$

Similarly, it can show $\mu^f(nx, q) \geq \mu^f[x, q)$. Hence G acts fuzzy left N-subgroup μ^f of f(R).

Theorem 3.6: Let μ is fuzzy left N-subgroup of a near-ring R acted by a group (G, Δ). Then the fuzzy set < μ >: is a fuzzy left N-subgroup of R generated by μ. Also G acts on <μ> as the smallest fuzzy left N-subgroup containing it.

Proof: Let $u, v \in R$; $\mu(x * u) = t_1$; $\mu(x * v) = t_2$; $\mu(x * n(u\text{-}v)) = t$.

Let it possible

$$\begin{aligned}t &= <\mu> (x * n(u\text{-}v))\\ &\leq T\ \{<\mu> (x * (nu)), <\mu> ((x * (nv))\\ &\leq T\ \{<\mu> (x * u), <\mu> ((x * v))\\ &= T\ \{t_1, t_2\} = t_1 \text{ (say).}\end{aligned}$$

Then $t_1 = <\mu> (x * u) = \sup\ \{k:\ (x * u) \in <\mu_k>\} \geq t$.

Therefore there exists k with $(x * u) \in <\mu_k>$.

Also $t_2 = <\mu> (x * v) = \sup\ \{k:\ (x * v) \in <\mu_k>\} \geq t$.

Therefore there exists $m > t$ with $(x * v) \in <\mu_l>$.

Without loss of generality, assume that k, m with $<\mu_k> \subset <\mu_m>$.
hen $u, v \in <\mu_k>$, which is a contradiction since $m > t$. Therefore $t \geq t_1$.

Consequently, $\mu\big(x * n(u - v)\big) \geq T\ \{<\mu> (x * u), \mu> (x * v)\}$ (1)

Now let, if possible $t_3 = \{<\mu> (x * (nu)) \leq <\mu> (x * u) = t_1$.
Then $t_1 = <\mu_k> (x * u) = Sup\ \{k: (x * u) \in <\mu_k>\} > t_3$.

So there exists k with $x * u \in <\mu_k>$, and $t_1 > k > t_3$ so that $n(x * u) \in <\mu_k> \subset <\mu_t>$, which is a contradiction. Hence $t_3 = \{<\mu>(x * (nu)) \geq <\mu>(x * u) = t2$ (2)

The equations (1) and (2) yield that G acts on fuzzy left N-subgroup < μ > of R.

Finally to show that $<\mu>$ is the smallest fuzzy left N-subgroup containing μ acted by G.

For this, assume that G acts on fuzzy left N-subgroup Q of R such that $\mu \subset Q$, and show that $<\mu> \subset Q$. Let it possible, $t = <\mu> (x * u) \geq Q(x * u)$ for some x in G, and u in R.

Let $\in > 0$ be given. Then $t = \mu_t = Sup\ \{k: (x * u) \in <\mu_k>$ and $t - \in \leq k < t\}$ implies that $(x * u) \in <\mu> \subset <\mu_{k\,t\,-\,\in}>$ for all $\in > 0$.

Now a = $a_1 (x * x_1) + a_2 (x * x_2) + a_3 (x * x_3) +\ldots+ a_n (x * x_n)$ where $a_i \in N$, and $(x * x_i)$ belongs to $(t-\in)$. $(x * x_i) \in \mu_{t-\in}$ implies that $\mu(x * x_i) \geq t - \in$. Thus $Q (x * u) \geq t - \in$ for all $\in$. 0

So that $Q (x * u) \geq T \{ Q (x * u_1), Q (x * u2), \ldots, Q (x * u_n\} \geq t - \in$ for all $\in > 0$.

Hence $Q (x * u) = t$ which is a contradiction to our supposition

Theorem 3.7: Let a group (G, Δ) acts a fuzzy left N-subgroup μ of a near ring R and μ^+ be a fuzzy set in R defined by $\mu^+ (x * u) = \mu(x * u) + 1-\mu(x * 0)$ for all u in R, and x in G. Then G acts on a normal fuzzy N-subgroup μ^+ of R containing $'\mu'$.

Proof: Let $u, v \in R$, and $x \in G$. We have

(i). $\mu^+ (x * n(u-v)) = \mu(x * n(u-v)) + 1 - \mu(x * 0))$

$\geq \{\mu(x * u) + 1 - \mu(x * nv)\} + 1 - \mu (x * 0)$

$\geq T\{\mu(x * u) +1 - \mu (x * 0, \mu(x * v)\} + 1 - \mu(x * 0))$

$\geq T\{\mu^+(x *u), \mu^+(x *v)\}$

(ii). $\mu^+ (x * nu) = \mu(x * nu) + 1 - \mu(x * 0)$

$\geq \mu(x * u) + 1 - \mu(x * 0)$

$= \mu^+(x * u)$

REFERENCES

1. Abou.Zoid, S., On Fuzzy sub near rings and ideals, Fuzzy sets and systems, 44,(1991),139-146.
2. Ajmal, N., Homomorphism of fuzzy groups, correspondence theorem and fuzzy quotient groups, Fuzzy Sets and Systems, Volume 61, (1994), 329-339.
3. Das: P.S., Fuzzy groups and level subgroups, J. Math. Anal. Appl. Volume 84, (1981), 264-269.
4. Fang: J.X., Fuzzy homomorphism and fuzzy isomorphism, Fuzzy Sets and Systems, Volume 63 (1994), 237-242*:*
5. Kim: J.G., Some characterizations of fuzzy subgroups, Fuzzy Sets and Systems, Volume 87 (1997), 243-249*:*
6. Kim, J.G., and Kim: H.D., A characterization of fuzzy subgroups of some abelian groups, Inform. Sci. Volume 80, (1994), 243- 252*:*
7. Kumar, I.J., Saxena, P.K., and Yadav, P., Fuzzy normal subgroups and fuzzy quotients, Fuzzy Sets and Systems, Volume 46, (1992), 121-132
8. Kumbhojkar, H.V, and Bapat,M.S., Correspondence theorem for fuzzy ideals, Fuzzy Sets and Systems, Volume 41, (1991), 213-219.
9. Liu, Y.L., Meng, J., and Xin, X.L., Quotient rings induced via fuzzy ideals, Korean J. Comput & Appl. Math. Volume 8, (2001), 631-643.
10. Makamba, B.B., Direct products and isomorphism of fuzzy subgroups, Inform. Sci. Volume 65 (1992), 33-43.
11. Morsi, N.N., & Yehia, S.E.B., Fuzzy-quotient groups, Inform. Sci. Vol. 81, (1994), 177-191.
12. Mukherjee, N.P., and Bhattacharya, P., Fuzzy groups: some group theoretic analogs, Inform. Sci. Volume 39, (1986), 247-268.
13. Mukherjee, N.P., and Bhattacharya,P., Fuzzy normal subgroups and fuzzy cosets, Inform. Sci. Volume 34, (1984), 225-239.
14. Nagarajan, R., and Manermaran, S.V., Contributions on M-fuzzy factor group, International Journal of Mathematical Archive, Volume 4 (7), (2013), 253-256.
15. Ray, S., Isomorphic fuzzy groups, Fuzzy Sets and Systems, Volume 50 (1992), 201-207.
16. Rosenfeld: A., Fuzzy groups, J. Math. Anal. Appl. Volume 35, (1971), 512-517.
17. Wu: W.M, Normal fuzzy subgroups, Fuzzy Math. Volume 1, (1981), 21-30.
18. Zadeh, L.A., Fuzzy sets, Inform. Control, Volume 8, (1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared.