

MHD FLOW THROUGH POROUS MEDIA PAST AN OSCILLATING VERTICAL PLATE WITH VARIABLE TEMPERATURE AND CONSTANT MASS DIFFUSION IN THE PRESENCE OF HALL CURRENT

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ABSTRACT

In the present paper, MHD flow through porous media past an oscillating vertical plate with variable temperature and constant mass diffusion in the presence of Hall current is studied. The fluid considered is an electrically conducting, absorbing-emitting radiation but a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile and skin friction. The velocity profile and skin friction have been studied for different parameters like Schmidt number, Hall parameter, magnetic parameter, mass Grashof number, thermal Grashof number, Prandtl number, and time. The effect of parameters are shown graphically and the value of the skin-friction for different parameters has been tabulated.

Keywords: MHD, Hall current, Skin friction.

INTRODUCTION

The study of MHD flow through porous media with Hall effect plays an important role in engineering and biological science. MHD flow models with Hall effect have been studied by a number of researchers, some of which are mentioned here. Datta and Jana [7] have studied oscillatory magneto hydrodynamic flow past a flat plate will Hall effects. Pop and Watanabe [5] have studied Hall effects on magneto hydrodynamic boundary layer flow over a continuous moving flat plate. Attia and Ahmed [1] have studied the Hall effect on unsteady MHD couette flow and heat transfer of a Bingham fluid with suction and injection. Sudhakar, et al. [6] have studied Hall effect on unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction. Deka [8] has analyzed Hall effects on MHD flow past an accelerated plate. Some research articles related to MHD flow with oscillating plate are mentioned here. Muthucumaraswamy and Janakiraman [9] have analyzed mass transfer effect on isothermal vertical oscillating plate in the presence of chemical reaction. Further, we would mention some of the research articles which are also related to my work. Attia [2] has analyzed unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Rajput and kumar [10] have studied radiation effects on MHD flow through porous media past an exponentially accelerated vertical plate with variable temperature. Rajput and Sahu [11] have studied transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. Uddin and Kumar [12] have investigated MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation. Raptis et al. [3] have studied flow of a viscous fluid through a porous medium bounded by a vertical surface. Raptis along with Kafousias et al. [4] have further studied heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of magnetic field. We are considering, MHD flow through porous media past an oscillating vertical plate with variable temperature and constant mass diffusion in the presence of Hall current. The effect of Hall current on the velocity have been observed with the help of graphs, and the skin friction has been tabulated.

MATHEMATICAL ANALYSIS

An unsteady viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plate is considered here. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \le 0$ the temperature of the fluid and the plate are at the same as temperature T_{∞} , and the concentration of the fluid is C_{∞} . At time t > 0, the plate starts oscillating in its own plane with frequency ω_{\perp} .

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temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w . Using the relation $\nabla \cdot B = 0$, for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Due to Hall effect, there will be two components of the momentum equation, which are as under. The usual assumptions have been taken in to consideration. The fluid model is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_{\infty}) + g\beta^*(C - C_{\infty}) - \frac{\sigma B_0^2}{\rho(1 + m^2)}(u + mw) - \frac{\upsilon}{K}u, \qquad (1)$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (w - mu) - \frac{\upsilon}{K} w, \qquad (2)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2},\tag{3}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2}.$$
(4)

The following boundary conditions have been assumed $t \le 0$: $u = 0, w = 0, C = C_{\infty}, T = T_{\infty}$, for all th

$$t \ge 0: u = 0, w = 0, C = C_{\infty}, T = T_{\infty}, \text{ for all the values of } y,$$

$$t > 0: u = u_0 \cos \omega t, w = 0, C = C_w, T = T_{\infty} + (T_w - T_{\infty}) \frac{u_0^2 t}{\upsilon} at \ y = 0,$$

$$u \to 0, w \to 0, C \to C_{\infty}, T \to T_{\infty} \quad as \ y \to \infty.$$
(5)

Here *u* is the velocity of the fluid in x- direction (primary velocity u) wis the velocity of the fluid in z- direction (secondary velocity w) *m* - Hall parameter, g- acceleration due to gravity, β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of concentration expansion, t - time, C_{∞} - the concentration in the fluid far away from the plate, C - species concentration in the fluid, C_w - species concentration at the plate, D - mass diffusion, T_{∞} - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid near the plate, γ_w - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, and C_p - specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\overline{u} = \frac{u}{u_0}, \overline{w} = \frac{w}{u_0}, \overline{y} = \frac{yu_0}{v}, Sc = \frac{v}{D}, Pr = \frac{\mu C_P}{k}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \overline{t} = \frac{t u_0^2}{v}, \overline{\omega} = \frac{\omega v}{u_0^2},$$

$$\overline{K} = \frac{K u_0}{v^2}, G_r = \frac{g\beta v (T_w - T_w)}{u_0^3}, Gm = \frac{g\beta v (C_w - C_w)}{u_0^3}, \overline{C} = \frac{C - C_w}{C_w - C_w}, \theta = \frac{(T - T_w)}{(T_w - T_w)}.$$
(6)

Here the symbols used are:

 \overline{u} is the dimensionless velocity of the fluid in x- direction (primary velocity), \overline{w} is the dimensionless velocity of the fluid in z- direction (secondary velocity) θ - the dimensionless temperature, \overline{C} - the dimensionless concentration, Gr - thermal Grashof number, Gm - mass Grashof number, μ - the coefficient of viscosity, Pr - the Prandtl number, Sc - the Schmidt number, M - the magnetic parameter.

The dimensionless forms of equations (1), (2), (3) and (4) with corresponding boundary conditions (11) are as follows

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + Gr\theta + Gm\overline{C} - \frac{M(\overline{u} + m\overline{w})}{(1+m^2)} - \frac{1}{K}u,$$
(7)

$$\frac{\partial \overline{w}}{\partial \overline{t}} = \frac{\partial^2 \overline{w}}{\partial \overline{y}^2} - \frac{M(\overline{w} - m\overline{u})}{(1+m^2)} - \frac{1}{K}w,$$
(8)

$$\frac{\partial \overline{C}}{\partial \overline{C}} = \frac{1}{2} \frac{\partial^2 \overline{C}}{\partial \overline{C}}$$
(9)

$$\frac{\partial \overline{t}}{\partial \theta} = \frac{1}{2} \frac{\partial^2 \theta}{\partial \theta}$$

$$\partial \bar{t} = \Pr \partial \bar{y}^2$$
 (10)

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$$\overline{t} \leq 0, \overline{u} = 0, \overline{C} = 0, \theta = 0, \overline{w} = 0, \text{ for all values of } \overline{y},$$

$$\overline{t} > 0, \overline{u} = \cos \overline{\omega} \overline{t}, \overline{w} = 0, \theta = \overline{t}, \overline{C} = 1 \text{ at } \overline{y} = 0,$$

$$\overline{u} \to 0, \overline{C} \to 0, \theta \to 0, \overline{w} \to 0 \text{ as } \overline{y} \to \infty.$$

$$(11)$$

Dropping the bars and combining the Equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta + GmC - \left(\frac{M}{1+m^2}(1-mi) + \frac{1}{K}\right)q,\tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_C} \frac{\partial^2 C}{\partial v^2},\tag{13}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2},\tag{14}$$

where q = u + iw, with corresponding boundary conditions

$$t \le 0: q = 0, \theta = 0, C = 0, \text{ for all values of } y,$$

$$t > 0: q = \cos \omega t, \theta = t, C = 1, at \ y = 0,$$

$$q \to 0, C \to 0, \theta \to 0, as \ y \to \infty.$$
(15)

$$\begin{split} q &= \frac{1}{4} e^{-itoy} + A_0 + \frac{1}{4a^2} y \ Gr\{A_{13}(1 - \Pr - at)\} + \sqrt{a} \ e^{-\sqrt{a}y} (A_1 - e^{2\sqrt{a}y}) + B_{13}\{A_{14}(1 - \Pr)\} \\ &+ \frac{1}{2a} Gm(-e^{-\sqrt{a}y}A_1 + e^{\frac{at}{-1+Sc}\sqrt{\frac{aSc}{-1+Sc}}}(1 + B_{11} + e^{2\sqrt{\frac{aSc}{-1+Sc}}}B_{12})) \\ &- \frac{1}{2a^2\sqrt{\pi}} Gr\sqrt{\Pr} y(-B_{14}\{-1 + \sqrt{\Pr} + at\} + a(2e^{\frac{-\Pr y^2}{4t}}\sqrt{t} - \sqrt{\pi}\sqrt{\Pr} y\{1 - Erf[\frac{\sqrt{\Pr} y}{2\sqrt{t}}]\}) \\ &+ \frac{1}{y} e^{\frac{at}{-1+\Pr}\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}} \sqrt{\pi} B_{17} \frac{1}{\sqrt{\Pr}} \Pr r - 1) \\ &- \frac{1}{2a} Gm[-2Erfc[\frac{\sqrt{Sc} y}{2\sqrt{t}}] + e^{\frac{at}{-1+Sc}\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}y} (1 + B_{18} + e^{2\sqrt{\frac{aSc}{-1+Sc}}y})] \\ \theta &= \left[\left(t + \frac{\Pr y^2}{2}\right) Erf\left(\frac{\sqrt{\Pr y}}{2\sqrt{t}}\right) - e^{-\frac{y^2}{4t}^2 \Pr} \frac{\sqrt{\Pr t} y}{\sqrt{\pi}} \right], \\ C &= Erfc\left[\frac{\sqrt{Sc} y}{2\sqrt{t}}\right]. \end{split}$$

SKIN FRICTION

The dimensionless skin friction at the plate y = 0 is computed by

$$\left(\frac{dq}{dy}\right)_{y=0} = \tau_x + i\tau_z$$

Separating real and imaginary parts in $\left(\frac{dq}{dy}\right)_{y=0}$, the dimensionless skin– friction components:

$$\tau_x = \left(\frac{du}{dy}\right)_{y=0}$$
 and $\tau_z = \left(\frac{dw}{dy}\right)_{y=0}$ are obtained.

RESULT AND DISCUSSIONS

The numerical values of velocity and skin friction are computed for different parameters like thermal Grashof number Gr, mass Grashof number Gm, magnetic field parameter M, Hall parameter m, Prandtl number Pr, Schmidt number Sc, permeability of the medium K and time t. The values of the main parameters considered are:

Gr = 10, 20, 30; *M* = 2, 3, 4; *m* = 1, 5; *Gm* = 10, 20, 30, *Pr* = 0.71, 7; *Sc* = 2.01, 5, 10; ωt = 30°, 45°, 90° *K*= 0.2, 0.5, 1; and *t* = 0.15, 0.2, 0.25.

Figures 1, 2, 6, 7 and 9 show that primary velocity increases when *m*, *Gm*, *Gr*, *t*, and *K* are increased. Figures 3 4, 5, and 8 show that primary velocity decreases when *M*, *Sc*, *Pr*, and $\mathcal{O}t$ are increased. And figures 11, 12, 15, 16, and 18 show that the secondary velocity increases when *Gm*, *M*, *Gr*, *t*, and *K* are increased. Figures 10, 13, 14 and 17 show that secondary velocity decreases when *m*, *Sc*, *Pr*, and $\mathcal{O}t$ are increased.



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Figure-13: Velocity w for different values of Sc

Figure-14: Velocity w for different values of Pr



Table for the skin friction										
т	Gr	Gm	М	Sc	Pr	K	<i>Wt</i> (in degree)	t	$ au_x$	$ au_z$
0.5	10	10	2	2.01	0.71	0.2	30	0.2	0.0183	0.1612
0.5	10	10	2	2.01	7.00	0.2	30	0.2	-0.1302	0.1580
0.5	20	10	2	2.01	0.71	0.2	30	0.2	0.3397	0.1656
0.5	30	10	2	2.01	0.71	0.2	30	0.2	0.6610	0.1700
1.0	10	10	2	2.01	0.71	0.2	30	0.2	0.1390	0.2078
5.0	10	10	2	2.01	0.71	0.2	30	0.2	0.3403	0.0840
0.5	10	20	2	2.01	0.71	0.2	30	0.2	1.7995	0.1908
0.5	10	30	2	2.01	0.71	0.2	30	0.2	3.5806	0.2205
0.5	10	10	3	2.01	0.71	0.2	30	0.2	-0.1433	0.2322
0.5	10	10	4	2.01	0.71	0.2	30	0.2	-0.2997	0.2977
0.5	10	10	2	5.00	0.71	0.2	30	0.2	-0.3785	0.1490
0.5	10	10	2	10.0	0.71	0.2	30	0.2	-0.6580	0.1424
0.5	10	10	2	2.01	0.71	0.2	30	0.15	-0.2763	0.1434
0.5	10	10	2	2.01	0.71	0.2	30	0.25	0.2721	0.1763
0.5	10	10	2	2.01	0.71	0.2	45	0.2	0.6695	0.1465
0.5	10	10	2	2.01	0.71	0.2	90	0.2	3.4076	0.0785
0.5	10	10	2	2.01	0.71	0.5	30	0.2	0.6733	0.1900
0.5	10	10	2	2.01	0.71	1	30	0.2	0.9178	0.2018

CONCLUSION

Some conclusions of the study are as under

- Primary velocity increases with the increase in thermal Grashof number, Hall parameter, mass Grashof number, permeability of the medium and time. However, it decreases with the increase in and Prandtl number, magnetic field parameter, Schmidt number and phase angle.
- Secondary velocity increases with increase in thermal Grashof number, mass Grashof number, time, permeability of the medium and magnetic field parameter. However, it decreases with the increase Hall parameter, Schmidt number, Prandtl number and phase angle.
- Skin fraction τ_x decreases with increase in Schmidt number, Prandtl number, magnetic field parameter and phase angle. It increases with thermal Grashof number, mass Grashof number, Hall parameter, time and permeability of the medium. τ_z increases with increase in thermal Grashof number, mass Grashof number, magnetic field parameter, time, phase angle, and permeability of the medium. It decreases with Prandtl number, Hall parameter and Schmidt number increased.

APPENDIX

$$\begin{split} A_{0} &= A + B + C_{0} + D_{0} - E_{0} - F_{0} - G_{0} - H_{0}, \quad A_{1} = A_{11} + A_{12}, \quad A = e^{y\sqrt{a-i\omega}}, \quad B = e^{y\sqrt{a-i\omega}}, \quad C_{0} = e^{-y\sqrt{a+i\omega}+2it\omega}, \\ D_{0} &= e^{y\sqrt{a+i\omega}+2it\omega}, \quad E_{0} = AErf\left[\frac{y-2t\sqrt{a-i\omega}}{2\sqrt{t}}\right], \quad F_{0} = BErf\left[\frac{y+2t\sqrt{a-i\omega}}{2\sqrt{t}}\right], \quad G_{0} = C_{0}Erf\left[\frac{y-2t\sqrt{a+i\omega}}{2\sqrt{t}}\right], \\ H_{0} &= D_{0}Erf\left[\frac{y+2t\sqrt{a+i\omega}}{2\sqrt{t}}\right], \quad A_{11} = 1 + Erf\left[\frac{2\sqrt{at-y}}{2\sqrt{t}}\right], \quad A_{12} = e^{2\sqrt{ay}}Erfc\left[\frac{2\sqrt{at+y}}{2\sqrt{t}}\right], \quad A_{13} = \frac{2e^{-\sqrt{ay}}(1 + e^{2\sqrt{ay}} + A_{11} - A_{12})}{y}, \\ A_{13} &= \frac{2e^{-\sqrt{ay}}(-1 - e^{2\sqrt{ay}} - A_{11} + A_{12})}{y}, \quad B_{14} = \frac{Efr\left[2\sqrt{\frac{aPr}{-1 + Pr}t - y}\right]}{2\sqrt{t}}, \quad B_{2} = \frac{Efr\left[2\sqrt{\frac{aPr}{-1 + Pr}t + y}\right]}{2\sqrt{t}}, \quad B_{13} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1 + Sc}t - y}\right]}{2\sqrt{t}}, \quad B_{16} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1 + Sc}t + \sqrt{Pr}y}\right]}{2\sqrt{t}}, \\ B_{13} &= \left(-1 - e^{\frac{2\sqrt{aPr}}{-1 + Pr}y} - B_{1} + e^{\frac{2\sqrt{aPr}}{-1 + Pr}y}B_{2}\right), \quad B_{14} = \frac{2\sqrt{\pi}\left(-1 + Erf\left[\frac{\sqrt{Pr}y}{2\sqrt{t}}\right]\right)}{\sqrt{Pr}y}, \quad B_{18} = \frac{Efr\left[2\sqrt{\frac{a}{-1 + Pr}t - \sqrt{Pr}y}\right]}{2\sqrt{t}}, \quad B_{16} = \frac{Efr\left[2\sqrt{\frac{a}{-1 + Pr}t + \sqrt{Pr}y}\right]}{2\sqrt{t}}, \\ B_{17} &= \left(1 + e^{2\sqrt{\frac{a}{-1 + Pr}\sqrt{Pr}y}} + B_{15} - e^{2\sqrt{\frac{a}{-1 + Pr}\sqrt{Pr}}}B_{16}\right), \quad B_{18} = Erf\left[\frac{2\sqrt{\frac{a}{1 + Sc}t - \sqrt{Sc}y}}{2\sqrt{t}}\right], \quad B_{18} = Erfc\left[\frac{2\sqrt{\frac{a}{1 + Sc}t + \sqrt{Sc}y}}{2\sqrt{t}}\right], \quad a = \frac{M}{1 + m^{2}}(1 - im) + \frac{1}{K}. \end{split}$$

REFERENCES

- 1. Attia Hazem Ali, Ahmed Mohamed Eissa Sayed ,"The Hall effect on unsteady MHD Couette flow with heat transfer of a Bingham fluid with suction and injection". Applied Mathematical Modelling 28, 2004, 1027-1045.
- 2. Attia Hazem Ali, "Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties". Applied Mathematical Modeling 26, 2002, 863–875.
- 3. A. Raptis and C. Perdikis, "Flow of a viscous fluid through a porous medium bounded by a vertical surface". Int. J. Eng. Sci. 21, 1983, 1327–1330.
- 4. A. Raptis, N. Kafousias, "Heat transfer in flow through a porous medium bounded by infinite vertical plate under the action of magnetic field". International Journal of Energy Research 6, 1982, 241-245.
- 5. I. Pop and T. Watanabe, "Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate". Acta Mech, Vol. 108, 1995, pp. 35–47.
- 6. K. Sudhakar, R. Srinivasa Raju and M. Rangamma, "Hall effect on unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction". International Journal of Physical and Mathematical Sciences, Vol. 4, No. 1, 2013.
- 7. N. Datta and R. N. Jana, "Oscillatory magneto hydrodynamic flow past a flat plate will Hall effects". J. Phys. Soc. Japan, 40, 1976, 14-69.
- 8. R. K. Deka, "Hall effects on MHD flow past an accelerated plate". Theoret. Appl. Mech, Vol.35, No.4, Belgrade, 2008 pp. 333-346.
- 9. R. Muthucumaraswamy and Janakiraman, "Mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction". International Journal of Applied Mathematics and Mechanics, vol. 4, no. 1, 2008, pp. 66–74.

- 10. U. S. Rajput and Surendra Kumar, "Radiation effects on MHD flow through porous media past an exponentially accelerated vertical plate with variable temperature". ISST Journal of Mathematics and Computing System, Vol. 1, No. 2, India, 2010, 1-7, ISSN 0976-9098.
- 11. U. S. Rajput and P. K. Sahu, "Transient Free Convection MHD Flow between two long vertical parallel plates with constant temperature and variable mass diffusion". Int. Journal of Math. analysis, vol. 5, no. 34, 2011, 1665-1671.
- 12. Ziya Uddin and Manoj Kumar, "MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation". Jordan Journal of Mechanical and Industrial Engineering, Vol. 5, no.2, April 2011, 133 138.

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